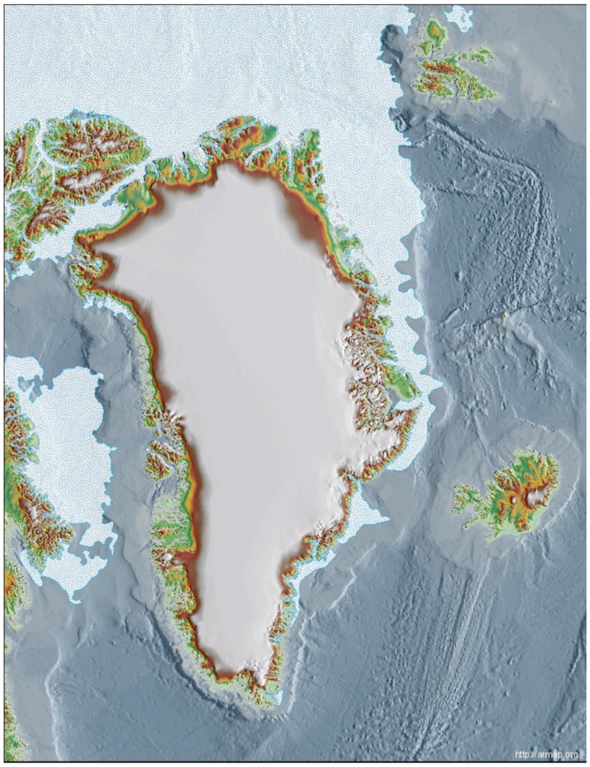
Ice Sheet Modeling

In this exercise, we will do some experiments with a simple ice sheet model based on a classic paper by Johannes Weertman, from 1976. Our goals are to understand some basic things about how these ice sheets grow and shrink, and how they can respond to sunlight variations caused by orbital changes of the Earth relative to the Sun.

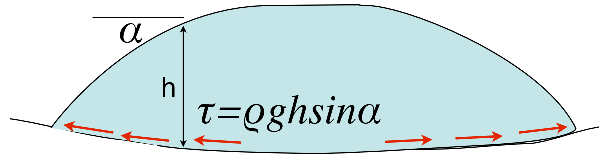


Large continental ice sheets such as Greenland (at left) are important components of the global climate system that play a critical role in altering the planetary albedo, which is connected to a potent positive feedback mechanism, and also in controlling the level of global sea level.

Their growth and decline has been one of the dominant features of the Pleistocene ice ages, and their current decline is of great importance to the rising global sea level. The timing of the ice ages and intervening warmer periods are largely controlled by orbital changes, and one of the goals of this modeling exercise is to see how this works.

To develop our model of an ice sheet, we have to start with a few basics of how ice forms and flows. Glacial ice begins as snowfall that accumulates over the years. As it gets buried under more snow, the snow crystals undergo a kind of metamorphism, eventually turning into solid ice. Ice, as a naturally occurring polycrystalline solid, is really a kind of rock, but unlike most other rocks, ice can actually flow at the surface without melting. This solid-state flow is quite fast relative to other geologic processes, enabling glaciers to be very dynamic features of the surface.

How do continental ice sheets flow?



The ice piles up, creating a surface slope (α), which generates a basal shear stress (τ) that causes the ice to flow. As the ice piles up, the crust subsides to achieve isostatic equilibrium.

It is common to assume that ice behaves as a deformable plastic material, which means that there is a critical shear stress, τ_0, below which no strain (deformation or flow) will occur, and above which, the strain is limitless. Stress is just a force acting on an area, and shear stress is a force applied parallel to a surface as opposed to a force applied perpendicular to a surface, which is called a normal stress. We talk about stresses rather than forces, since stresses are what can cause materials to deform (whether by flow or by fracture). The shear stress at the base of a pile of ice is a function of the surface slope, the thickness, gravity, and density:

$\tau_b = \rho g h \sin\alpha$ *where* α *is the slope angle* (1)

This means that where the thickness of the ice is greater, the slope can be smaller and still achieve the critical shear stress. Where the ice is thinner, you need a higher slope to get the critical shear stress. Considering that the height or thickness of the ice must taper to 0 at the edge, you can see that the slope of the glacier has to be greatest right at the edge (which is illustrated in a schematic way in the drawing above).

