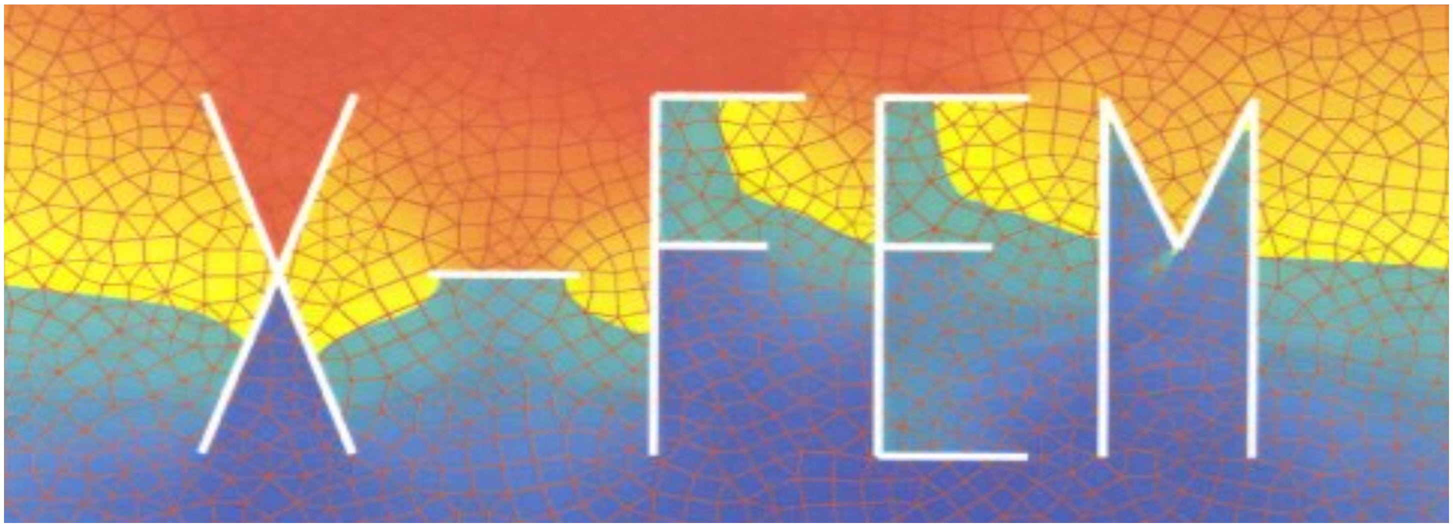
Extended Finite Element Method



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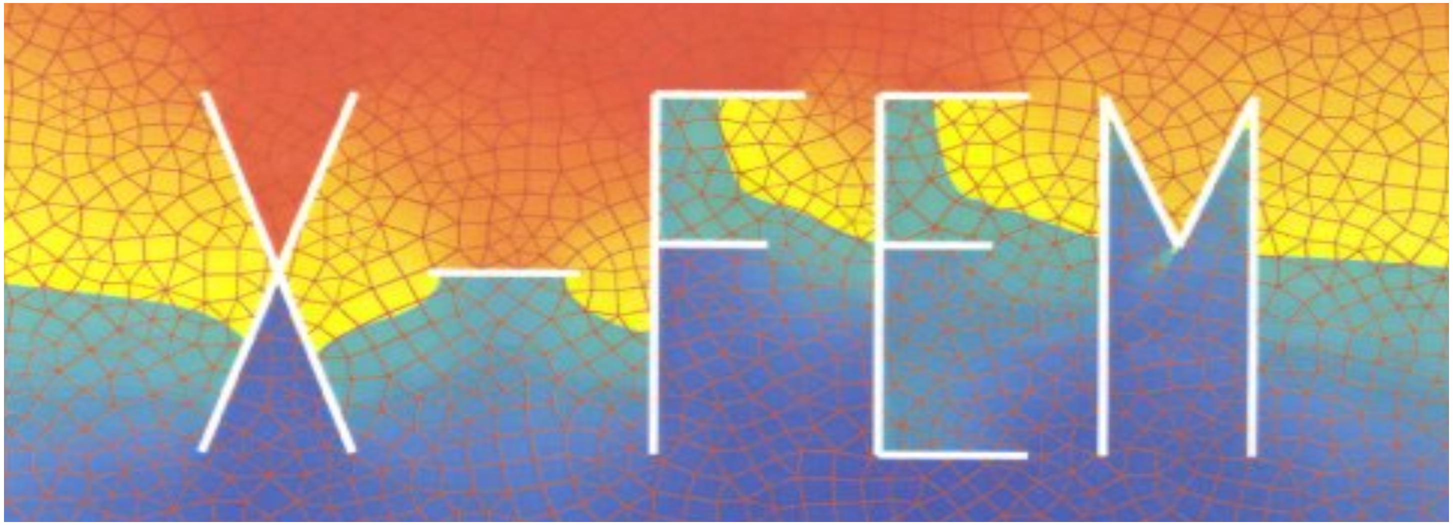
Introduction



