Meteo 431 Atmospheric Thermodynamics
Problem set #1

assigned: 11 January 2002
due: 18 January 2002

1. How much energy is required, in Joules, to carry a backpack weighting 25 lbs from the 1st to 6th floors of Walker Building?

- Energy required equals the change in potential energy.
- $\Delta E = mg(h_6 - h_1)$
- $h_2 - h_1 = (19.5)(8'' \text{ per tile}) \times 0.025\,m) = 3.9\,m$ } total $= 18.3\,m = h_6 - h_1$
- $\sum_{i=2} h_{i+1} - h_i = 18 \cdot 8'' \cdot 0.025 \cdot 4 = 14.4\,m$
  - $m = 25/2.2 = 11.4\,kg \Rightarrow \Delta E = 11.4 \cdot 9.8 \cdot (18.3\,m) = 2050\,J$

2. How many Joules are equivalent to:
   a.) 100 horsepower, for 1 hour; $100 \cdot 746 \cdot 3600 = 2.7 \times 10^8\,J$

   b.) 100 kilowatt hours; $100 \cdot 1000 \cdot 3600 = 3.6 \times 10^8\,J$

   c.) 10,000 BTU's $10^4 \cdot 1.06 \times 10^3 = 1.06 \times 10^7\,J$

   d.) 250 calories? $250 \cdot 4.18 = 1047\,J$

3. Plot the graph of the energy that must be dissipated to stop a car as a function of velocity from 0 to 80 MHP. Plot energy in Joules on the y-axis and velocity in MHP on the x-axis. Where does the energy go?

Assume mass of car: ~3000 lbs. = 1400 kg

The energy required to stop the car: $\frac{1}{2} m v^2$

$1\ mph = 1.6\ km \times 10^3\ \frac{m}{km} / 3600\ \frac{sec}{hr} = 0.45\ m\,s^{-1}$

see attached Excel plot

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4. Suppose an air parcel that is 100 meters in diameter and 100 meters high ascends from the surface to an altitude of 2 km before it stops. Make the unrealistic assumption that the parcel volume and temperature remains constant. How much energy was required to raise the parcel?

- Energy required $= mgh = (\rho\pi(\frac{d}{2})^2 \cdot \Delta z)gh$
- $\rho = 1.29 \cdot (\frac{p}{1013})(\frac{273}{T})$ kg m^{-3} assume $T = 290$ K & $p = 1000$ hPa
  $= 1.2$ kg m^{-3}
- $m = 1.2 \cdot \pi(\frac{100}{2})^2 \cdot 100 = 9.4 \times 10^5$ kg

Energy required: $9.4 \times 10^5 \cdot 9.8 \cdot 2 \times 10^3 = 2 \times 10^{10}$ J

5. Consider a balloon filled to a pressure p and volume V. What is the total work that the balloon must do to expel essentially all the air? (hint: What do you think the relationship is between p and V?) Explain the reasons for your assumptions.

$W_{tot} = -\int_{V_i}^{V_f} p\,dV$. assume Hook's Law applies in both directions on surface. ∴ Force ∝ Area

∴ $p \equiv$ constant. $W_{tot} = -p\,V_i - V_f \sim -pV_i$

6. What is the total work done by surroundings on an air parcel if the volume changes from $V_i = 1$ m^3 to $V_f = 0.2$ m^3 for an initial pressure, $p_i = 900$ hPa for the following processes:

a.) p = constant; $W_{tot} = -\int_{V_i}^{V_f} p\,dV = p(V_i - V_f) = 900 \times 10^2 \cdot 0.8 = 7.2 \times 10^4$ J

b.) pV = constant; $p_iV_i = \text{constant} = 900 \times 10^2\ \text{Pa} \cdot 1\ \text{m}^3 = 9 \times 10^4$ J

$W_{tot} = -9 \times 10^4 \int_{V_i}^{V_f} \frac{1}{V} dV = 9 \times 10^4 \ln(V_i/V_f) = 1.4 \times 10^5$ J

c.) $p(V)^{\gamma}$ = constant; ($\gamma = 1.4$) $p_i(V_i)^{\gamma} = \text{Constant} = 9 \times 10^4$ J

$W_{tot} = -9 \times 10^4 \int_{V_i}^{V_f} \frac{1}{V^{\gamma}} dV = -9 \times 10^4 \frac{1}{1-\gamma}(V_f^{1-\gamma} - V_i^{1-\gamma})$

$= \frac{9 \times 10^4}{1.4 - 1}(0.2^{-.4} - 1^{-.4}) = 2.0 \times 10^5$ J

You can easily calculate all the constants. Rank the processes in order greatest to least work required.

c.) > b.) > a.)