If the slope is too low, the basal shear stress will not match the critical shear stress τ_0, but as snow piles up, creating more ice, the thickness will increase until τ_0 is reached, at which point, flow will begin. As flow begins, the slope will decrease; this causes the basal shear stress to drop below τ_0 and flow will stop, but then snow piles up again and τ_0 is met. The result of this is that the glacier evolves to the point where the basal shear stress hovers right around the critical shear stress τ_0 and a steady state condition occurs. The result of this is that a glacier has an *equilibrium profile*, which is described by the following equation:

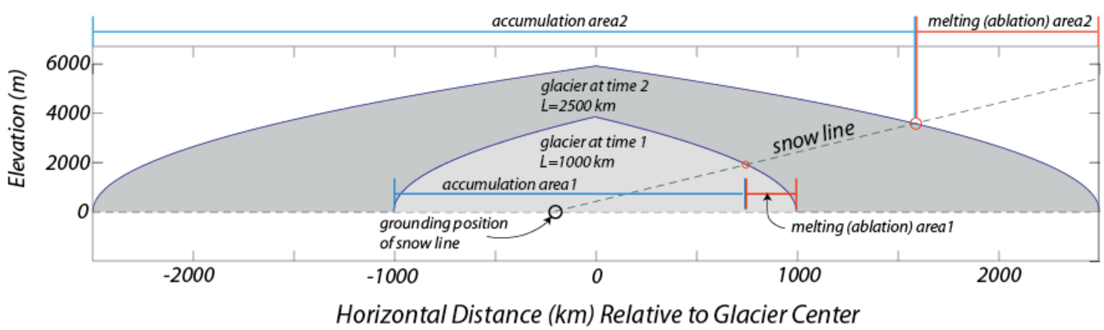
$$h(x) = \sqrt{\frac{2\tau_0}{\rho g}\left(L - |x|\right)} = \left(\lambda\left(L - |x|\right)\right)^{\frac{1}{2}} \text{ where } \lambda = \left(\frac{2\tau_0}{\rho g}\right) \quad (2)$$

Here, h is the height or thickness of the ice at values of x, which is distance along the surface; x=0 is the center of the ice mass and L is the distance from the center of the ice to the edge. The ice sheet is considered to be perfectly symmetrical so it looks the same in the +x and –x regions. Weertman says that typical values for λ are 8 - 15. If you integrate this equation (2) from x=-L to x=L, you get the cross-sectional area, and you can also flip this around to get the length from the cross-sectional area:

$$A_X = \frac{4}{3}\lambda^{\frac{1}{2}}L^{\frac{3}{2}} \text{ and conversely, } L = \left(\frac{\left(\frac{3}{4}A_X\right)^2}{\lambda}\right)^{\frac{1}{3}} \quad (3)$$

Here is what the shape of the glacier looks like, at two different times, with different cross-sectional areas:

Weertman's Glacier Model



Also shown in this diagram is the snow line, which separates colder areas where snow will accumulate to form ice from warmer regions where the melting exceeds snowfall and the glacier will experience a loss of ice. The snow line slopes gently up to the right towards the warmer side of the diagram. Where this snowline intersects the surface of the glacier (red circles above), we divide the glacier into its accumulation zone and its melting zone. The grounding position of the snowline (black circle above) marks the place where it intersects an elevation of zero.

The model starts with an initial glacier length, and from that, we can calculate the profile of the glacier and its cross-sectional area. Once we have the profile, we can find the intersection with the snow line, which allows us to separate the glacier into the regions above the snow line where accumulation can occur and below the snowline where melting will occur. We get the snow line by setting the equation for the snow line equal to the equation for the shape of the ice surface, which leads to a quadratic equation. Once we have the snow line, we can calculate the change in the cross sectional area as follows:

