

Atomic scale modeling of austenite-martensite transformation

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Martensite transformation (MT)

Introduction

A martensitic transformation:

- a first order phase transition that proceeds under conditions where the initial phase maintains metastability.
- The tendency toward stress relaxation leads to the formation of new phase plates with regular orientations and internal substructure (polysynthetic twins or slip packets).
- The plates form regular groups in such a way that the free energy should be at a minimum.
- The austenite/martensite interfaces are invariant under the transformation ->habit plane



Multidomain structure -> two twin-related orientation variants

Self-accommodation by variants

A typical feature of martensitic transformations is that each colony of martensite laths/plates consists of a stack in which different variants alternate. This allows large shears to be accommodated with minimal macroscopic shear.

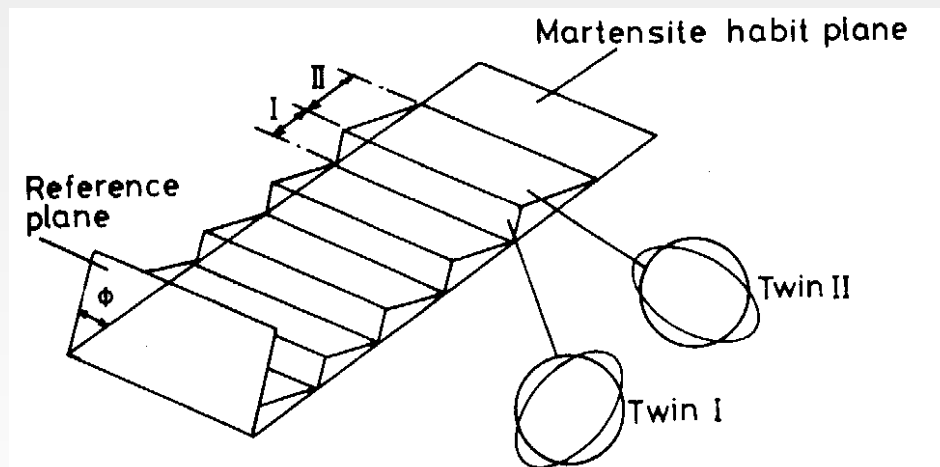


Fig. 6.10 Twins in martensite may be self-accommodating and reduce energy by having alternate regions of the austenite undergo the Bain strain along different axes.

Martensite transformation

Introduction

Nucleation mechanism of martensite

G.B. Olson and A.L. Roitburd, Martensite (1992).

Y. Wang, A.G. Khachaturyan, Acta.Mat. (1997)

W. Zang, Y.M. Jin, A.G. Khachaturyan, Acta Mat. (2007)

- The strain energy contribution to the nucleation barrier of a single variant martensitic particle is too high to be overcome by thermal fluctuations -> pre-existing **martensitic embryos** around a dislocation defect is a **complex nanoscale assemblages of martensitic microdomains** rather than a homogeneous single-domain particle.

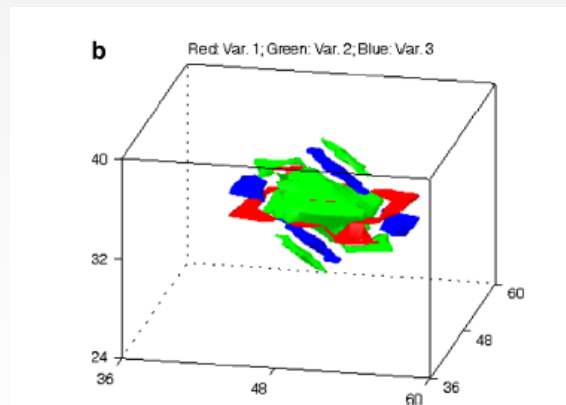


Fig. 3. Metastable martensitic embryos induced by a single dislocation loop ($m = 1$) at the time $\tau = 2000$: (a) $\Delta f_L^* = 0.056$ ($\Delta T \approx 200$ K); (b) $\Delta f_L^* = 0.058$ ($\Delta T \approx 207$ K). The same visualization method is used for all plots: parent phase is transparent, $\Delta f_L^* = \eta_{|1|} \geq 0.7$ in red, $|\eta_2| \geq 0.7$ in green, and $|\eta_3| \geq 0.7$ in blue.

W. Zang, Y.M. Jin,
A.G. Khachaturyan, (2007)

Objectives

Understand the first stages of MT at atomic scale:

- The microstructural sequence of the development of a martensitic nucleus from the parent phase

Approach:

3D atomic density function modeling

Atomic density function model

Static concentration waves method (A. G. Khachaturyan, 1971)

Occupation probability: $p = \bar{c}$
 $P(r) = \bar{c} + \Delta c(r)$

$$P(r) = \bar{c} + \frac{1}{2} \sum_j [Q(\vec{k}_j) e^{i\vec{k}_j \cdot \vec{r}} + Q^*(\vec{k}_j) e^{-i\vec{k}_j \cdot \vec{r}}]$$

Amplitudes of static concentration waves \rightarrow $Q(k_j) = \eta(k_j) \gamma(k_j)$



Internal energy \rightarrow $U = \frac{N}{2} V(0) c^2 + \frac{N}{2} \sum_j V(k_j) \eta_j^2$

$$V(k) = \sum_r V(r) e^{-ikr}$$

$$F_{chem} = \frac{1}{2} \sum_{r,r'} V(r-r') p(r) p(r') + k_B T \sum_{\vec{r}} \{ p(\vec{r}) \text{Ln}(p(\vec{r})) + [1 - p(\vec{r})] \text{Ln}[1 - (p(\vec{r}))] \}$$

Atomic density function model

$$F = F_{chem} + E_{elast}$$

$$E_{elast} = \frac{1}{2} \sum_{pq} \int \frac{d^3k}{(2\pi)^3} B_{pq}(\mathbf{n}) \theta_p(\mathbf{k}) \theta_q^*(\mathbf{k})$$

Microscopic diffusion equation

$$\frac{dP(r,t)}{dt} = \frac{1}{k_B T} \sum_{\alpha,\beta} \sum_{r'} L_{\alpha\beta}(r-r') c_\alpha c_\beta \frac{\partial F}{\partial P(r',t)}$$

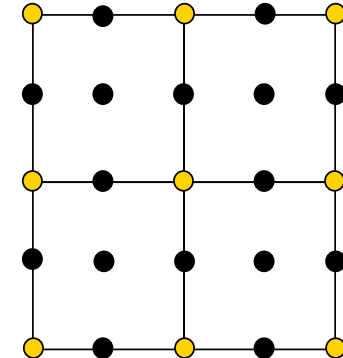
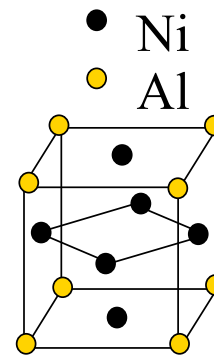
Conserved dynamic with the condition:

$$\sum_r L(r-r') = 0$$

Phases presentation in ADF model on constrained lattice

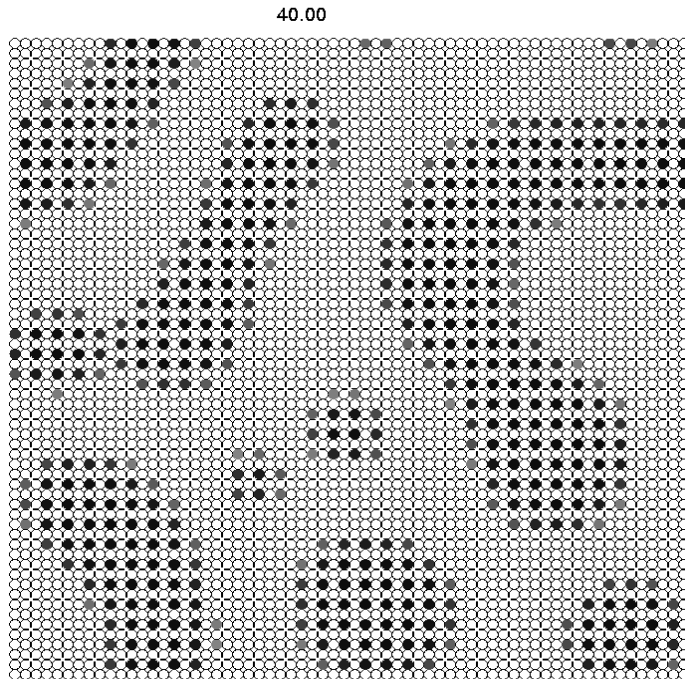
2D Simulation :

