Meteo 431 Atmospheric Thermodynamics Spring 2002

Problem set # 5

assigned: 8 February 2002

due: 15 February 2002

1. Consider an air parcel with the following characteristics: T = 300 K; p =950 hPa; z = 0 km.

a. What is the parcel's potential temperature?

$$\Theta = T\left(\frac{P_0}{P}\right)^{.286} = 300\left(\frac{1000}{950}\right)^{.286} = 304\,K$$

b. What is its dry static energy?

$$h_s = h + gz = c_pT + gz = C_pT = 1005 \cdot 300 = 3.02 \times 10^5\ J/kg$$

Suppose this air parcel rises adiabatically to 4 km (e.g., the top of the PBL in Arizona in summer).

c. Find the air parcel's temperature at 4 km by using conservation of dry static energy.

$$\frac{dh_s}{dz} = 0 \Rightarrow h_s(z=0) = h_s(z=4) \Rightarrow 3.02 \times 10^5 = C_pT + 9.8(4 \times 10^3)$$

$$T = 261\,K$$

d. Find the air parcel's temperature at 4 km by using the dry adiabatic lapse rate.

$$T = T_0 - \Gamma_d z \Rightarrow T = 300 - 9.8(4) = 261\,K$$

2. Suppose an air parcel has a temperature of 310 K when the environment has a temperature of 290 K. The pressure is 960 hPa.

a. What are the densities of the parcel and its environment?

$$\rho_p = 1.29\ kg/m^3 \cdot \left(\frac{273}{310}\right)\left(\frac{960}{1013}\right) = 1.08\ kg\,m^{-3}$$

$$\rho_e = 1.29 \cdot \left(\frac{273}{290}\right)\left(\frac{960}{1013}\right) = 1.15\ kg\,m^{-3}$$

(Can also use $\rho = \frac{P}{RT}$)

b. What is the upward acceleration (buoyancy) of the air parcel?

$$B = \frac{T_p - T_e}{T_e} \cdot g = \frac{(310-290)}{290} 9.8 = 0.67 \ ms^{-2}$$

c. Assumes that the air parcel ascends adiabatically. Explain the behavior of the air parcel for a typical vertical temperature profile.

The parcel:
- accelerates upward until $\rho_p = \rho_e$
- overshoots because of momentum
- Comes back down because $B<0$
- oscillates about the stable point where $\rho_p = \rho_e$
- With friction, eventually settles to $\rho_p = \rho_e$
- mixes with environment.

3. Consider two air parcels with the following characteristics:

parcel 1: T = 310 K; p = 900 hPa; e = 40 hPa; V = 10^5 m^3

parcel 2: T = 290 K; p = 900 hPa; e = 10 hPa; V = 3×10^5 m^3

a. What are the enthalpies of the two air parcels?

$$H = C_p T = VT\{C_{pd}\rho_d + C_{pv}\rho_v\} = VT\left\{\frac{C_{pd}(p-e)}{R_d T} + \frac{C_{pv}e}{R_v T}\right\}$$

$$= V\left\{\frac{C_{pd}(p-e)}{R_d} + \frac{C_{pv}e}{R_v}\right\}$$

$$H_1 = 10^5\left\{\frac{1005 \cdot 86000}{287} + \frac{1850 \cdot 4000}{462}\right\} = 3.2\times10^{10} \ J$$

$$H_2 = 3\times10^5\left\{\frac{1005 \cdot 89000}{287} + \frac{1850 \cdot 1000}{462}\right\} = 9.5\times10^{10} \ J$$

b. What are the number of molecules of dry air and of water vapor in the two air parcels?

$$p_i V_i = n_i k T_i \Rightarrow n_i = p_i V_i / k T_i$$

	n_d	n_v	n_{total}
parcel 1	2.01×10^{30}	9.35×10^{28}	2.10×10^{30}
parcel 2	6.67×10^{30}	7.50×10^{28}	6.75×10^{30}

c. If the two parcels mix isobarically, what are the final T and e?

$$T = \frac{(H_1/T_1)}{(H_1/T_1 + H_2/T_2)} T_1 + \frac{H_2/T_2}{(H_1/T_1 + H_2/T_2)} T_2$$

$$= \frac{1.03 \times 10^8 \, T_1}{1.03 \times 10^8 + 3.27 \times 10^8} + \frac{3.27 \times 10^8 \, T_2}{4.30 \times 10^8} = 295 \text{ K}$$

$$e = \frac{n_{t1}}{n_{t1} + n_{t2}} e_1 + \frac{n_{t2}}{n_{t1} + n_{t2}} e_2 = 17 \text{ hPa}$$

4. B&A, Chapter 3, #34. The keys to the solution are:

① At 11,000 ft, $\rho_{system} = \rho_{environment}$; $p_{balloons} = p_{atmosphere}$
② ρ_{system} = (Mass of man, chair, beer + Mass of He) / Volume
③ Volume ~ Volume of balloons. (man can be no more than ½ m³)
④ At 11 kft, $T_{env} = T_{system}$

$$\therefore \quad \rho_{system} = \rho_{env.} = \frac{p_{env}}{R_d T_{env}}$$

Environment: $\rho_{env} = p_{11kft} / R_d T_{11kft}$ 11 kft = 3.4 km

assume $T_{11} \sim 275\text{ K}$ $p_{11} \sim 1000 \exp\left(-\frac{3.4}{8}\right) \sim 660 \text{ hPa}$

$$\rho_{env} = \frac{66000}{287 \cdot 275} = 0.84 \text{ kg m}^{3}$$

System: method #1 $\rho_{sys} = (70 + V_{He} @ 0\text{ km} \cdot \rho_{He}) / V_{11kft}$

assume at surface $V = 45 \cdot \pi/6 \cdot (d)^3 = 45 \cdot \pi/6 \cdot (4.5 \cdot .305)^3 = 61 \text{ m}^3$

$$\rho_{He} = 10^5 / R_{He} \cdot 298 = (10^5 \cdot 0.004)/(8.314 \cdot 298) = .16 \text{ kg m}^{-3}$$

$$\therefore \quad V_{11kft} = (70 + 61 \cdot 0.16)/\rho_{env} \Rightarrow 95 \text{ m}^3 \Rightarrow \boxed{d_{11kft} = \left(\frac{V_{11kft}}{45(\frac{\pi}{6})}\right)^{1/3} = 1.6 \text{ m}}$$

→ <u>method #2.</u> $\rho_{env} = \frac{(70 + \rho_{He} @ 11kft \cdot V_{11kft})}{V_{11kft}}$

$$\text{or} \quad V_{11} = \frac{70}{\rho_{env} - \rho_{He}} = \frac{T_{11}}{p_{11kft}} \left(\frac{70}{1/R_d - 1/R_e}\right) = \frac{R^* \cdot T_{11}}{p_{11kft}} \left(\frac{70}{M_d - M_{He}}\right)$$

$$= 97 \text{ m}^3 \Rightarrow d_{11kft} = \left(\frac{97}{45(\pi/6)}\right)^{1/3} \quad \boxed{d_{11kft} = 1.6 \text{ m}}$$

We do not even need to know the initial diameter & helium pressure. But He is <15% of the total mass.