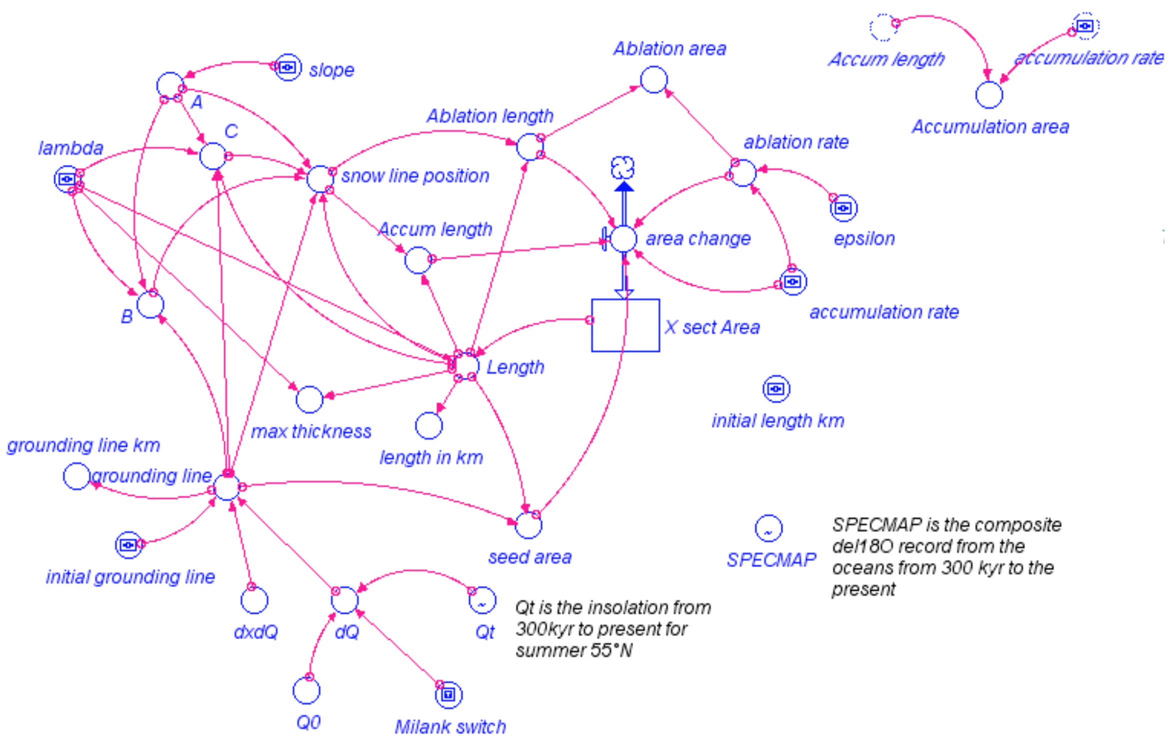
$$\frac{dA_X}{dt} = L_{ac}v_{ac} + L_{ab}v_{ab} \qquad (4)$$

Here, L_{ac} is the length over which accumulation occurs, and L_{ab} is the length over which melting or ablation occurs. These lengths are multiplied by their corresponding rates v_{ac} and v_{ab} (the ablation rate is negative) summed to give the change in cross-sectional (A_X) over a given interval of time. The balance of accumulation and ablation — the sign of equation 4 — then determines if the glacier will shrink or grow; in either case, we assume that it maintains the equilibrium profile. In the model, the accumulation rate (v_{ac}) and ablation rate (v_{ab}) are related by a parameter called epsilon:

$$\varepsilon = \frac{v_{ac}}{v_{ab}} \qquad (5)$$

If warming occurs, the grounding line moves to the left (-x is considered to be toward the North), whereas cooling moves it to the South (right in the diagram). Based on observations of the present, Weertman calculated that the grounding position of the snowline changes by 17.7 km for every W/m^2 of mean summer insolation change. In this way, we can make a connection between the orbitally-driven changes in summer insolation to the model as a way of forcing the glacier to grow and shrink.

Here is what the model looks like:



Qt is the time-varying summer insolation (= *in*coming *sol*ar radi*ation*) for 55°N and **Q0** is the present day summer insolation for the same region; **dQ** is just the difference between **Qt** and **Q0** and **dxdQ** tells how much the grounding line moves given the change in insolation (**dQ**). **Qt** and **Q0** are connected to the model via a switch so that we can disable them or enable them. The switch allows us to do a experiments without the complications of orbital forcing. **SPECMAP** is the oxygen isotope record from the oceans that gives us a sense of the timing and magnitude of ice volume changes over time; this is just something we can plot to see the extent to which our little ice sheet model mimics the actual record of ice growth and melting. Both SPECMAP and Qt go back to 300 kyr. Time begins at -300,000 years and ends at 0. The model also includes a converter called **seed area**, which comes into play when there is no glacier and the grounding line moves into the positive realm, indicating cooling; this just allows the glacier to get going again.

Experiments

These experiments can either be done by constructing your own model using STELLA, or by downloading a pre-made version, or by working with a version that runs online.

To begin with, make sure that the Milankovitch orbital variations of insolation are turned off so they do not impact the model.

Experiment 1: Steady State? Response Time?

In this first experiment, let's see what happens to the glacier's length over time with some reasonable initial conditions.

Time Specs:

Run from -300,000 to -200,000 years, with a DT of 200 and Runge-Kutta 4.

