Meteo 300 Sp2001 Problem Set #2

1. a. $pV^{\gamma} = (10^5\,Pa)(1000\,m^3)^{1.4} \Rightarrow V = \left(\frac{(10^5)(1000)^{1.4}}{5\times10^4}\right)^{\frac{1}{1.4}} = 1640\,m^3$

$T/p^{(\gamma-1)/\gamma} = \frac{300}{(10^5)^{.286}} \Rightarrow T = 300\left(\frac{5\times10^4}{10^5}\right)^{.286} = 246\,K$

b. Isothermal $\Rightarrow$ T = constant pV = constant

$T = 300\,K$ @ 500 hPa

$pV = (10^5)(10^3\,m^3) \Rightarrow V = \frac{(10^5)(10^3)}{5\times10^4} = 2000\,m^3$

c. Energy must be added to keep the parcel isothermal on ascent. By the ideal gas law, since $T_{isothermal} > T_{adiabatic}$, $V_{isothermal} > V_{adiabatic}$ for the same pressure.

2. Isothermal ascent $\Rightarrow$ T = constant $\Rightarrow$ pV = constant

- Work $= \int_{p_1}^{p_2} p\,dV$. Can integrate over volume or pressure
- $pV = (10^5)(10^3) = 10^8 \Rightarrow V = 10^8/p \Rightarrow dV = -10^8(dp/p^2)$
- $\therefore$ Work $= -\int_{10^5}^{5\times10^4} p(10^8)\left(\frac{dp}{p^2}\right) = -\int 10^8 (dp/p) = 10^8 \ln(p)\Big|_{5\times10^4}^{10^5}$

Work $= 10^8 \ln(10^5/5\times10^4) = 7\times10^7$ Joules

or Work $= +\int_{10^3}^{2\times10^3} \frac{10^8}{V}\,dV = 10^8 \ln(V)\Big|_{10^3}^{2\times10^3} = 7\times10^7$ Joules

3. Adiabatic ascent. pV^{γ} = constant. Integrate by volume

$p = \text{const}/V^{\gamma} = \text{Const}\cdot V^{-\gamma}$

$W = \int_1^2 \text{const}\, V^{-\gamma}\,dV = \text{const}\int_1^2 \frac{1}{(1-\gamma)}\,d(V^{1-\gamma})$

$= \frac{\text{Const}}{1-\gamma} V^{1-\gamma}\Big|_{1000}^{1640} = \frac{10^5\cdot(1000)^{1.4}}{1-1.4}\left[(1640)^{-.4} - (1000)^{-.4}\right]$

Work $= 4.5\times10^7$ Joules

In this case, the work comes at the expense of the internal energy, since $dq = 0 \Rightarrow du = -dw$