Meteo 431 Atmospheric Thermodynamics Spring 2002

Problem set # 8

assigned: 8 April 2002

due: 15 April 2002

1. Assuming Raoult's law applies, how much salt would you have to add to 4 liters of water in order to bring the boiling temperature of the water up to 100^{o} C? Note that we must multiply the solute mixing ratio, c, by 2 because salt dissociates into chlorine and sodium ions.
2. B&A, Chapter 5, #34.
3. B&A, Chapter 5, #37. (hint: e falls proportional to the pressure, while e_s falls because T drops in adiabatic expansion. Write a computer program to find the altitude at which a fog does not form.)
4. B&A, Chapter 5, #50. (hint: Liquid water is used. To get sensitivity, consider equation 5.48.)
5. B&A, Chapter 5, #55.

P.S.8

1. $p = e = 830$ hPa
$e_s(100) = 1000$ hPa

$$e = e_s(1-2\chi) \Rightarrow \chi = \tfrac{1}{2}\left(1 - \frac{e}{e_s}\right)$$

but $\chi = \frac{N_{NaCl}}{N_{H_2O} + N_{NaCl}} \Rightarrow N_{NaCl} = \frac{\chi}{1-\chi} N_{H_2O} = \frac{\frac{1}{2}(1-e/e_s)}{1-\frac{1}{2}(1-e/e_s)} N_{H_2O}$

$$= \frac{(1-e/e_s)}{(1+e/e_s)} N_{H_2O}$$

But $M_{NaCl} = m_{NaCl} N_{NaCl} = m_{NaCl} N_{H_2O} \frac{(1-e/e_s)}{(1+e/e_s)}$
mass per mole

and
$N_{H_2O} = \frac{\rho_e V}{m_{H_2O}}$
so, putting it all together, $M_{NaCl} = m_{NaCl} \cdot \frac{\rho V}{m_{H_2O}} \cdot \frac{(1-e/e_s)}{(1+e/e_s)}$

$m_{NaCl} = 0.055$ kg mole^{-1}

$$M_{NaCl} = 0.055 \cdot \frac{1000 \cdot 4 \times 10^{-3}}{.018} \frac{(1-830/1000)}{(1+830/1000)} = 1.1 \text{ kg } (2.4 \text{ lb})$$

This is a lot of salt!

2. B & A #34

a. $\Delta E = M_e l_v = (0.5)(2.5 \times 10^6) = 1.25 \times 10^6$ J

b. $C_b \Delta T = \Delta E \Rightarrow \Delta T = \Delta E / C_B = \Delta E / C_L M$

$$\Delta T = 1.25 \times 10^6 / (4218 \cdot 80) = 3.7°C$$

Body T = 98.6°F (37°C).

Drops to $37 - 3.7 \sim 33°C$

Take a raincoat.

P.S. 8

(3.) B&A Ch5, 37

Rapid decompression (adiabatic) means $T_f = T_i\left(\frac{p_f}{p_i}\right)^{.286}$