Model Parameters:

accumulation_rate = 1.2 { m/yr}
epsilon = .24 { ratio of rates }
initial_grounding_line = -400 {km}
initial_length_km = 400 {km starting length}
lambda = 14 {ice strength parameter }
slope = 0.002

Be sure that the **Milank switch** is turned off for this experiment.

a) Before running the model, try to predict what will happen to the ice sheet. Will it find a steady state, or will it just shrink to nothing or will it grow indefinitely? Then run the model and explain what happens.

b) Now, change the initial length to 3000 km — a very large ice sheet in this case. Will it find a steady state again? Will it have the same steady state length as in the first case? At the start, do you think that the accumulation rate times accumulation area will be greater than or less than the ablation rate times the ablation area?

c) How quickly does the ice sheet get into its steady state? How fast can the glacier grow and shrink? In systems analysis, this is called the *response time*, and is often defined as the time it takes a system to accomplish about 2/3 of its change to the eventual steady state (so it really only applies to systems that tend toward a steady state). In the case of our glacier, you can find the difference in length between the steady state length and the starting length — then find the point in time when about 2/3 of this change has been accomplished; that is your response time.

Use the model set-up and results from the first experiments (a & b) to estimate the response time, giving the result in kyr.

Experiment 2: Crossing the Threshold to Rapid Melting

In the above experiment, we looked at 2 initial lengths and found that in both cases, the glacier evolved into a steady state length, where the accumulation area added was equal to the ablation area removed. Now, let's explore a wider range of initial lengths, which will reveal an interesting change. Start with the same model set-up as in 1a, where the initial length was 400 km. Be sure that the **Milank switch** is turned off for this experiment.

a) Run the model, and you should see the glacier grow to a length of about 2100 km and then level off, having reached a steady state.

b) Now, decrease the initial length to 300 km. What length does the glacier end up at?

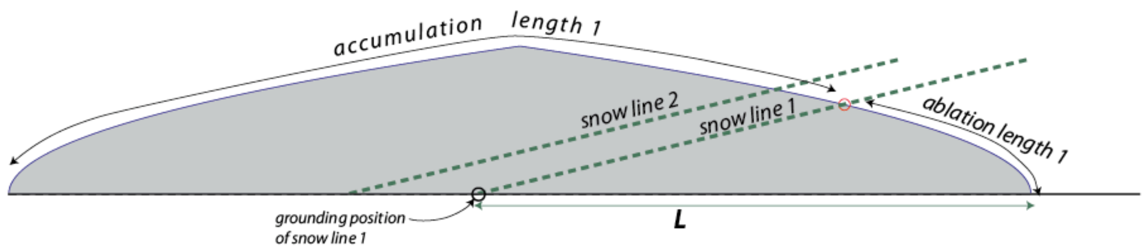
c) Now, decrease the initial length to 200 km. What length does the glacier end up at? Describe, briefly, what happens to the glacier in this case. Note that in (a), the glacier decelerates as it approaches the steady state — it gets there very gradually. How does this deceleration of (a) compare with the behavior in this case?

d) Now increase the initial length to 210 km. What length does the glacier end up at? Describe, briefly, what happens to the glacier in this case.

e) It should be clear to you that there is a threshold in the initial length that separates two very different behaviors and different outcomes. Fiddle around with the initial length until you find the threshold (within 1 km is fine).

Experiment 3: Changing the Grounding Line

Now let's see what happens if we change the position of the grounding line, which is shown graphically below, shifted to the left (towards more negative values):



This is kind of like imposing a warming on the glacier. Start with the same model set-up as in 1a:

Time Specs:

Run from -300,000 to -200,000 years, with a DT of 200 and Runge-Kutta 4.

Model Parameters:

accumulation_rate = 1.2 { m/yr}
epsilon = .24 { ratio of rates }
initial_grounding_line = -400 {km}
initial_length_km = 400 {km starting length}
lambda = 14 {ice strength parameter }
slope = 0.002

Be sure that the **Milank switch** is turned off for this experiment.

a) Run this model to act as a control, taking note of the ending length and the general behavior. Then shift the grounding line to -500 km. Make a prediction about what will happen, then run the model and describe how this change has affected the glacier.

b) Now shift the grounding line to -300 km and make a prediction about how this will affect the glacier, then run the model and describe how this change has affected the glacier.

Experiment 4: Changing the Ice Strength (λ)

Now we will investigate the affect of changing the ice strength parameter (λ), which has as its main variable the critical shear stress for flow of the ice. If we lower λ, then we are effectively lowering the critical shear stress, making it easier for the ice to flow. This would mean that with a lesser thickness and/or a shallower slope, the ice will flow. To begin, we will use the standard set-up from experiment 1a, where λ is set to a value of 14. Run this "control" model first, and take note of the ending length and maximum thickness of the glacier. Be sure that the **Milank switch** is turned off for this experiment.

a) Change λ to 12, thus making the ice flow more easily. Make a prediction about what this will do to the glacier in comparison with our control. Will the glacier grow to a greater or lesser length relative to the control? Will the height be lesser or greater?

b) Run the model and describe what happens and how the results compare with your predictions.

c) Now change to 10, and make a prediction. Then run the model and describe what happens and attempt to explain why it happens.