fcc lattice => square lattice



L1₂

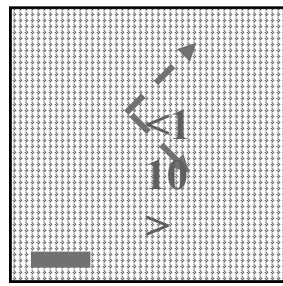
2D Projection of L1₂



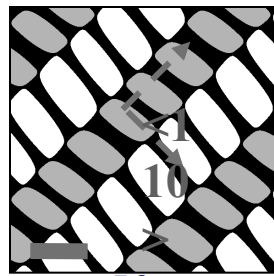
Site probability presentation



Order parameter presentation

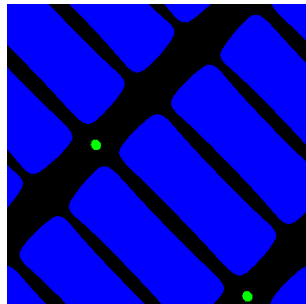
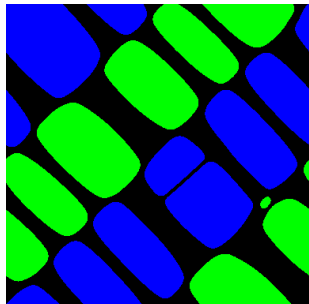
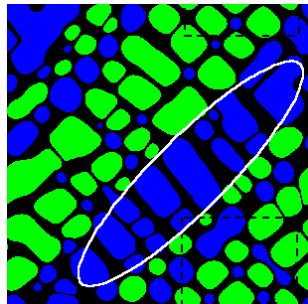


50 nm



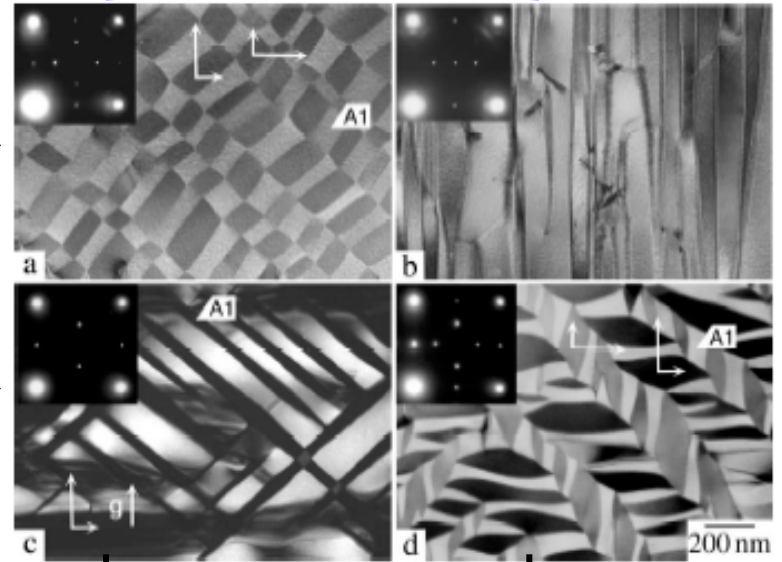
50 nm

$$d = \epsilon_{11} / \epsilon_{33} = -0.35$$



$$d = -0.5$$

Ni-20V-10Co

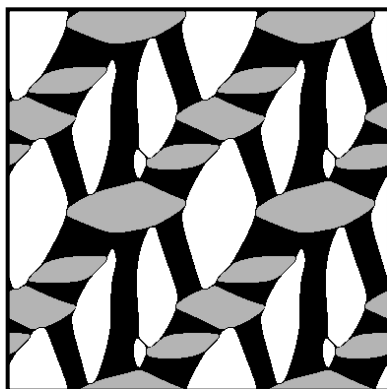


Ni-19V-4Fe

Ni-15V-5Nb

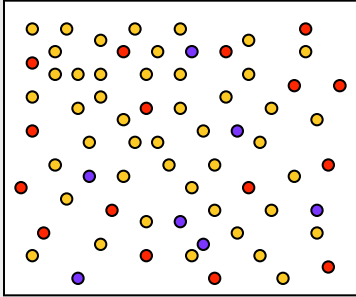
TEM images of ternaries alloys aging at 1073°K
864 ks, A1/DO₂₂ microstructure
(A. Suzuki, H. Kojima, T. Matsuo
and M. Takeyama, *Intermetallics* 12 (2004) 969-975)

d=0



H. Zapolsky, S. Ferry, X. Sauvage, D. Blavette, L.Q. Chen, *Phil. Mag.* (2011)
J. Boisse, H. Zapolsky, A.G. Khachatryan *Acta Mat.* (2011)

Continuum version of the ADF model (Y. Jin, A.G. Khachatryan APL(2006))



$$\frac{a}{R_{W\alpha\beta}} \ll 1$$

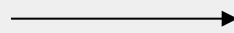
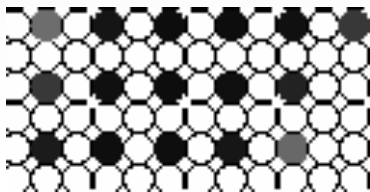
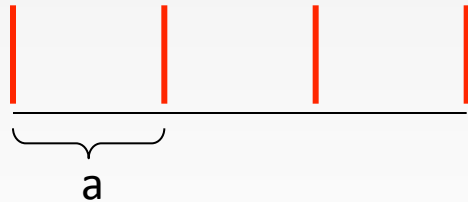
A small parameter determining the transition discrete continuum version of ADF model is:

a - Ising lattice parameter

$R_{W\alpha\beta}$ - characteristic distance of interatomic interaction

ADF on constrained lattice

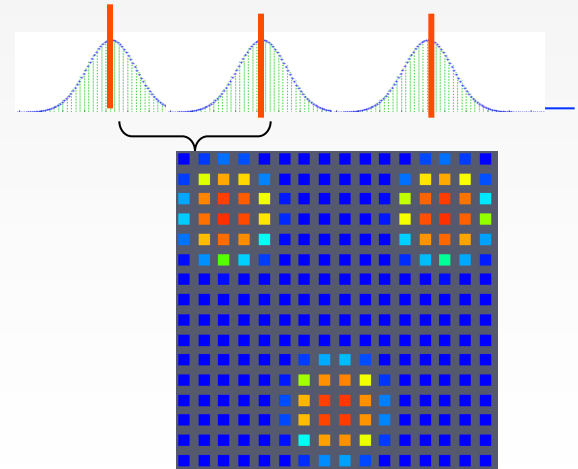
$P(r)$ - probability to find atom at position r
Microscopic diffusion equation



ADF on unconstrained lattice

$\rho(r)$ - atomic density

$$\{\rho(\mathbf{r})_{\alpha}\} = (\rho_1, \rho_2, \dots, \rho_{\alpha}, \dots, \rho_n)$$



- There is no Ising lattice constraint: atoms are free to continuously move to relax the free energy.
- The n -component system is described by the n **atomic density functions**:

$$\{\rho(\mathbf{r})_\alpha\} = (\rho_1, \rho_2, \dots, \rho_\alpha, \dots, \rho_n)$$

- **The ADF kinetic** equations are essentially the same as a microscopic diffusion equation (A.G. Khachaturyan 1976) but the integration over continuum space is substituted for summation over lattice sites:

$$\frac{\partial \rho_\alpha(\mathbf{r}, t)}{\partial t} = \sum_{\beta=1}^{\beta=n} \int_V L_{\alpha\beta}(\mathbf{r}, \mathbf{r}') \frac{\delta F}{\delta \rho_\beta(\mathbf{r}', t)} d^3 r' \quad \alpha = 1, 2, \dots, n$$

$F(\{\rho_\alpha(\mathbf{r})\})$ is a non-local free energy functional of n atomic density functions, $L_{\alpha\beta}(\mathbf{r}, \mathbf{r}')$ is the mobility matrix.