Introduction

- Finite Element Method (FEM)

  - popular numerical method in practical engineering applications
  - not successful in simulating the propagation of fracture

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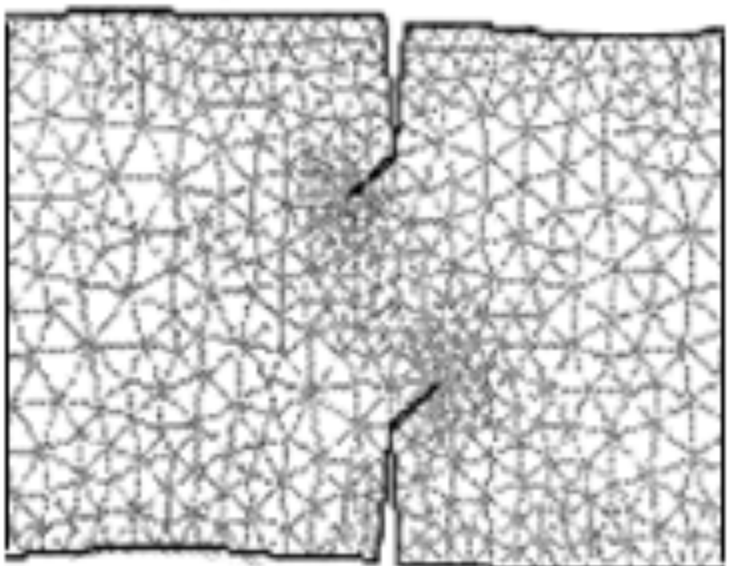
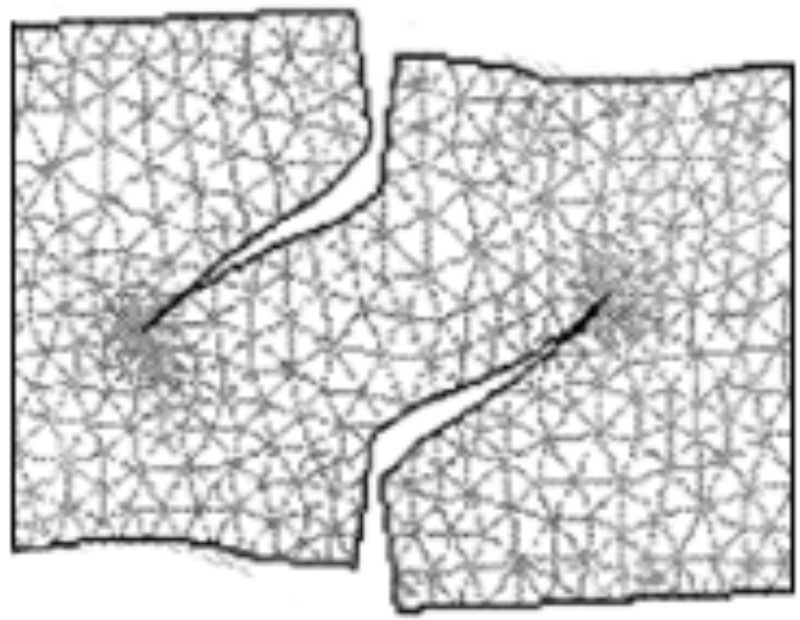