7. In class, I worked a problem concerning the increase in the temperature of the surface layer on a sunny day in Texas. Assume that the solar heating rate peaks at 800 W m^{-2}, that the sun is up for 12 hours, and that the surface layer is 0.2 km deep. However, in this case, assume that the solar energy can go into other processes: 30% is reflected back to space; 15% goes into the ground, and 25% goes into evaporating the moisture in the ground and plants. The rest goes into raising the air temperature in the surface layer. What is the resulting air temperature?

a total of 70% of the energy goes into processes other than heating the air.

$\int Q\,dt = 9 \times 10^6$ J $\cdot 0.3 = 2.7 \times 10^6$ J in the air

$dT = \frac{2.7 \times 10^6}{(1005)(240)} = 11°C$; start at $80°F = 27°C \rightarrow 38°C = 100°F$

This temperature change is much more likely.

8. I worked problem 5 from B&A, Chapter 1 in class. Do problem 6 using the results from problem 5.

- Kinetic energy $= \frac{1}{2} m V_\infty^2$. Power $= \frac{\text{energy}}{\text{time}} =$ watts
- We already have the mass flux. We get kinetic energy flux by multiplying by $\frac{1}{2} V_\infty^2$
- Power $= \frac{1}{2} \rho R V_\infty^2 \cdot$ area of storm

$= \frac{1}{2} (1000)(1''/hr \cdot 2.54 \times 10^{-2} \times 3600) \cdot (4\ m/s)^2 \cdot 500\ km^2 \cdot 10^6$

$\sim 4 \times 10^8$ J/s. Storms occur at random places & times.

9. B&A, problem 12

a) Power $= F_D \cdot v = \frac{1}{2} \rho v^3 A C_D$

$\cdot C_D = 0.25$; $\rho \sim 1.2$ kg m^{-3} ; $V = 60 \cdot 0.44 = 27$ m s^{-1} ; $A = 2 \cdot 1.5 \sim 3\ m^2$

$\therefore$ power $= \frac{1}{2}(1.2)(27)^3(3)(0.25) \sim 9000$ W $\Rightarrow \frac{9000}{746} = 12$ hp

b) To accelerate, $a = \frac{60-0}{10} = 2.7\ m s^{-1}$ $\quad m \sim 1400$ kg

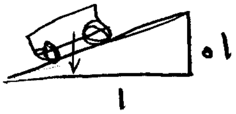
assume $a =$ constant

$E = F \cdot d \Rightarrow d = \frac{1}{2} a t^2 \Rightarrow E = \frac{1}{2} m a^2 t^2 \Rightarrow P = \frac{E}{T} = \frac{1}{2} m a^2 t$

$\therefore P = 5 \times 10^4$ W $= 68$ hp.

10. B&A, problem 13

$$V = 60 \text{ mph} = 27 \text{ ms}^{-1}$$



$$p = F \cdot V = m \cdot a \cdot V = 1400 \cdot (g) \cdot (0.1 \cdot 27)$$

$$P = 1400 \cdot (.98) \cdot 27 = 3.7 \times 10^4$$

$$= 50 \text{ hp}$$

Cars have power for both acceleration and climbing grades.

11. B&A, problem 15

Compare the power when we have wind to the power required when the wind is on average zero.

$$P_w = F_D v_{car} = \frac{1}{2}\rho (v_{car} - v_w)^2 A C_D v_{car} \quad \text{with wind}$$

$$P_a = \frac{1}{2}\rho (v_{car} + v_w)^2 A C_D v_{car} \quad \text{against wind}$$

$$\text{Is } P_w + P_a = 2P_0 = 2 \cdot \frac{1}{2}\rho (v_{car})^2 A C_D v_{car}$$

$$\therefore (v_{car} - v_w)^2 + (v_{car} + v_w)^2 \stackrel{?}{=} 2 v_{car} \quad (v_w = \text{wind velocity})$$

$$v_{car}^2 - 2 v_w v_{car} + v_w^2 + v_{car}^2 + 2 v_w v_{car} + v_w^2 = 2 v_{car}^2$$

$$2 v_w^2 > 0$$

$\therefore$ total power required is greater than with no wind, even though the average wind = 0.