Meteo 431 Atmospheric Thermodynamics Spring 2002

Problem set # 4

assigned: 1 February 2002

due: 8 February 2002

1. Consider the heating of an air parcel. What is the ratio of the temperature change of a dry air parcel in a constant volume process compared to a constant pressure process? You should be able to obtain both an analytical expression and a numerical value.

Same energy in both cases. $\Delta T_v C_v = \Delta T_p C_p$

$$\Rightarrow \frac{\Delta T_p}{\Delta T_v} = \frac{C_v}{C_p} = \frac{1}{\gamma} = \frac{1}{1.4} = 0.71$$

2. What is the air temperature change with time in a closed room (5 m x 4 m x 3.5 m) after the 10 computers in the room are turned on? Assume that each computer dumps 200 Watts of energy into the air? In reality, the temperature does not just keep increasing. Why?

$$Q\Delta t = C_v \Delta T = \rho V c_v \Delta T \Rightarrow \frac{\Delta T}{\Delta t} = \frac{Q}{\rho V c_v}$$

$$\frac{\Delta T}{\Delta t} = \frac{200 \cdot 10}{(1.2)(70)(718)} = 0.033°/\text{sec} = 12°/\text{hr}$$

Walls & ceiling & floor is also heated. Energy is transfered through these surfaces.

3. What is the density of an air parcel that contains 1% water vapor at a pressure of 1000 hPa and temperature of 293K?

$$\langle \rho \rangle = \frac{\sum N_i \rho_i}{\sum N_i} = \frac{N_d \rho_d}{N_d + N_v} + \frac{N_v}{N_d + N_v} = (1-f_v)\left(\frac{P}{R_d T}\right) + f_v\left(\frac{P}{R_v T}\right)$$

where $f_v = 0.01$

$$\langle \rho \rangle = (.99)\left(\frac{10^5}{287 \cdot 293}\right) + (0.01)\left(\frac{10^5}{461 \cdot 293}\right) = 1.19 \text{ kg m}^{-3}$$

4. What is the potential temperature of an air parcel that has a pressure of 200 hPa and a temperature of 200 K?

$$\Theta = T\left(\frac{P_0}{P}\right)^{R/C_p} \Rightarrow \Theta = 200\left(\frac{1000}{200}\right)^{.286} \Rightarrow \Theta = 317\,K$$

5. What is the temperature of this air parcel as it is brought adiabatically to 500 hPa?

$$T = \Theta\left(\frac{P}{P_0}\right)^{.286} \Rightarrow T = (317)\left(\frac{500}{1000}\right)^{.286} \Rightarrow T = 260\,K$$

6. B&A, Chapter 3, # 2.

Diesel engine. Assume adiabatic compression.

$$T V^{\gamma-1} = \text{constant} \Rightarrow T^{\frac{1}{\gamma-1}} V = \text{constant}$$

$$\text{compression ratio} \equiv \frac{V_1}{V_2} = \left(\frac{T_2}{T_1}\right)^{\frac{1}{\gamma-1}}$$

$$\frac{V_1}{V_2} = \left(\frac{800}{300}\right)^{\frac{1}{1.4-1}} \Rightarrow \frac{V_1}{V_2} = \sim 12$$

7. B&A, Chapter 3, # 7.

Commercial aircraft. Assume volume is constant

a.) $P_{cabin} > P_{air}$; $T_{cabin} > T_{air}$ $T_{air} \sim 300 - 8 \cdot (.3)(30\,kft) \sim 230\,K$

Suppose the air is brought in and compressed

$$P_{cabin}^{(1-\gamma)/\gamma} T_{cabin} = P_{air}^{(1-\gamma)/\gamma} T_{air}$$

$$\frac{T_{cabin}}{T_{air}} = \left(\frac{P_{cabin}}{P_{air}}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{.8}{.26}\right)^{.286} \Rightarrow T_{cabin} = 1.4\,T_{air}$$

If $T_{air} = 230\,K$, $T_{cabin} = 322\,K$!!!

The air must be cooled on the way into the cabin.