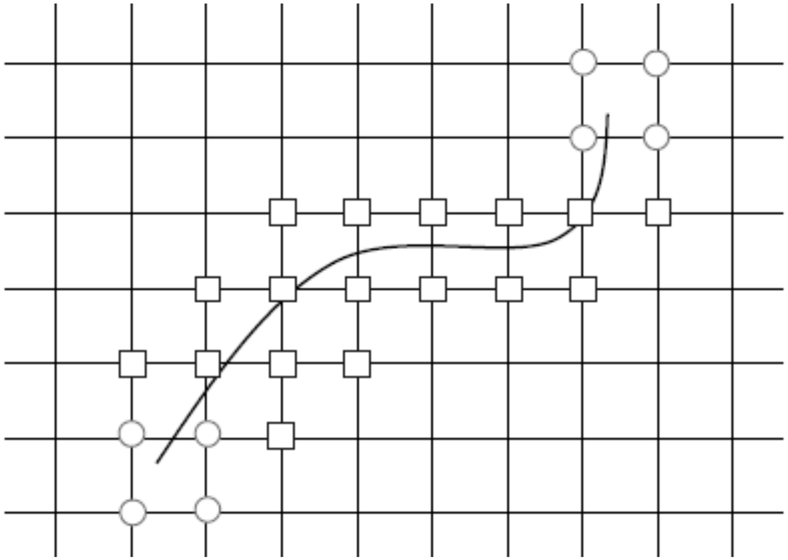


Introduction

- Extended Finite Element Method (X-FEM)
  - Introduced by **Belytsckho and Black** in 1999
  - Based on the concept of **partition of unity**, (the sum of the shape functions must be unity).

- The FE method is used as the building block in the XFEM!

- Local enrichment functions are added to the finite element shape functions
  - Jumped function
  - crack tip function