The conservation of the number of atoms:

$$\int_V L_{\alpha\beta}(\mathbf{r}, \mathbf{r}') d^3 r' = 0.$$

Atomic density function approach

Free energy :

$$F[\{\rho_\alpha\}] = \frac{1}{2} \iint_V \sum_{\alpha\beta} \underbrace{W_{\alpha\beta}(\mathbf{r}-\mathbf{r}') \rho_\alpha(\mathbf{r}) \rho_\beta(\mathbf{r}')}_{\text{Non-local term}} d^3r d^3r' + \int_V \underbrace{f(\{\rho_\alpha(\mathbf{r})\})}_{\text{Local term}} d^3r$$

The local free energy density $f(\rho_\alpha(r))$ can be approximated by the Landau polynomial expansion with respect to density $\rho_\alpha(r)$ or by entropy term.

$$W_{\alpha\beta}(\mathbf{r}-\mathbf{r}')$$

the effective interaction potentials



Embedded potential, effective pair potential, ...

Limit transition to the Landau theory

$$F = \int \frac{1}{2} \sum_{\alpha\beta} V_{\alpha\beta}(k) \Phi_{\alpha}(k) \Phi_{\beta}^*(k) \frac{d^3k}{(2\pi)^3} + \int_V f(\{\rho_{\alpha}(r)\}) d^3r$$

Where $V_{\alpha\beta}(\mathbf{k})$ is the Fourier transforms of the effective potentials $W_{\alpha\beta}(\mathbf{r})$:

$$V(k) = \int_V W(r) \exp(-ikr) d^3r$$

Using Taylor expansion of $V_{\alpha\beta}(\mathbf{k})$ in k

$$V_{\alpha\beta}(k) = A_0^{\alpha\beta} + \frac{1}{2!} A_2^{\alpha\beta} k^2 + \frac{1}{4!} A_4^{\alpha\beta} k^4 + \dots + \frac{1}{n!} A_n^{\alpha\beta}$$

Eq.(1) is a generalized **Landau gradient expression**:

$$F = \int_V \sum_{\alpha\beta} \left(\frac{1}{2!} A_0^{\alpha\beta} \rho_{\alpha}(\mathbf{r}) \rho_{\beta}(\mathbf{r}) + \frac{1}{2!} A_2^{\alpha\beta} \nabla \rho_{\alpha}(\mathbf{r}) \nabla \rho_{\beta}(\mathbf{r}) + \frac{1}{4!} A_4^{\alpha\beta} \nabla^2 \rho_{\alpha}(\mathbf{r}) \nabla^2 \rho_{\beta}(\mathbf{r}) + \dots \right) d^3r$$

$$+ \int_V f(\{\rho(\mathbf{r})_{\alpha}\}) d^3r$$

The **Phase Field Crystal model** has used two first terms of the gradient expansion of the Landau theory.

To investigate the stability of the homogeneous state with respect to the density modulations, we can expand the free energy (1) into the Taylor series with respect to the spatial variations of the atomic densities

$$\Delta\rho_\alpha(\mathbf{r}) = \rho_\alpha(\mathbf{r}) - \bar{\rho}_\alpha$$

Free energy change with respect to homogeneous state is :

$$\Delta F = \frac{1}{2} \iint_V \sum_{\alpha\beta} \left[W_{\alpha\beta}(\mathbf{r} - \mathbf{r}') + \frac{\partial^2 f(\{\rho_\alpha\})}{\partial\rho_\alpha\partial\rho_\beta} \right] \Delta\rho_\alpha(\mathbf{r}) \Delta\rho_\beta(\mathbf{r}') d^3r d^3r'$$

$$D_{\alpha\beta}(k, T, \{\rho_\alpha\}) \quad \text{Response function}$$

Response function can be estimated from the elastic diffuse scattering

$$I(\mathbf{k}) \sim |\Phi(\mathbf{k})|^2$$

where $\Phi(\mathbf{k})$ Fourier transform of the $\rho(\mathbf{r})$

$$\langle \Phi_\alpha(k) \Phi_\beta^*(k) \rangle = k_B T D_{\alpha\beta}^{-1}(\mathbf{k})$$

Atomic density function modeling of atomic structure of grain boundaries

$$f(\{\rho(\mathbf{r})\}) = \int \frac{d^3k}{(2\pi)^3} V(k) \rho_{\mathbf{k}} \rho_{-\mathbf{k}} + f_{\text{loc.}},$$

$$f_{\text{loc.}} = k_B T \int \frac{d^3r}{V} [\rho(\mathbf{r}) \ln \rho(\mathbf{r}) + (1 - \rho(\mathbf{r})) \ln(1 - \rho(\mathbf{r}))].$$

$$V(k) = V_0 \left(1 - k^4 / [(k^2 - k_1^2)^2 + k_2^4] \right)$$

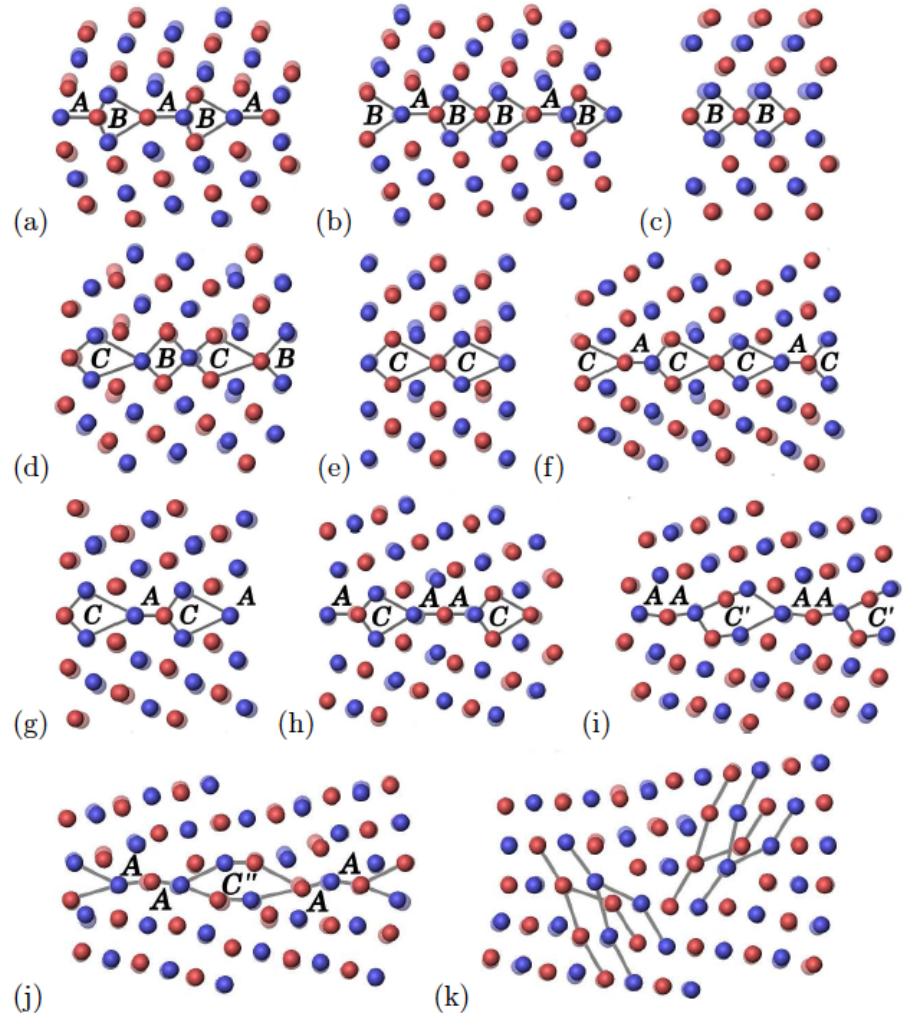


Figure 2: The structural units in $\langle 110 \rangle$ GBs modeled by the ADF: a) $38.94^\circ, \Sigma 9(114)$; b) $50.48^\circ, \Sigma 11(113)$; c) $70.53^\circ, \Sigma 3(112)$; d) $93.37^\circ, \Sigma 17(334)$; e) $109.53^\circ, \Sigma 3(111)$; f) $121.01^\circ, \Sigma 33(554)$; g) $129.52^\circ, \Sigma 11(332)$; h) $141.06^\circ, \Sigma 9(221)$; i) $148.41^\circ, \Sigma 27(552)$; j) $153.47^\circ, \Sigma 19(331)$; k) $163.9^\circ, \Sigma 51(551)$. The equilibrium atomic positions after a relaxation by MD simulation are indicated in the background with faded colors; different colors refer to two adjacent atomic planes. The last image represents dislocations.

