Meteo 431 -- Atmospheric Thermodynamics
Second Mid-term Exam
5 April 2002

You have the entire class period to complete this closed-book examination. I have given you all the equations that you will need, except those that I asked you to learn on the study guide. If you have any questions about the meaning of the words, please ask the person proctoring the exam. Good luck.

- Poisson's relations: $TV^{\gamma-1}$ = const.; pV^{γ} = const.; $p^{(1-\gamma)/\gamma}T$ = const., for $\gamma = c_p/c_v$=1.4
- Specific heat capacity of dry air at 0°C: c_{vd} = 720 J kg^{-1} K^{-1}; c_{pd} = 1005 J kg^{-1} K^{-1}
- Specific heat capacity of dry air at 0°C: c_{vv} = 1390 J kg^{-1} K^{-1}; c_{pv} = 1850 J kg^{-1} K^{-1}
- enthalpy of vaporization, l_v=2.5x10^6 J kg^{-1}; enthalpy of sublimation, l_s=2.8x10^6 J kg^{-1}
- R_v = 461 J kg^{-1} K^{-1}; R_d = 287 J kg^{-1} K^{-1}
- Clausius-Clapeyron Equation (useful approximation):
  liquid: e_s = 6.11 hPa exp[6808(1/273 -1/T) - 5.09 ln(T/273)]
  ice: e_{si} = 6.11 hPa exp[6293(1/273 -1/T) – 0.555 ln(T/273)]

1. (15 points) Define and explain the following concepts:
   3 a. Gibbs free energy: equation and how we use it

   G = H - TS ; it is the energy available to do work
   We equate changes in g for different phases at equilibrium to get useful relations.

   3 b. two assumptions that make the Ideal Gas Law different from the van der Waals equation of state

   (1) molecules occupy no volume
   (2) molecules do not attract each other

   3 c. Clausius-Clapeyron Equation: differential equation (given on study sheet)

   $$\frac{d\ln e_s}{dT} = \frac{l_v}{R_v T^2}$$

   3 d. boiling: the concept

   Boiling occurs when $e_s(T_b) = p$

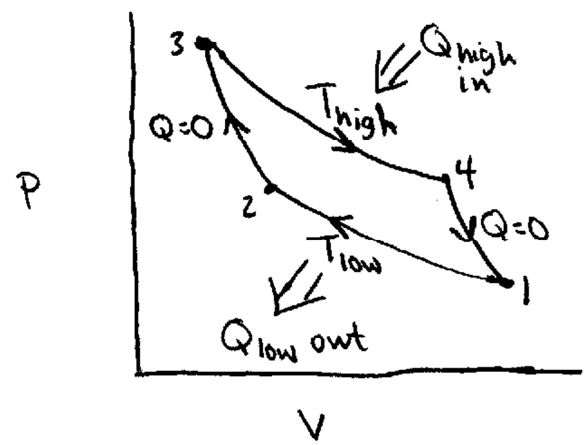
   3 e. specific humidity, q, and water vapor mixing ratio, w

   $q = \rho_v / \rho_{total}$ $\quad$ $w = \rho_v / \rho_d$

20

2. (~~15~~ points) The Carnot cycle for a system.

10 a. Draw the Carnot cycle on a diagram of pressure (y-axis) vs. volume (x-axis) and describe briefly the processes occurring during the four legs.



3 b. Show on which legs heat energy enters and leaves the volume and state why heat energy must enter and leave the system.

Heat energy must be added at T_{high} to keep T constant during an expansion
Heat energy must be removed at T_{low} to keep T constant during a compression.

3 c. What is the entropy change for an entire reversible cycle? In a few words, explain your answer.

$\Delta S = 0$. Entropy is a state variable, and so must have a given value at a given P & V at a point in the cycle. Therefore, $\Delta S = 0$ over the cycle.

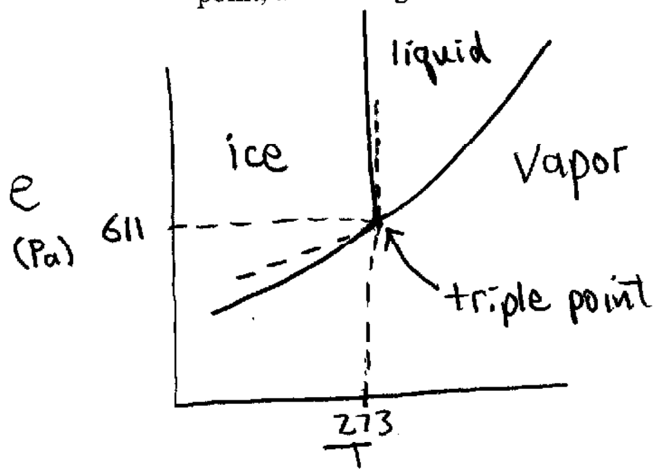
4 d. What are ways to increase the efficiency of processes and devices that operate using the Carnot cycle?

$$\eta = 1 - \frac{T_{low}}{T_{high}}$$

We can decrease T_{low}, or increase T_{high}
Decreasing T_{low} has the greater effect.

