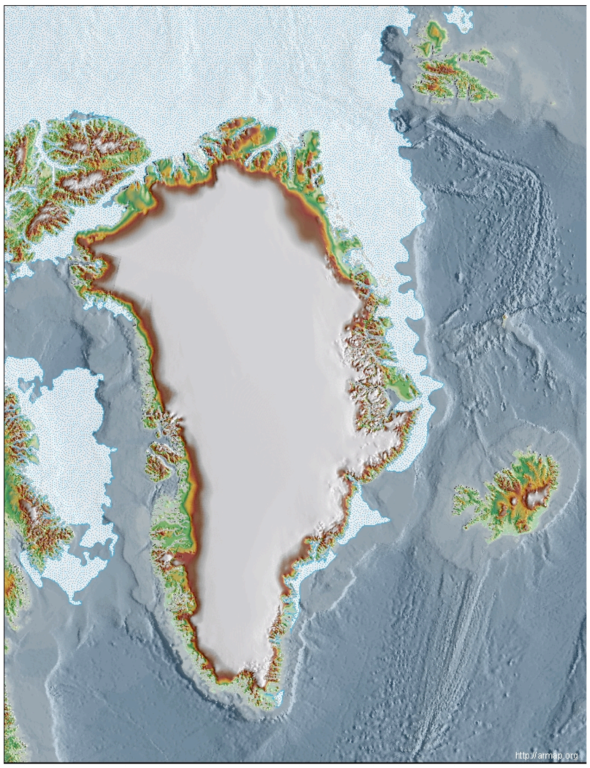
Ice Sheet Modeling

In this exercise, we will do some experiments with a simple ice sheet model based on a classic paper by Johannes Weertman, from 1976. Our goals are to understand some basic things about how these ice sheets grow and shrink, and how they can respond to sunlight variations related to orbital changes of the Earth relative to the Sun.

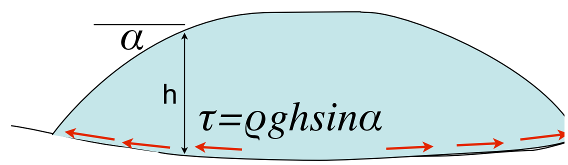


Large continental ice sheets such as Greenland are important components of the global climate system that play a critical role in altering the planetary albedo, which is connected to a potent positive feedback mechanism, and also in controlling the level of global sea level.

Their growth and decline has been one of the dominant features of the Pleistocene ice ages, and their current decline is of great importance to the rising global sea level. The timing of the ice ages and intervening warmer periods are largely controlled by orbital changes, and one of the goals of this modeling exercise is to see how this works.

To develop our model of an ice sheet, we have to start with a few basics of how ice forms and flows. Glacial ice begins as snowfall that accumulates over the years. As it gets buried under more snow, the snow crystals undergo a kind of metamorphism, eventually turning into nearly solid ice. Ice, as a naturally occurring polycrystalline solid, is really a kind of rock. But unlike most other rocks, ice can actually flow at the surface without melting. This solid-state flow is quite fast relative to other geologic processes, enabling glaciers to be very dynamic features of the surface.

How do continental ice sheets flow?



The ice piles up, creating a surface slope (α), which generates a basal shear stress (τ) that causes the ice to flow. As the ice piles up, the crust subsides to achieve isostatic equilibrium.

It is common to assume that ice behaves as a deformable plastic material, there means that there is a critical shear stress τ_0 below which no strain (deformation or flow) will occur, and above which, the strain is limitless. Stress is just a force acting on an area, and shear stress is a force applied parallel to a surface as opposed to a force applied perpendicular to a surface, which is called a normal stress. We talk about stresses rather than forces, since stresses are what can cause materials to deform (whether by flow or by fracture). The shear stress at the base of a pile of ice is a function of the surface slope times the height times gravity times density:

$\tau_b = \rho g h \sin\alpha$ *where α is the slope angle* (1)

This means that if the height of the ice is greater, the slope can be smaller and still achieve the critical shear stress. Where the ice is thinner, you need a higher slope to get the critical shear stress. Considering that the height or thickness of the ice must taper to 0 at the edge, you can see that the slope of the glacier has to be greatest right at the edge (which is illustrated in a schematic way in the drawing above).