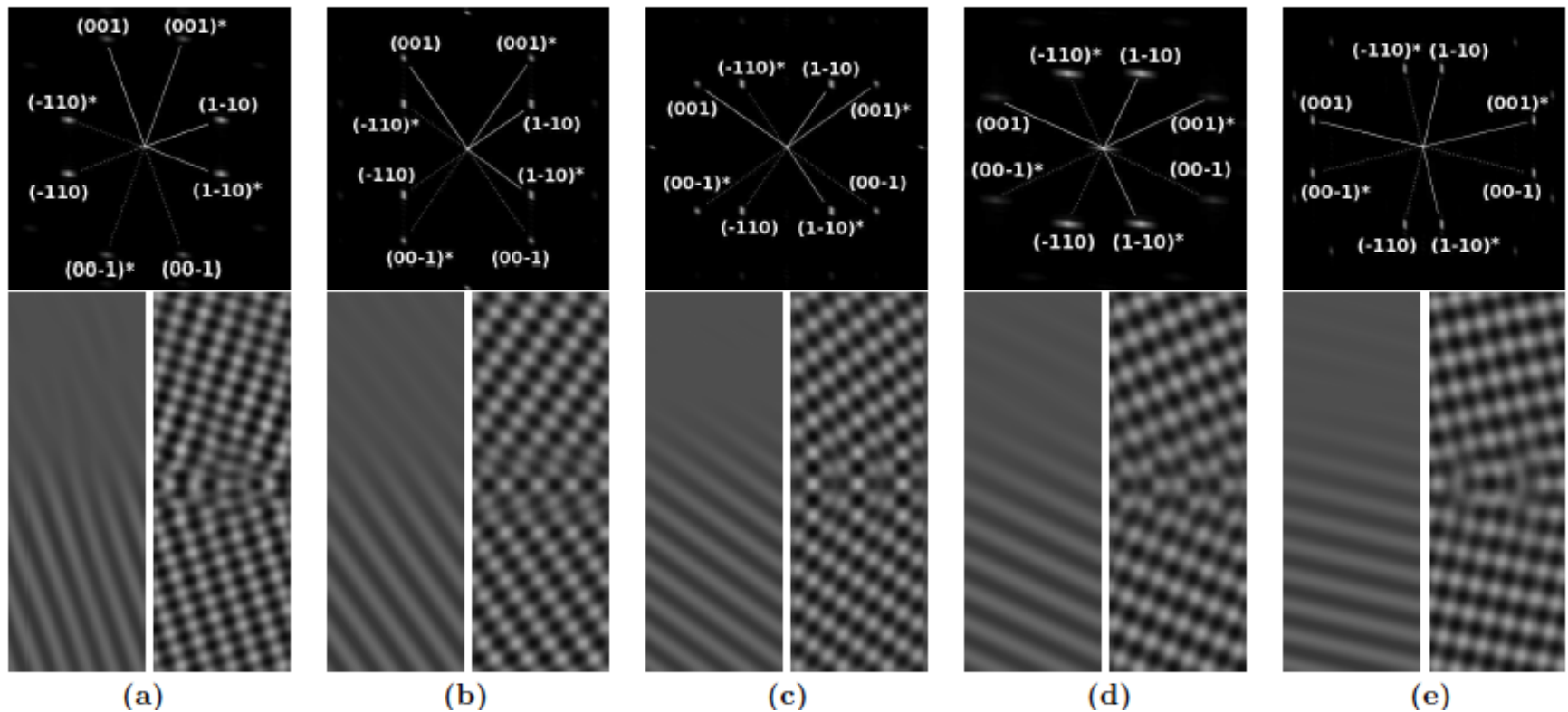
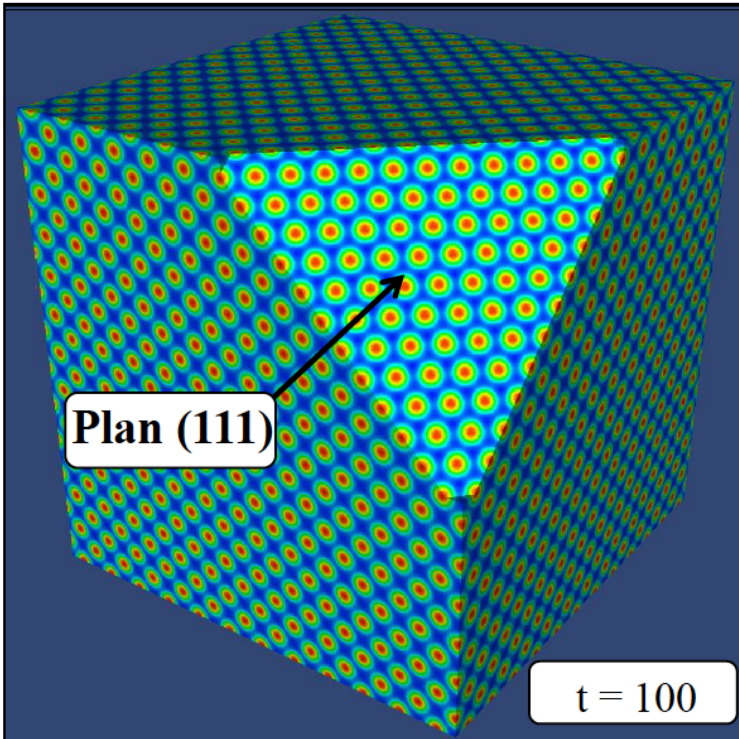


FIG. 8. Diffraction patterns (upper row), and the real space reconstruction based on the brightest spot ($1\bar{1}0$) and its conjugate ($\bar{1}10$) (on the left of each column) or on all the spots indicated in the diffraction patterns (on the right of each column) for a $[110]$ tilt GB of (a) 38° ($\Sigma 9(114)$); (b) 70° ($\Sigma 3(112)$); (c) 109.53° ($\Sigma 3(111)$); (d) 129° ($\Sigma 11(332)$); (e) 153° , ($\Sigma 19(221)$).

FCC structure



$$V_{\text{CFC}}(\mathbf{k}) = \min \begin{cases} V_0 \left[1 - \frac{(k/\epsilon_1)^4}{((k/\epsilon_1)^2 - \kappa_1^2) + \kappa_2^4} \right] \\ V_0 \left[1 - \frac{(k/\epsilon_2)^4}{((k/\epsilon_2)^2 - \kappa_1^2) + \kappa_2^4} \right] \end{cases}$$

$$\epsilon_1 = 1 \quad \epsilon_2 = 2/\sqrt{3}$$

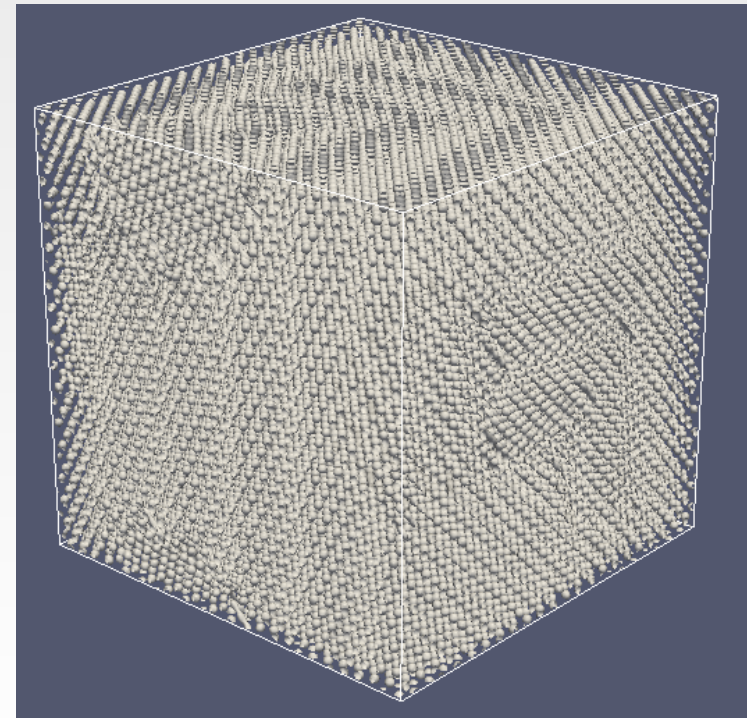
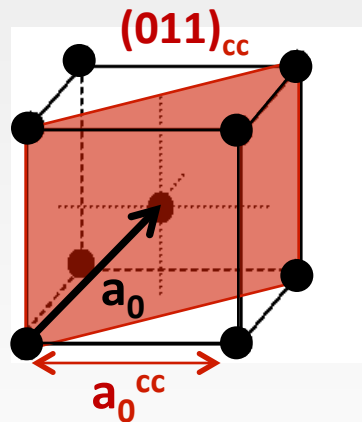
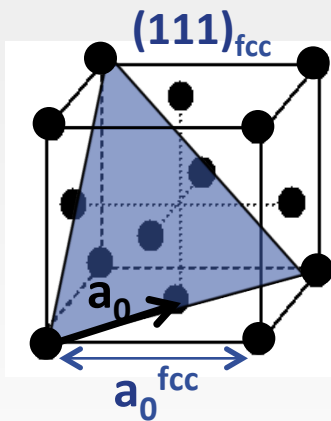
$$\kappa_1 = 0.97 \quad \kappa_2 = 0.485 \quad V_0 = 1/17$$