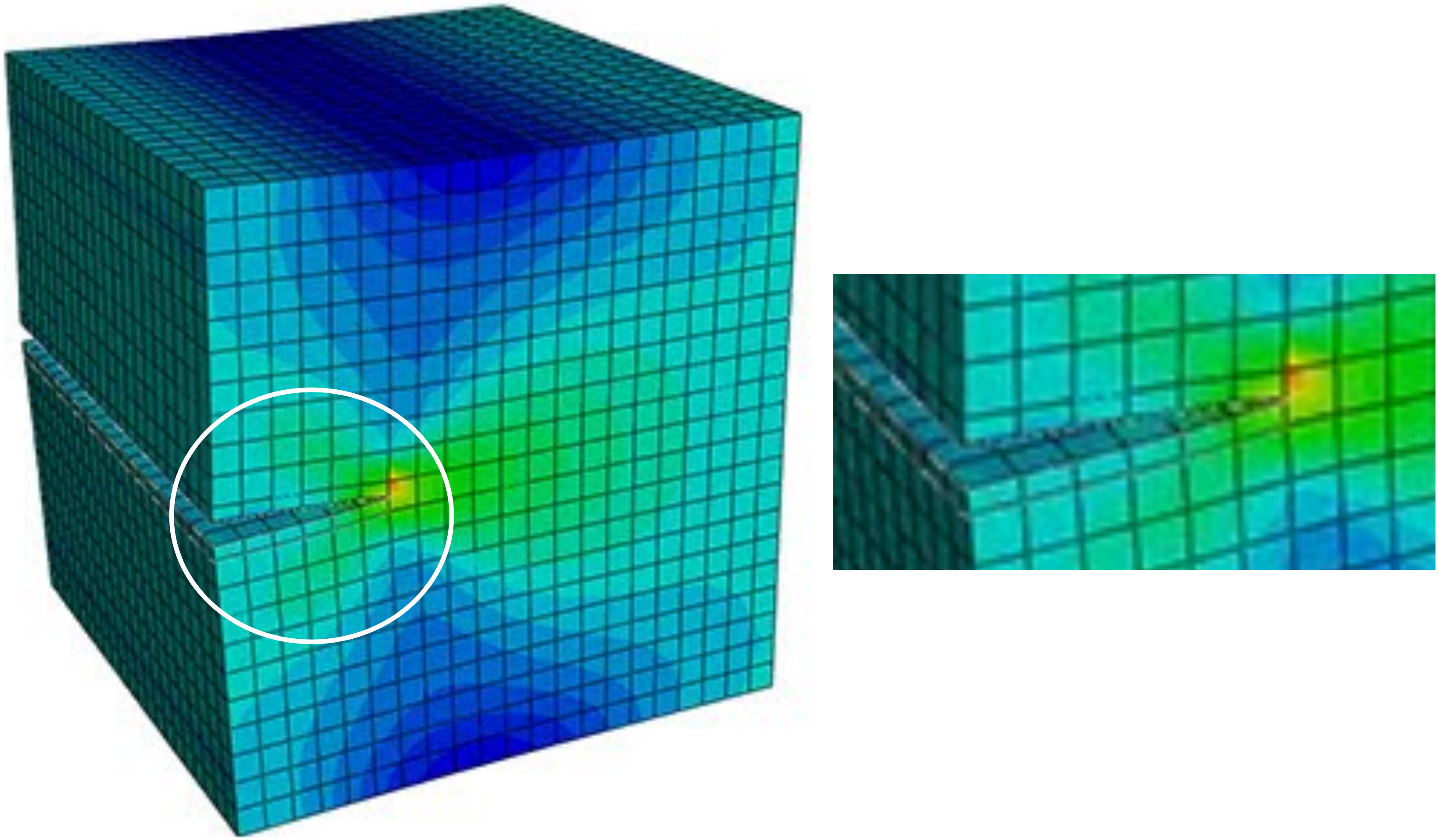


A crack on a uniform mesh

Introduction

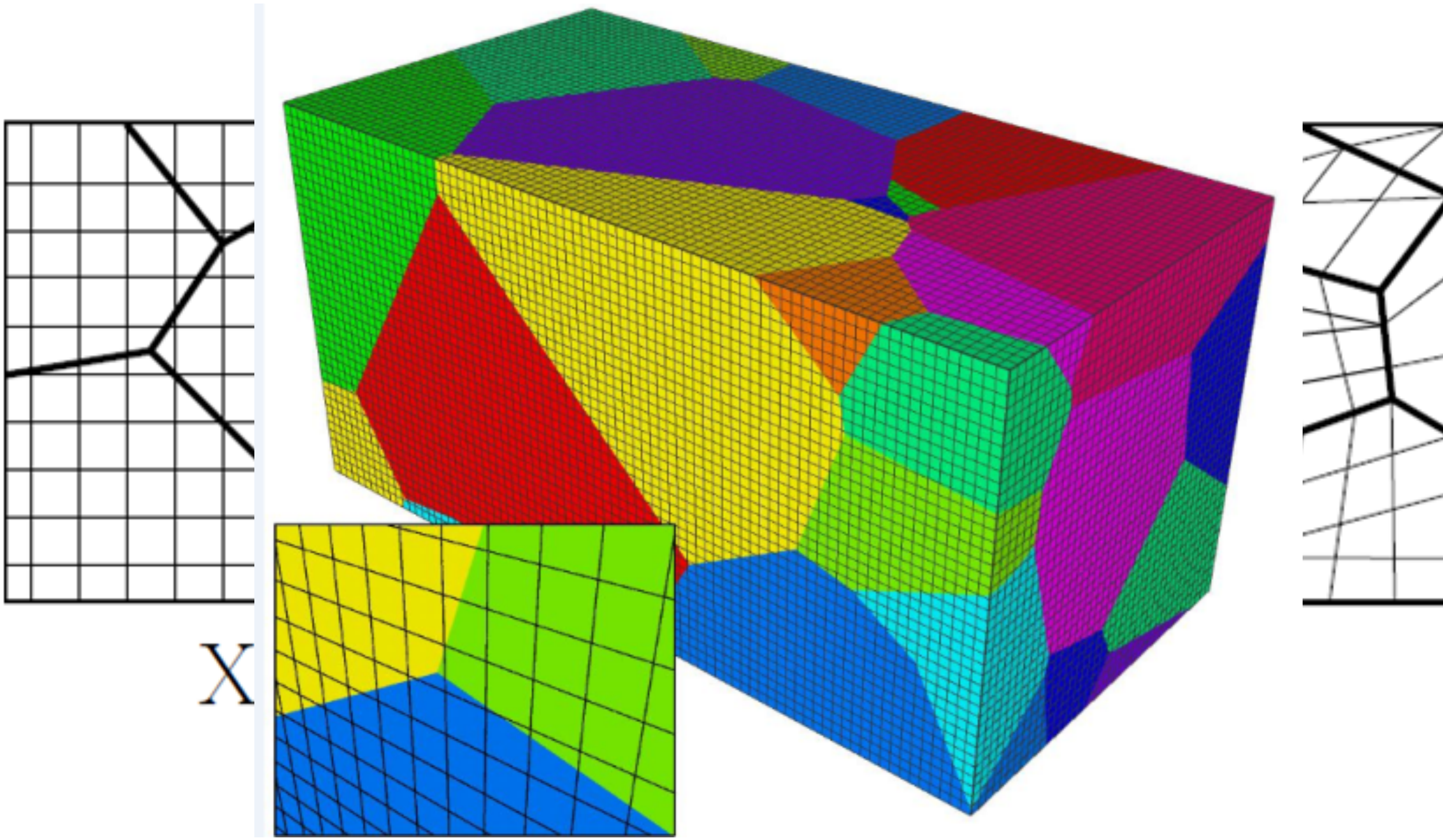
- Extended Finite Element Method (X-FEM)
  - A conventional FE mesh is generated
  - Defects (i.e. cracks, voids and inhomogeneities) are added
    - ✓ mesh is independent of discontinuities
    - ✓ By enriching the standard displacement approximation with additional functions

