Grain Growth in Porous Oxides: Diffuse-Interface Modeling and Experiments Karim Ahmed¹, Anter El-Azab¹, Janne Pakarinen², Lingfeng He², Todd Allen²

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- All physical properties of ceramics depend strongly on the grain size. Furthermore, the grain size affects their behavior under extreme conditions such as irradiation, high temperatures, and stress.
- Modeling the process of grain growth in porous ceramics is complicated by the interaction between the pore and the grain boundary.
- The classical models by Nichols, Brook, and Carpay give only a qualitative description of the problem since they assume homogeneous microstructures and rigid body motion of the pore

The Sharp-Interface Model

• In the sharp-interface description by Evans, Riedel and Svoboda, the grain boundary migrates under the influence of its mean curvature, while the pore moves via surface diffusion.

$$\gamma_b = -\gamma_b M_b \kappa_b, \qquad ($$

(motion by mean curvature flow)

$$v_p = \frac{\gamma_s D_s \,\delta \Omega}{K_B T} \nabla_s^2 \kappa_s.$$

(motion by surface diffusion)

• At triple-junctions, the balance of forces, fluxes, and continuity of chemical potential must hold. Solving this problem for general geometries is a cumbersome task, and only 2D simulations of this model have been performed.

• We carried out a formal asymptotic analysis of the phase field model in the limit where the interface thickness vanishes. The equations of motions of the grain boundary and the pore (free) surface in the diffuse-interface description are given by,

$$v_b = -L\varepsilon^2 \kappa_b, \quad v_p = (1+q)M_s\varepsilon^2 \nabla_s^2 \kappa_b$$





A typical microstructure of a porous ceramic.

Sharp- and Diffuse-Interface Models of Grain Growth in Porous Ceramics



Asymptotic Matching and Model Implementation



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ke into account the details of the pore and and Svoboda proposed the sharp-interface description of the problem. We have recently introduced the phase field (diffuse-interface) description of the problem. the phase field model alleviates all the unrealistic assumptions of the classical models and obviates all the numerical difficulties of the sharp-interface model. • We present here for the first time 3D simulations of the grain growth process in porous ceramics.

The Diffuse-Interface Model

- A schematic illustration of the order parameters used in the diffuse-interface model. K. Ahmed et al., JNM, 2014
- The conserved density field (η) and non-conserved orientation field (ρ) are used to fully represent the microstructure of a porous polycrystalline ceramic. The free energy of the non-uniform medium is given by, $F = \int_{\Omega} [f(\rho, \eta_1, \dots, \eta_{\alpha}, \dots, \eta_p) + \frac{q\varepsilon^2}{2} |\nabla \rho|]$

$$\frac{\partial \rho}{\partial t} = \nabla \cdot \left[M_s \rho^2 (1 - \rho)^2 (\mathbf{I} - \hat{\mathbf{n}}_s \otimes \mathbf{I}) \right]$$
$$\frac{\partial \eta_\alpha}{\partial t} = -L \frac{\delta F}{\delta \eta_\alpha}; \qquad \alpha = 1, 2....p.$$

• Therefore, the phase field model parameters are directly related to the regular thermodynamic and kinetic (sharpinterface) parameters as follows,

$$L\varepsilon^2 = \gamma_b M_b, \quad (1+q)M_s\varepsilon^2 = \frac{\gamma_s D_s \delta \Omega}{K T}$$

• The kinetic equations are solved using a standard explicit finite difference scheme . In order to solve the problem efficiently in **3D**, we have utilized parallel computing.





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$$p|^2 + \frac{\varepsilon^2}{2} \sum_{\alpha=1}^p |\nabla \eta_{\alpha}|^2] d^3r.$$

quations of the order parameters are derived as,

$$[\mathbf{\hat{s}}_{\mathbf{s}})]\nabla \frac{\delta F}{\delta \rho},$$