FCC to BCC transition

ADF modeling

Initial configuration → bcc nucleus ($r=20\Delta$) with KS orientation relation with the fcc parent phase

$$\frac{a_0^{FCC}}{a_0^{BCC}} = \sqrt{\frac{3}{2}} \approx 1.225$$



$$\left. \begin{array}{l} \text{Fe}_\alpha : a_0 = 2,886 \\ \text{Fe}_\gamma : a_0 = 3,6 \end{array} \right\} \approx 1,247$$

$$\left. \begin{array}{l} \text{Fe-Ni-C} : - \text{BCC} : a_0 = 2,845 \\ - \text{FCC} : a_0 = 3,592 \end{array} \right\} \approx 1,263$$

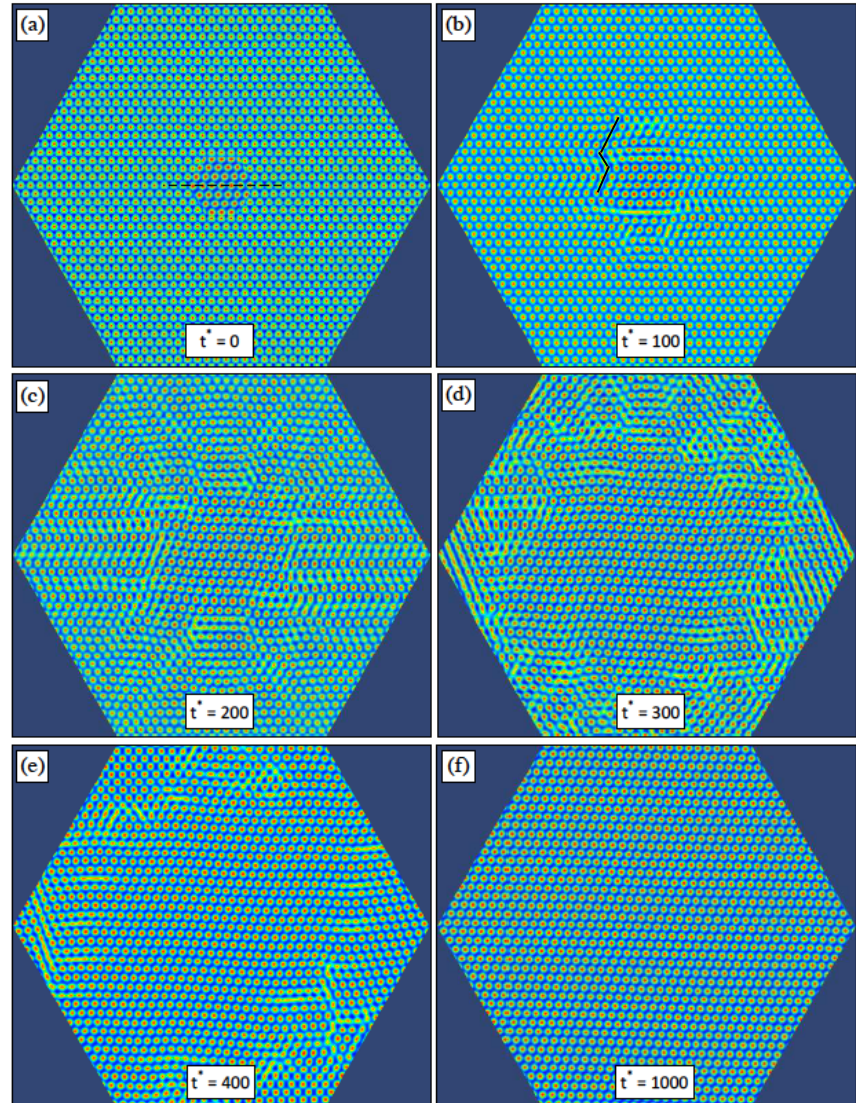
$t^*=9000$

Simulation box: 256^3

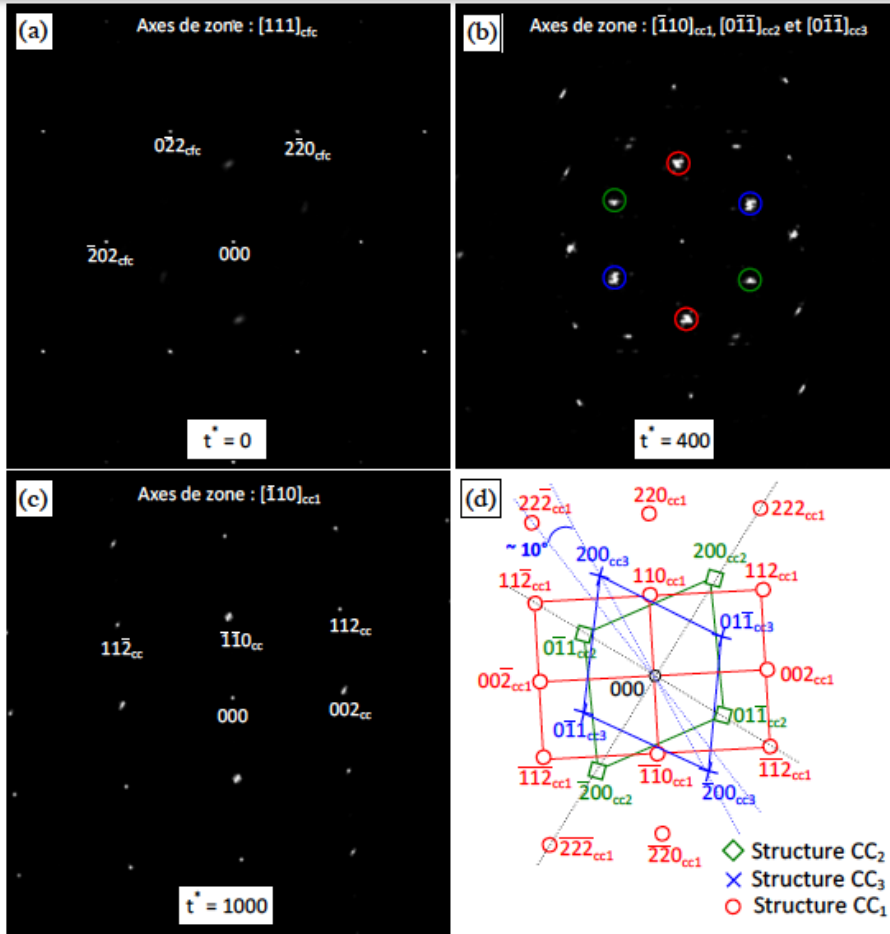
FCC to BCC transition. ADF modeling