$p_f = 300$ hPa
$p_i = 840$ hPa
$T_i = 293$

$e_i = RH\, e_s(T_i)$

$e_f = \frac{p_f}{p_i} RH\, e_s(T_i)$

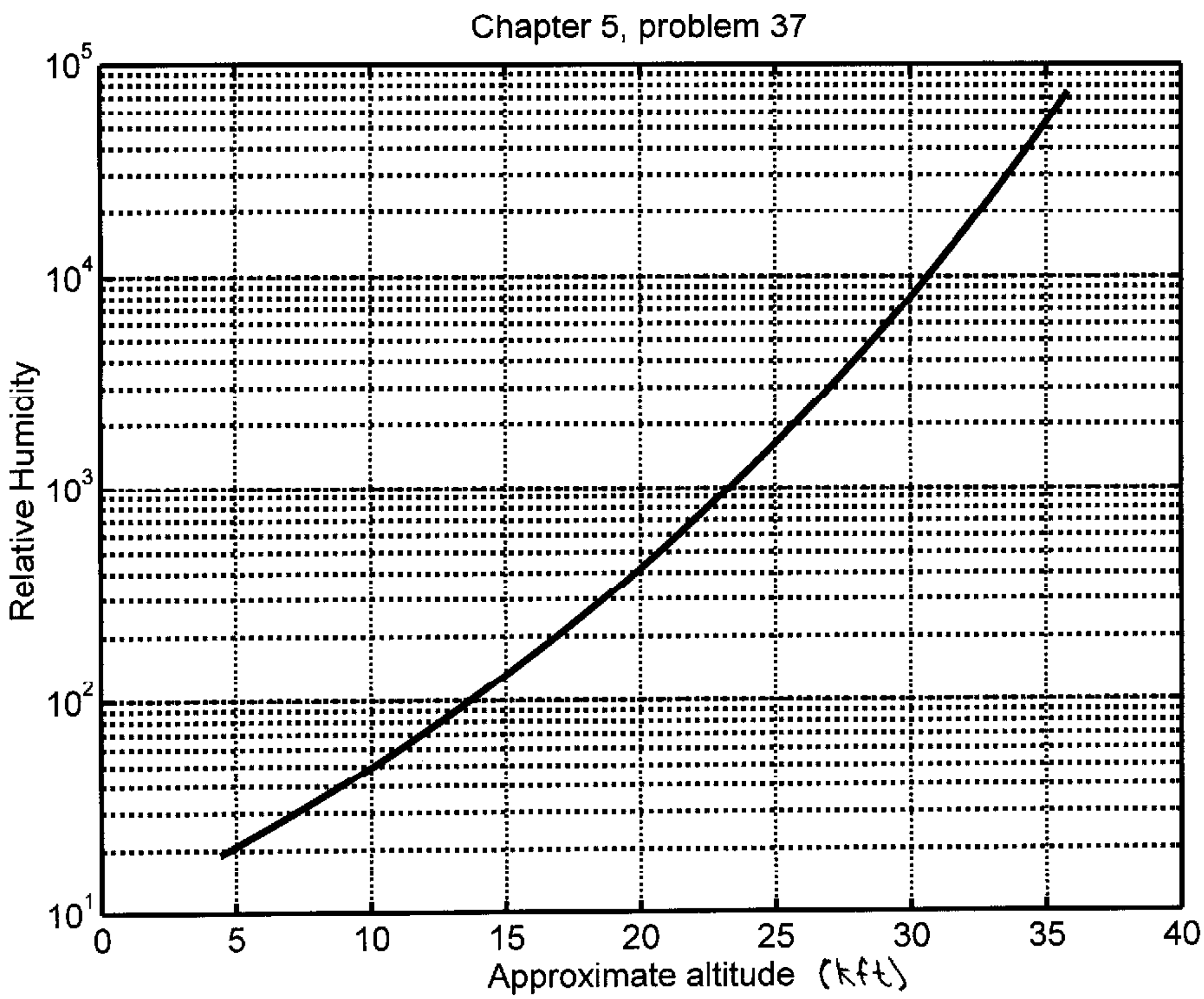
$$RH_f = \frac{e_f}{e_{sf}} \simeq \frac{(p_f/p_i)\, RH_i\, e_s(T_i)}{e_s(T_i(p_f/p_i)^{.286})}$$

$$RH_f = \frac{(300/840)(0.20)(23.6)}{e_s(293(300/840)^{.286})} = \frac{1.68}{e_s(218)} = \frac{1.68}{.0035} > 100\%$$

Yes, a fog will form.

The flight altitude below which a fog will <u>not</u> form can be found from the following program and diagram. When $RH < 10^2$ (100%), a fog will not form. This altitude is about 13,000 ft.

```
% c5p37.m solves the non-linear equation to determine the possibility of a ↙
fog forming
% during rapid decompression.
%
pi = 840;
Ti = 293;
ri = 0.20;
%
for i = 1:650
    p(i)=200 + i;
end
esi = 6.11*exp((6808*(1/273-1/Ti))-5.09*log(Ti/273));
Tf = Ti*(p/pi).^.286;
%
ef = (p/pi)*ri*esi;
esf = 6.11*exp((6808*(1/273-1./Tf))-5.09.*log(Tf/273));
%
rf = 100*ef./esf;
%
gamma = 9.8/1000;
To = 290;
Rd = 287;
g = 9.8;
po = 1000;
%
z = (To/gamma)*(1-(p/po).^(Rd*gamma/g))/305;
%
semilogy(z,rf)
grid
xlabel('Approximate altitude')
ylabel('Relative Humidity')
title('Chapter 5, problem 37')
```


Chapter 5, problem 37
Relative Humidity
Approximate altitude (kft)
10^5
10^4
10^3
10^2
10^1
0
5
10
15
20
25
30
35
40

P.S. 8

(4) B&A, Ch 5, #50.

Use boiling point of water to determine p, since $p = e_s(T_b)$. Consider the Clausius Clapeyron Equation.

$$de_s = \frac{l_v}{R_v}\frac{e_s}{T^2}\,dT = dp$$

but $e_s \sim 1000$ hPa and $T \sim T_b \sim 373$

$$\therefore\ de_s = \frac{2.5\times10^6}{461}\,\frac{(1000)}{(373)^2}\,(0.1) \sim 4\,\text{hPa}.$$

$dp \sim 4$ hPa. The precision of pressure measurement this way is about 4 hPa.

(5) B&A, Ch 5, #55.

$$\frac{Q}{A} = \frac{l_v \rho_v\, dz\, A}{A\, dt} = l_v \rho_v \frac{dz}{dt}$$

$$Q/A = (1365/3) \Rightarrow \frac{dz}{dt} = \frac{(1365/3)}{2.5\times10^6 \cdot 1000} = 2\times10^{-7}\,\frac{m}{s}$$

$$\frac{dz}{dt} = (2\times10^{-7})\,\frac{m}{s}\cdot\frac{1000\,mm}{m}\cdot\frac{3600\,s}{hr} = 0.66\,\frac{mm}{hr}$$

Net <u>daytime</u> evaporation is $\sim 3000\ \frac{mm}{yr} \sim .7\ \frac{mm}{hr}$

(net evaporation is $\sim 1000\ \frac{mm}{yr}$)

The two answers are close.