3. (20 points) Water vapor temperature, pressure diagram

15 a. Draw the diagram for the equilibrium vapor pressure (y-axis) vs. T (x-axis) for water near its triple point. Be sure to label all of the lines, the triple point, and the regions for vapor, liquid, and ice.



5 b. It is possible to have liquid water cooled to less than 273 K? Explain an effect of the difference between the saturation vapor pressure over ice and over liquid water.

Yes, we can have supercooled liquid. The saturation vapor pressure over liquid is greater than over ice, so if both are present, water will go from the liquid to the ice. (the Bergeron effect).

4. (25 points) Consider calm, humid air on a clear night. Suppose that the temperature at 8 PM is 295 K and that the dew point temperature is 285 K. Show how you get the answers to the questions below.

5 a. What is the relative humidity?

$$R.H = \frac{e}{e_s} = \frac{e_s(T_d)}{e_s(T)} = \frac{e_s(285)}{e_s(295)} = \frac{14.0}{26.5} = 52.9\%$$

5 b. What is the water vapor density, or total mass of water in 1 m^3?

Ideal Gas Law $e = \rho_v R_v T \Rightarrow \rho_v = e / R_v T$

$$\rho_v = 1400/(461 \cdot 295) = 0.010 \text{ kg m}^{-3}$$

5 c. Suppose the temperature falls at 2 K per hour. At what time does a fog form?

We want T to fall to T_d.

$$T_d = T_o - \frac{\Delta T}{\Delta t}\Delta t \Rightarrow \Delta t = \frac{T_o - T_d}{\Delta T/\Delta t}$$

$$\Delta t = \frac{295-285}{2} = 5 \text{ hrs.} \Rightarrow \text{Dew forms @ 1 AM.}$$

5 d. What is the radiative cooling rate per unit area (J m^{-2} s^{-1}) required to cool an airmass that is 20 m thick at a rate of 2 K per hour? (Assume dry air and constant pressure for this calculation. Remember, the calculation is per unit area.)

$$\frac{Q}{A} = C_p \frac{dT}{dt} = \rho_d \frac{V}{A} c_p \frac{dT}{dt} = \frac{P}{R_d T_i} \cdot z \cdot c_p \cdot \frac{dT}{dt}$$

$$= \frac{10^5}{287 \cdot 295} \cdot 20 \cdot 1005 \cdot \frac{2}{3600} = 13 \text{ W m}^{-2}$$

5 e. Assume that after the fog forms that the radiative cooling rate remains the same. However, we find that the air cools more slowly after the fog forms. Explain why.

Because as condensation occurs, energy is released as latent heat, thus warming the air even as it cools radiatively.

5. (~~25~~ 20 points) Consider a shallow pool that is 0.001 m deep and 6 m in diameter. The airmass directly above the pond is well mixed vertically, has a dry air mass of M_d = 10,000 kg, T = 300 K, p = 1000 hPa, and an initial water mixing ratio, w, of 0.005. (Remember that $w_s = \varepsilon\, e_s/(p-e_s)$, where $\varepsilon = 0.622$. Hint: Think about what w is.)

7 a. What is the initial relative humidity?

Find $e_s(300K)$, then w_s, then $RH = 100 \cdot \frac{w}{w_s}$

$e_s(300K) = 35.7\ hPa \Rightarrow w_s = (.622 \cdot 35.7)/(1000-35.7)$

$w_s = 0.0230 \Rightarrow RH = 100 \cdot \frac{.005}{.0230} = 21.7\%$

7 b. If all the energy for evaporating the water comes from the air in a constant pressure process, what is the temperature change of the air mass when all the water has evaporated? (Heating and cooling of the water vapor is not important)

$C_p \Delta T = M_\ell l_v \Rightarrow c_p M_d \Delta T = \rho_\ell A_\ell d_\ell l_v$

$$\Delta T = \frac{\rho_\ell A_\ell d_\ell l_v}{c_p M_d}$$

$$= \frac{1000 \cdot \pi (3)^2 \cdot .001 \cdot 2.5\times10^6}{1005 \cdot 10^4} = 7\ K$$

6 c. What is the final relative humidity when all the water vapor has evaporated?