Case II

Bcc embryo $r=20\Delta$
fcc/bcc K-S relation



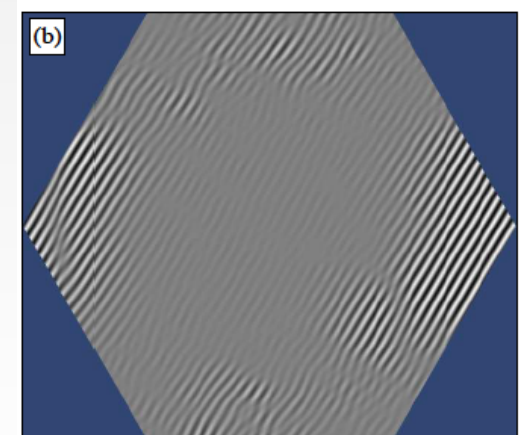
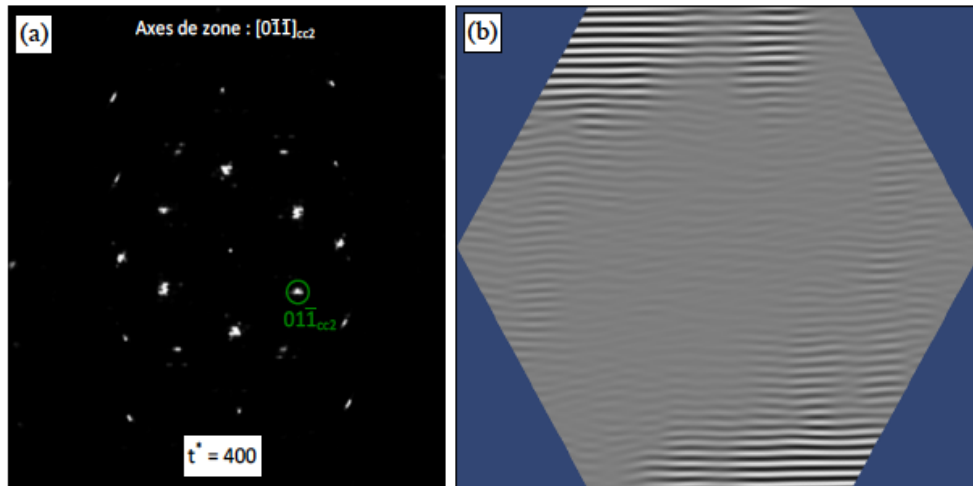
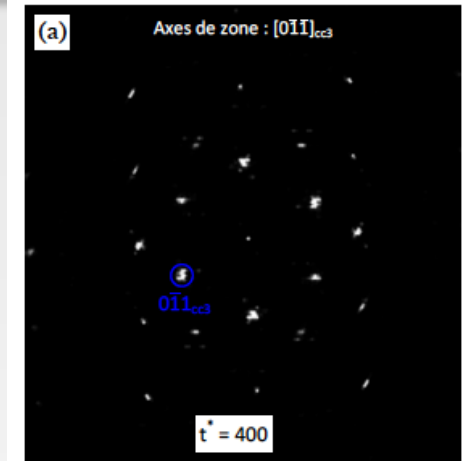
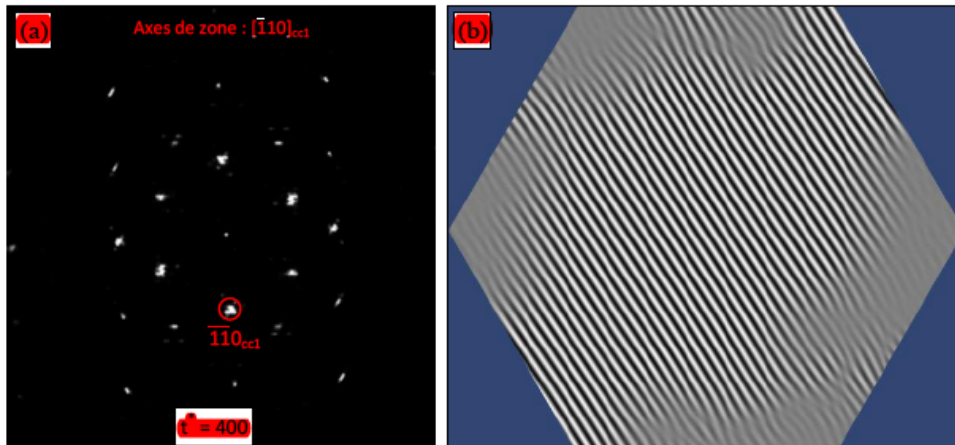
FCC to BCC transition. ADF modeling



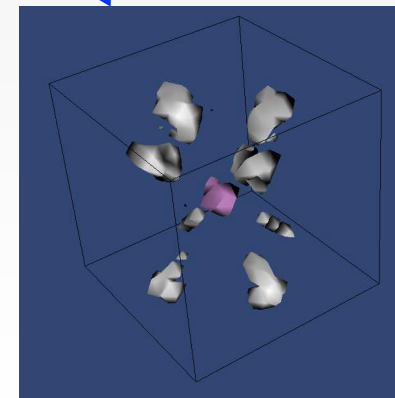
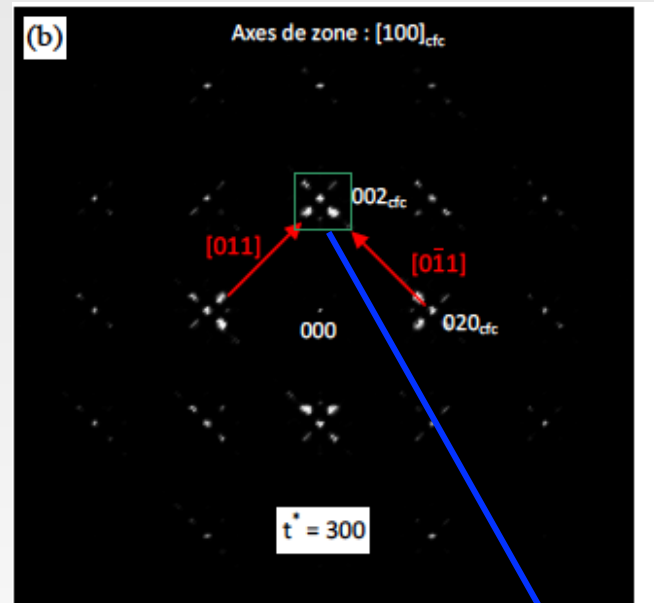
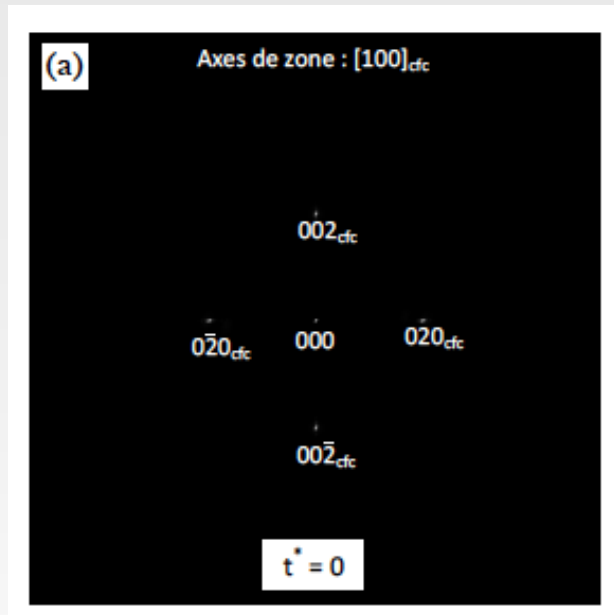
$$([\bar{1}10]_{cc1}, [0\bar{1}\bar{1}]_{cc2}) \approx 10^\circ \text{ et } [\bar{1}10]_{cc1} // [0\bar{1}\bar{1}]_{cc3}$$

Simulated diffraction pattern

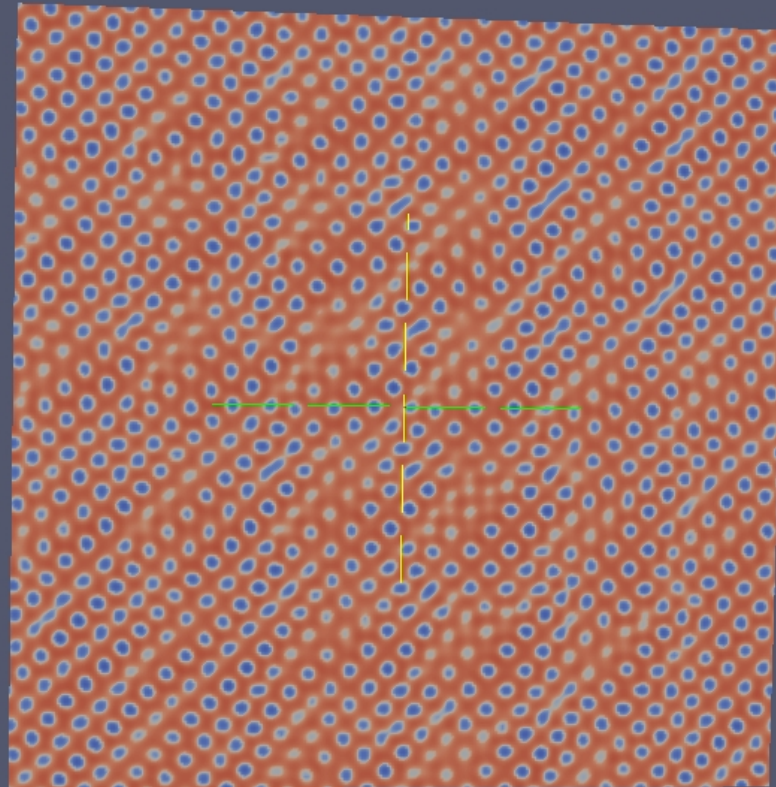
FCC to BCC transition. ADF modeling Dark-field images