Experiment 5: Ratio of Accumulation and Ablation (ε)

How will changing the ratio of accumulation and ablation (melting) rates affect the growth of the ice sheet? The model parameter called epsilon (ε) controls this ratio. We will again use the model set-up from 1a as our control; here ε is set at 0.24. First run this model to recall what happens to the length. Be sure that the **Milank switch** is turned off for this experiment.

a) Now change epsilon to 0.28. Remember that the ablation rate is equal to the accumulation rate divided by epsilon. What will changing epsilon to a larger value do to the ablation rate — make it greater or lesser than the control? Predict how this change will affect the equilibrium length of the glacier, and explain your reasoning.

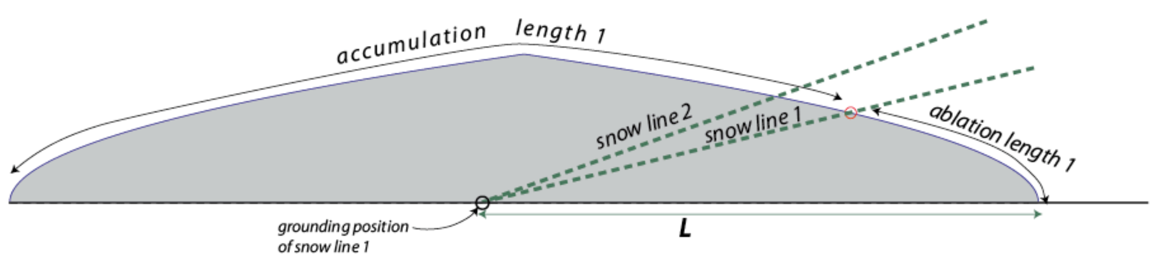
b) Then, run the model and describe what happens, and explain why the glacier responds this way.

c) Now change epsilon to 0.20. How will this change the ablation rate relative to the control, and how will this affect the growth of the glacier?

d) Run the model and then describe what happens, and explain why the glacier responds this way.

Experiment 6: Changing the Slope of the Snowline

What will happen if the slope of the snowline increases? It is already set to a very low value of 0.002. First, let's visualize what this would do to the glacier:



You can see that it will shorten the accumulation length and increase the ablation length. So, what will this do to the growth of the glacier?

As before, we begin with the model set-up for 1a, and then run this model to remind ourselves of the control case. *Take note of the beginning ablation length in the control.*

a) Change the slope slightly to 0.0022. First make a prediction about how this will affect the growth of the glacier relative to the control.

b) Run the model and explain what happens and why it happens. How does the beginning ablation length of this model compare with the control? How did the results compare with your prediction?

c) Now increase the slope even more to 0.0024. Run the model and describe what happens and why.

Experiment 7: Orbital Forcing

Now, we will connect the orbital forcing to the model by turning on the **Milank switch**. For the web-based version, we will now shift to a different model that runs for the full 300 kyr (we've just been running 100 kyr so far). First restore all the parameters to the way they were for experiment 1a. Now, with the Milank switch turned on, the changing summer insolation due to orbital variations will force the grounding line position to move back and forth. Higher insolation pushes the groundling line position to the north (toward more negative values, while a decrease in insolation moves the grounding line to the south (more positive values). As you should know by now, moving the grounding line position will cause the glacier to advance and retreat.

Run the model and plot the length in km and Qt (the orbitally controlled variation in summer insolation), and study the relationship between the peaks and troughs in Qt and the size of the ice sheet.

a) Study the relationship between the glacier's length and Qt (the insolation over time). Are they perfectly in sync, or does one seem to lag the other?

b) What is the lag time in kyr of the ice sheet relative to Qt?

c) How consistent is this lagtime?

d) What is the range of variation in the length of the glacier in km? For comparison, the Laurentide ice sheet expanded and contracted about 25° of latitude from it's center of mass (x=0) and there are 111 km per degree of latitude.

e) Now compare the ice sheet length with the SPECMAP record of del18O, which is partly a measure of ice volume and partly a measure of temperature — higher values represent more ice and colder temperatures. How well do they agree? How similar or dissimilar are the times of the peaks and troughs?

f) Look at the most recent 10 kyr of the model. How is Qt changing during this time, and how does the model glacier respond? What does this suggest might be happening at the present time if we were not increasing the greenhouse effect through elevated CO_2 levels — entering another small glaciation or holding steady or moving to a warmer interglacial?