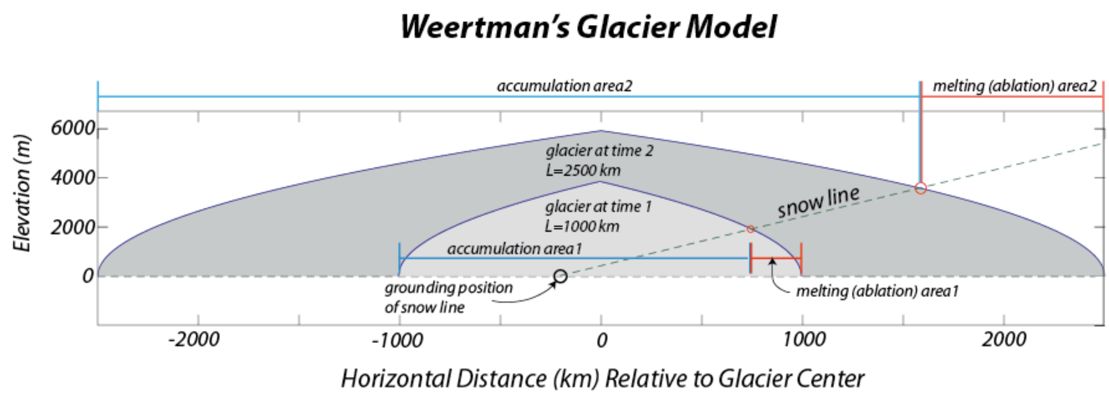
If the slope is too low, the basal shear stress will not match the critical shear stress τ_0, but as snow piles up, creating more ice, the slope will increase until τ_0 is reached, at which point, flow will begin. As flow begins, the slope will decrease; this causes the basal shear stress to drop below τ_0 and flow will stop, but then snow piles up again and τ_0 is met. The result of this is that the glacier evolves to the point where the basal shear stress hovers right around the critical shear stress τ_0 and a steady state condition occurs. The result of this is that a glacier has an equilibrium profile, which is described by the following equation:

$$h(x) = \sqrt{\frac{2\tau_0}{\rho g}\left(L - |x|\right)} = \left(\lambda\left(L - |x|\right)\right)^{1/2} \text{ where } \lambda = \left(\frac{2\tau_0}{\rho g}\right) \quad (2)$$

Here, h is the height or thickness of the ice at values of x, which is distance along the surface; x=0 is the center of the ice mass and L is the distance from the center of the ice to the edge. The ice sheet is considered to be perfectly symmetrical so it looks the same in the +x and –x regions. Weertman says that typical values for λ are 8 - 15. If you integrate this equation (2) from x=-L to x=L, you get the cross-sectional area, and you can also flip this around to get the length from the cross-sectional area:

$$A_X = \frac{4}{3}\lambda^{1/2}L^{3/2} \text{ and conversely, } L = \left(\frac{\left(\frac{3}{4}A_X\right)^2}{\lambda}\right)^{1/3} \quad (3)$$

Here is what the shape of the glacier looks like, at two different times, with different cross-sectional areas:



Also shown in this diagram is the snow line, which separates colder areas where snow will accumulate to form ice from warmer regions where the melting exceeds snowfall and the glacier will experience a loss of ice. The snow line slopes gently up to the right towards the warmer side of the diagram. Where this snowline intersects the surface of the glacier (red circles above), we divide the glacier into its accumulation zone and its melting zone. The grounding position of the snowline (black circle above) marks the place where it intersects an elevation of zero.

The model starts with an initial glacier length, and from that, we can calculate the profile of the glacier and its cross-sectional area. Once we have the profile, we can find the intersection with the snow line, which allows us to separate the glacier into the regions above the snow line where accumulation can occur and below the snowline where melting will occur. We get the snow line by setting the equation for the snow line equal to the equation for the shape of the ice surface, which leads to a quadratic equation. Once we have the snow line, we can calculate the change in the cross sectional area as follows:

$$\frac{dA_X}{dt} = L_{ac}v_{ac} + L_{ab}v_{ab} \qquad (4)$$

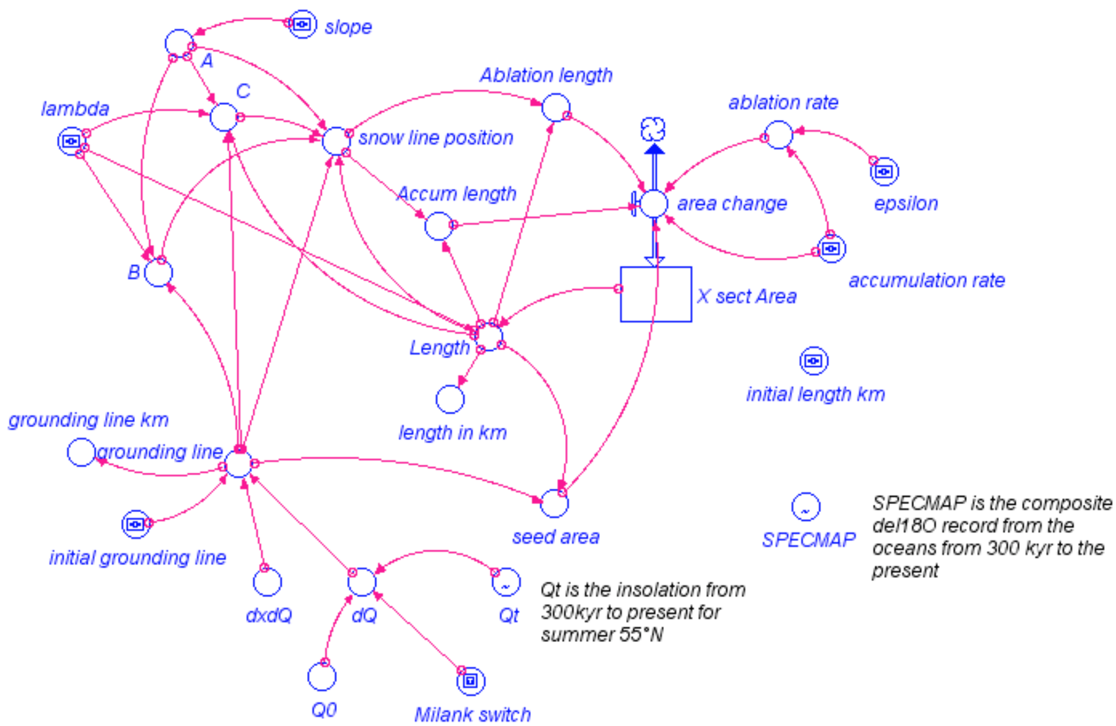
Here, L_{ac} is the length over which accumulation occurs, and L_{ab} is the length over which melting or ablation occurs. These lengths are multiplied by their corresponding rates v_{ac} and v_{ab} (the ablation rate is negative) summed to give the change in cross-sectional (A_X) over a given interval of time. The balance of accumulation and ablation — the sign of equation 4 — then determines if the glacier will shrink or grow; in either case, we assume that it maintains the equilibrium profile. In the model, the accumulation rate (v_{ac}) and ablation rate (v_{ab}) are related by a parameter called epsilon:

$$\varepsilon = \frac{v_{ac}}{v_{ab}} \qquad (5)$$

If warming occurs, the grounding line moves to the left (-x is considered to be toward the North), whereas cooling moves it to the South (right in the diagram).

Based on observations of the present, Weertman calculated that the grounding position of the snowline changes by 17.7 km for every W/m^2 of mean summer insolation change. In this way, we can make a connection between the orbitally-driven changes in summer insolation to the model as a way of forcing the glacier to grow and shrink.

Here is what the model looks like:



Qt is the time-varying summer insolation (= *in*coming *sol*ar radi*ation*) for 55°N and **Q0** is the present day summer insolation for the same region; **dQ** is just the difference between **Qt** and **Q0** and **dxdQ** tells how much the grounding line moves given the change in insolation (**dQ**). **Qt** and **Q0** are connected to the model via a switch so that we can disable them or enable them. The switch allows us to do a experiments without the complications of orbital forcing. **SPECMAP** is the oxygen isotope record from the oceans that gives us a sense of the timing and magnitude of ice volume changes over time; this is just something we can plot to see the extent to which our little ice sheet model mimics the actual record of ice growth and melting. Both SPECMAP and Qt go back to 300 kyr. Time begins at -300,000 years and ends at 0. The model also includes a converter called **seed area**, which comes into play when there is no glacier and the grounding line moves into the positive realm, indicating cooling; this just allows the glacier to get going again.