FCC to BCC transition. ADF modeling Slip planes



*FCC to BCC transition. ADF modeling.
110 slip planes*

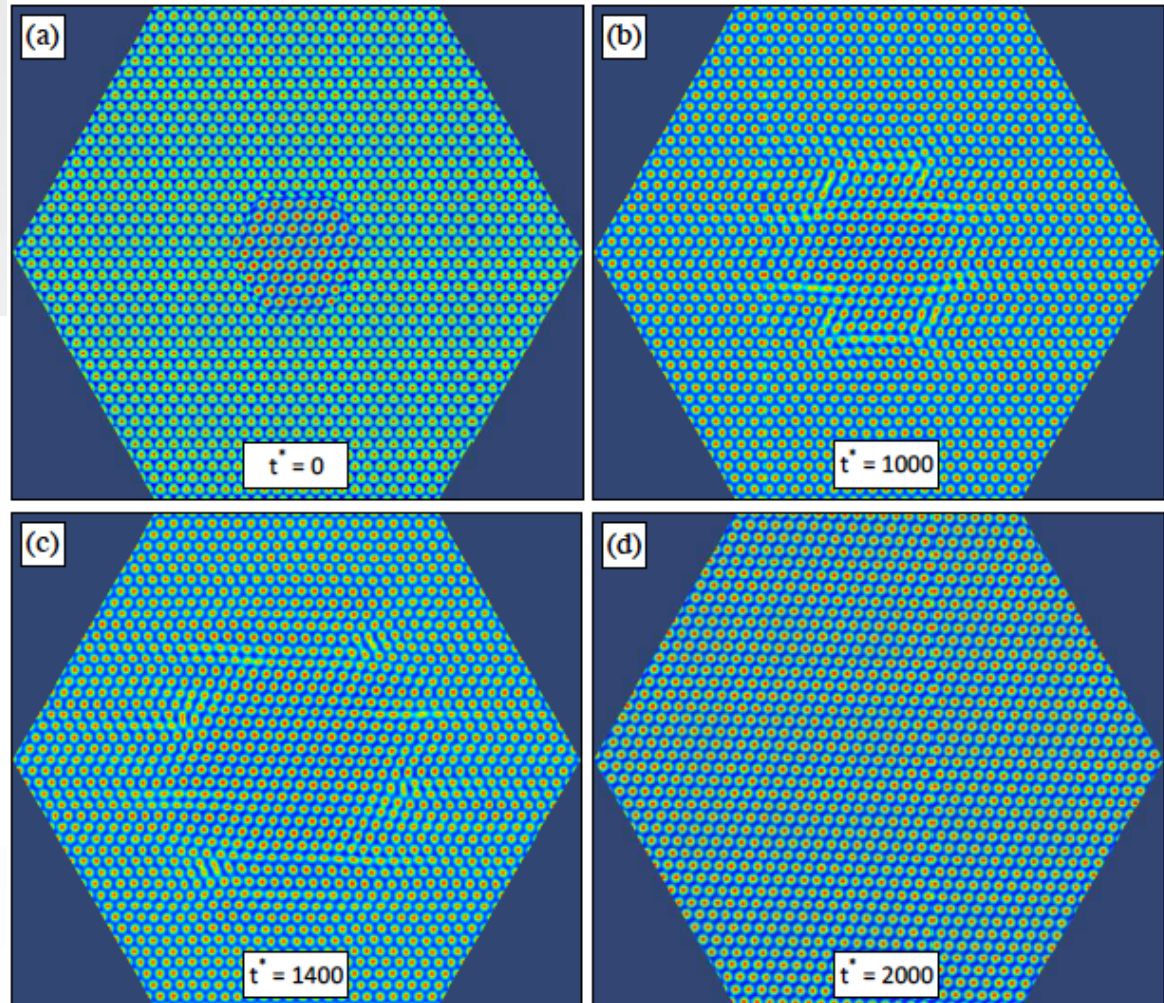
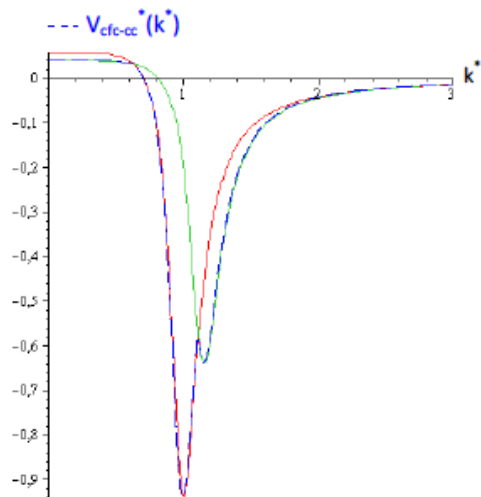


FCC to BCC transition. ADF modeling

FCC stable phase

$$V_1(k) = V_0 \left[1 - \left(\frac{(k/\varepsilon_1)^4}{((k/\varepsilon_1)^2 - \kappa_1^2)^2 + \kappa_2^4} \right) \right]$$

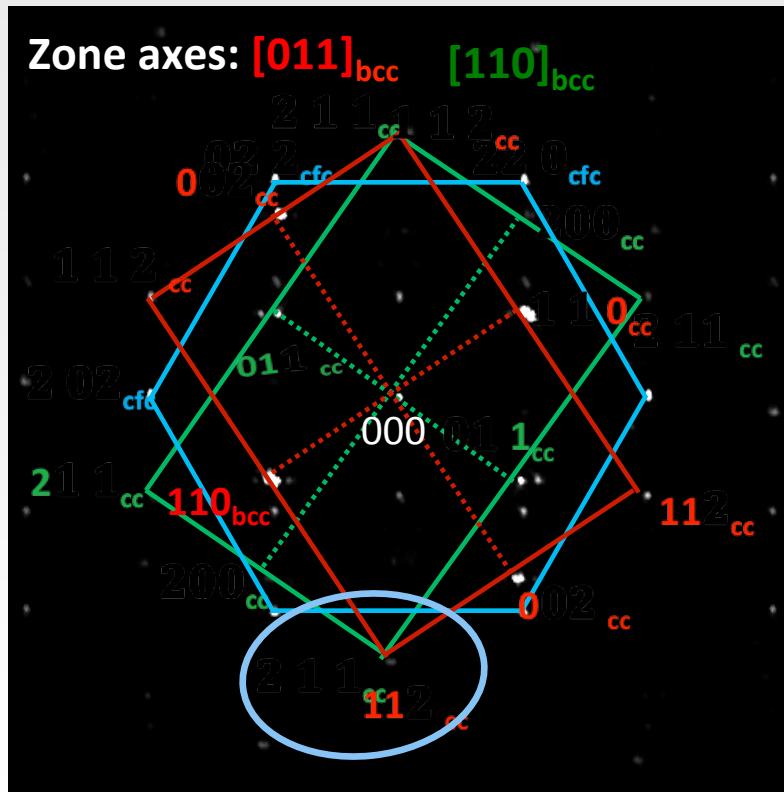
$$V_2(k) = V_1 \left[1 - \left(\frac{(k/\varepsilon_2)^4}{((k/\varepsilon_2)^2 - \kappa_1^2)^2 + \kappa_2^4} \right) \right]$$



FCC to BCC transition. ADF modeling

Diffraction at $t = 400$

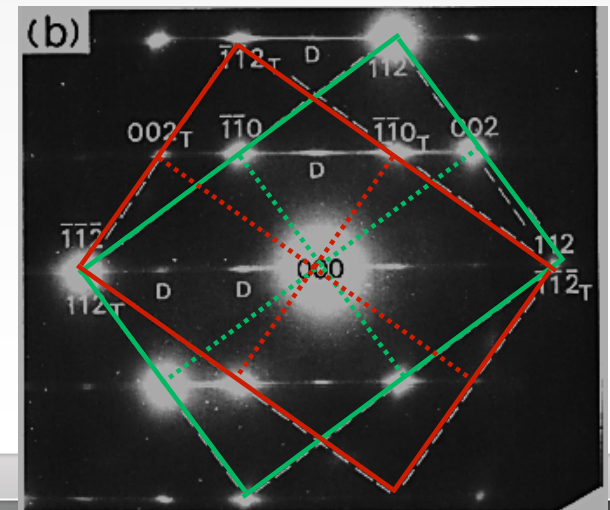
Zone axes: $[111]_{fcc}$



- FCC structure
- BCC (nucleus)
- second variant of the BCC structure
- Two variants with twin relation $\{112\}$