We must add the addition water vapor and take into account the lower temperature

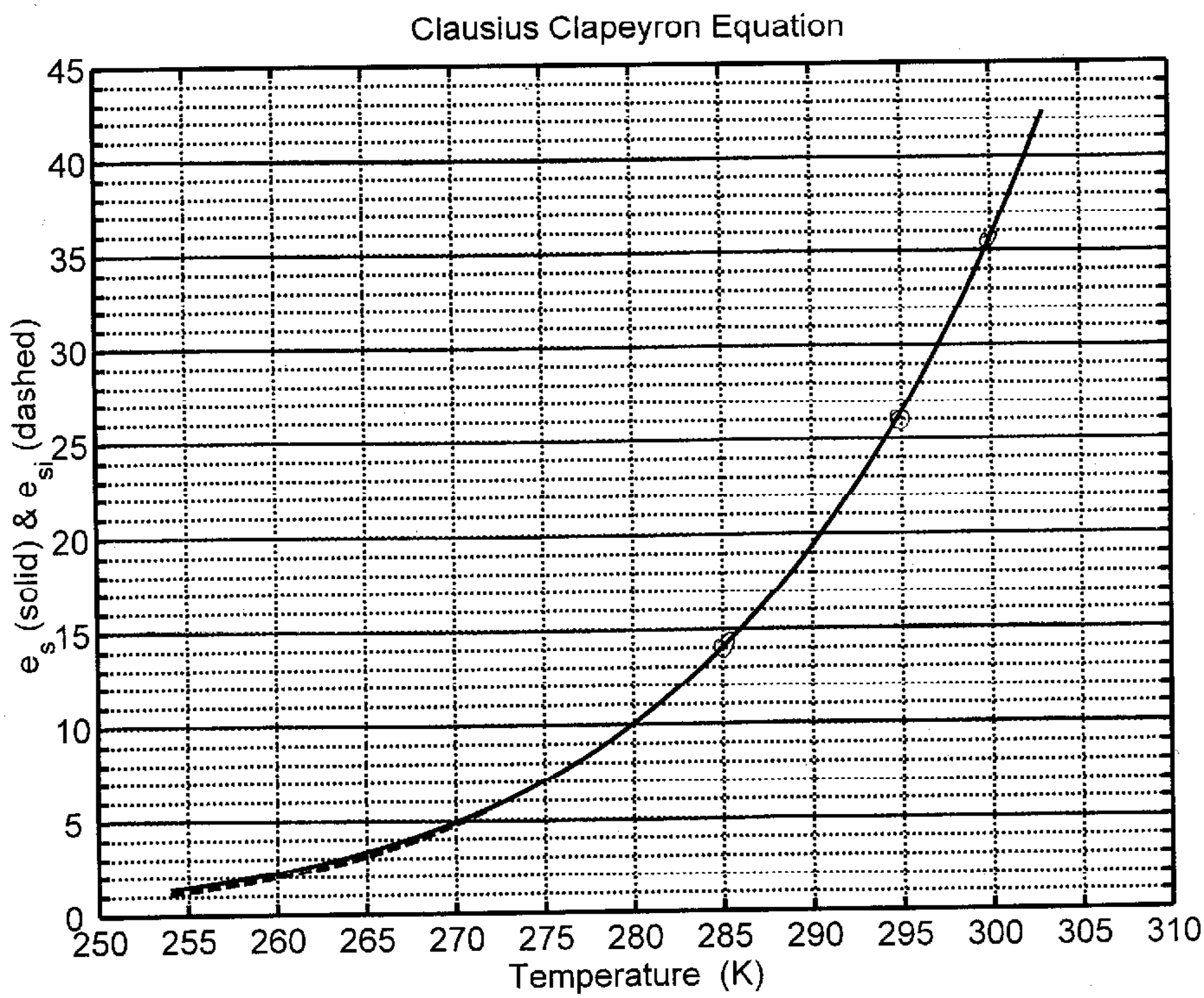
$$w_{added} = \frac{M_\ell}{M_d} = \frac{\rho_\ell V_\ell}{10^4} = \frac{1000 \cdot \pi (3)^2 (.001)}{10^4} = .0028$$

$$w_{total} = .005 + .0028 = 0.0078$$

Now T is 7 K lower, so new $e_s(293) = 23.4\ hPa$

$$w_s^{new} = \frac{.622 \cdot 23.4}{1000-23.4} = 0.0149$$

$$\therefore \text{new } R.H. = \frac{w_{total}}{w_s^{new}} = \frac{.0078}{.0149} = 52.3\%$$


Clausius Clapeyron Equation
e_s (solid) & e_{si} (dashed)
Temperature (K)
0
5
10
15
20
25
30
35
40
45
250
255
260
265
270
275
280
285
290
295
300
305
310