→ Good agreement with experimental data

Fe-Ni alloy



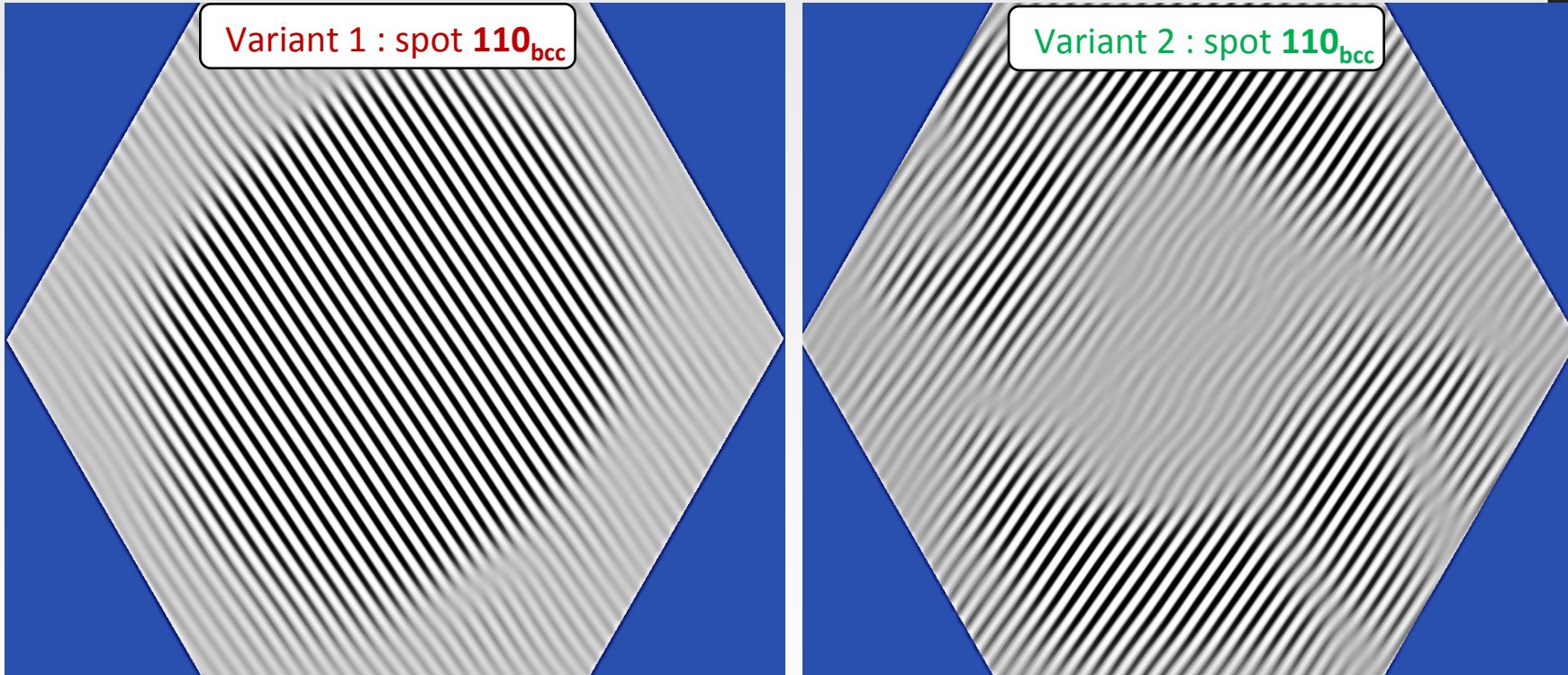
Localisation of two BCC variants

→ Evolution of dark field images:

(111) plane

Variant 1 : spot 110_{bcc}

Variant 2 : spot 110_{bcc}



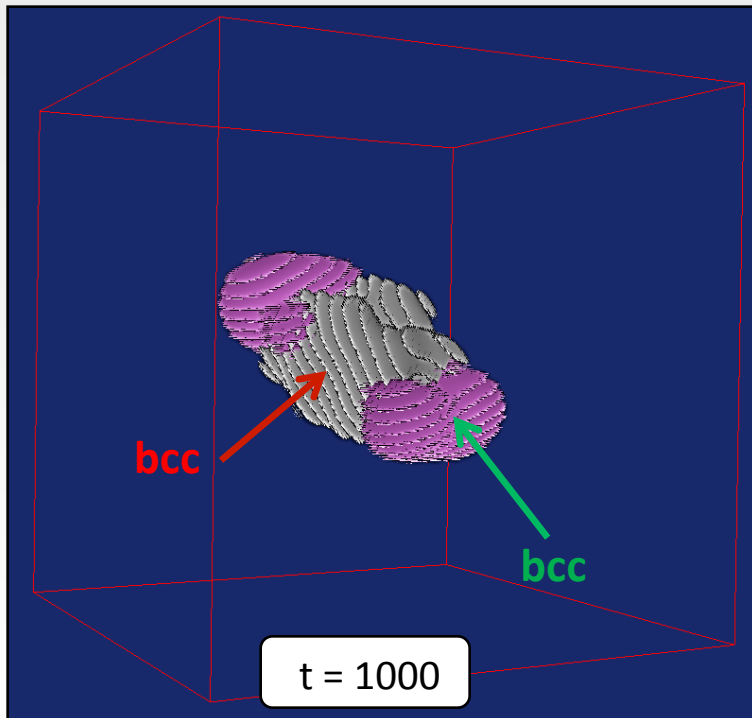
t = 1600

→ Second **BCC** variant is situated at the interface between the first variant and the matrix

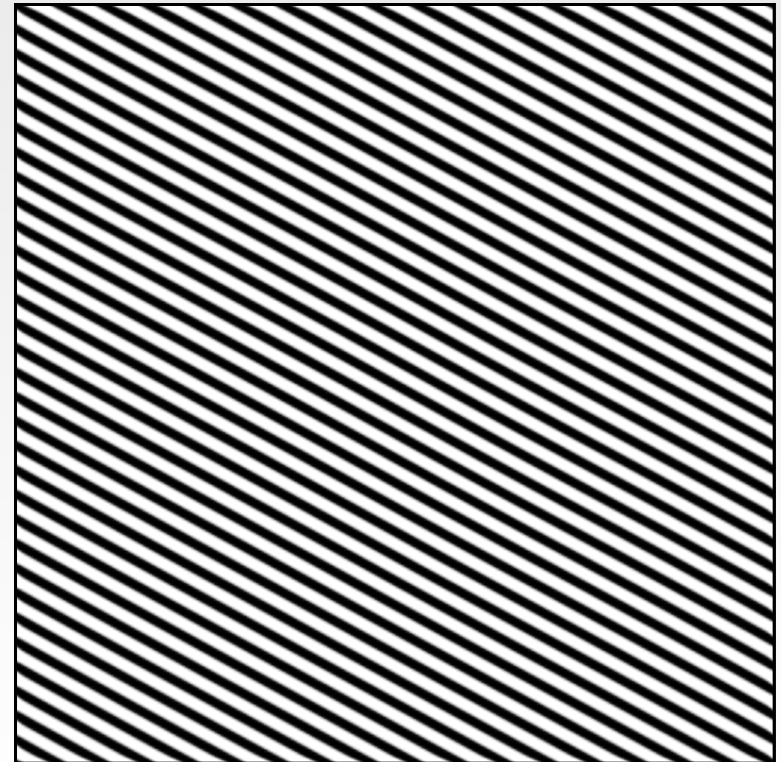
Shape of two variants martensite nucleus



3D dark field image



plan (100)



The nucleus growth in a plate form

CONCLUSIONS

- *The ADF model previously developed to describe the decomposition and ordering on the Ising lattice is applied to the general case of the microstructural evolution with arbitrary atomic movements.*
- *The main results of this modeling is the finding that the pre-existing martensitic nucleus never had a single-domain structure. The martensite nucleus is the multivariant structure which depends from the stability of the fcc phase.*

THANK YOU FOR ATTENTION