Introduction



3D XFEM model of a crack

Introduction



Micromodel of a polycrystal

Introduction

- XFEM - Advantages:

  - Mesh is independent the shape of the entities
  - Mesh does not need to conform the crack geometry
  - Requires minimal remeshing and computational efficient.
  - Combining XFEM with level sets, XFEM is good in modelling a growing crack or moving phase boundaries.

Introduction

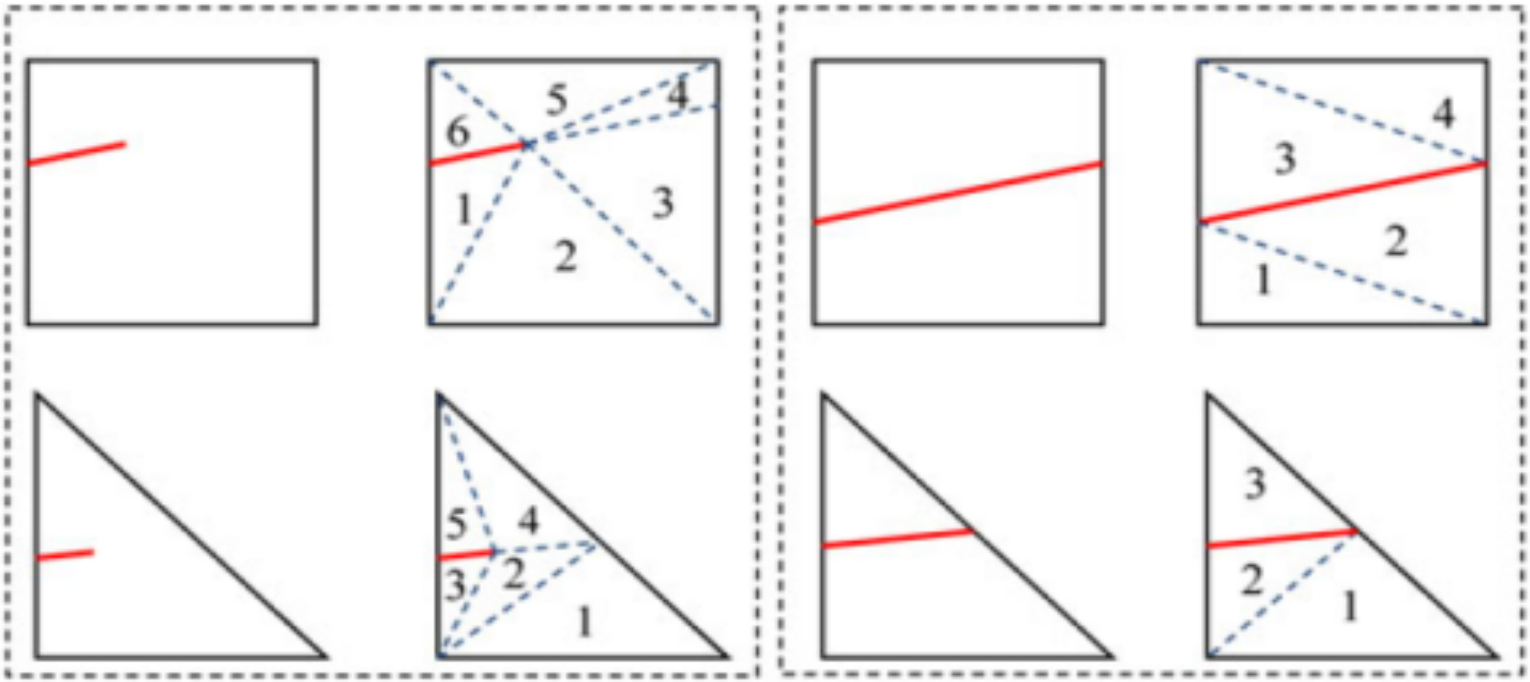
- Improving the accuracy of the results in XFEM:

  - By applying relevant enriching functions based on known fields behavior.

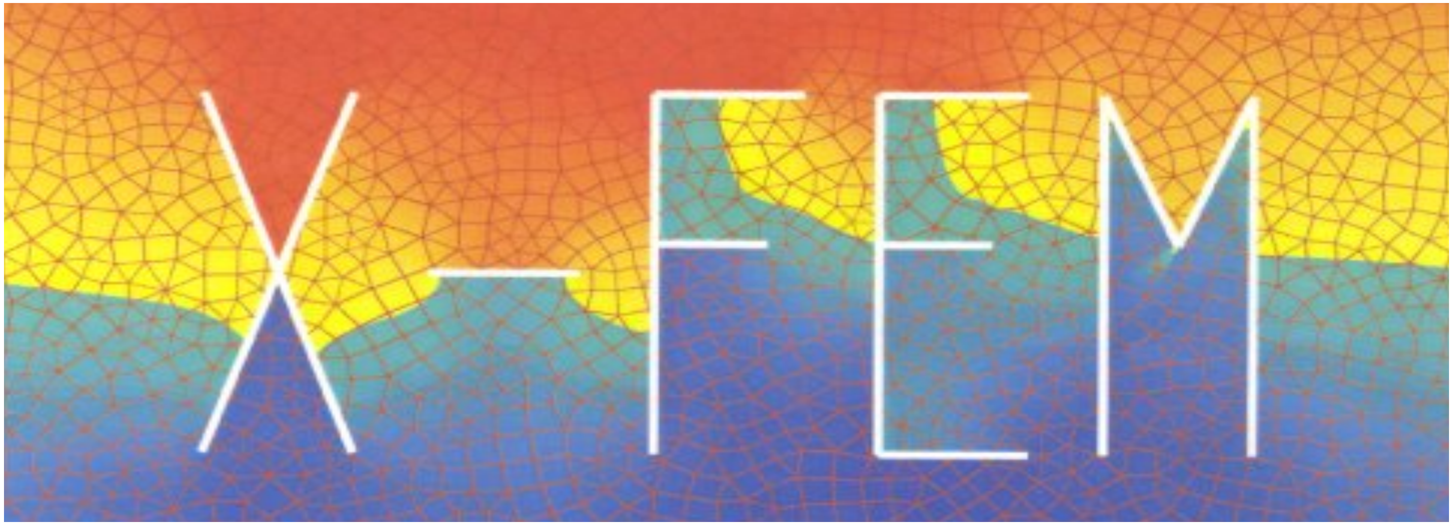
  - For cracks and dislocations in elastic media, well-known asymptotic singular near-field solutions for cracks in elastic media can be added as enrichments.

Introduction

- XFEM – Drawback
  - Using standard Gaussian or Monte Carlo integration results in numerical inaccuracies and creates ill-conditioned system.



Historical Perspective



Historical Perspective (XFEM)

- Belytsckho and Black (1999)
  - Solve cracks growth problems with minimal remeshing
  - Allowed the crack to be arbitrarily aligned within the mesh
  - Enriching FEM approximations with the two-dimensional plane strain asymptotic crack tip fields
- Moes et al. (1999) and Dolbow (1999)
  - Improvement and called as eXtended Finite Element Method (XFEM)
  - Introduced a much more elegant technique by adapting an enrichment (the asymptotic near-tip field and a Heaviside function)

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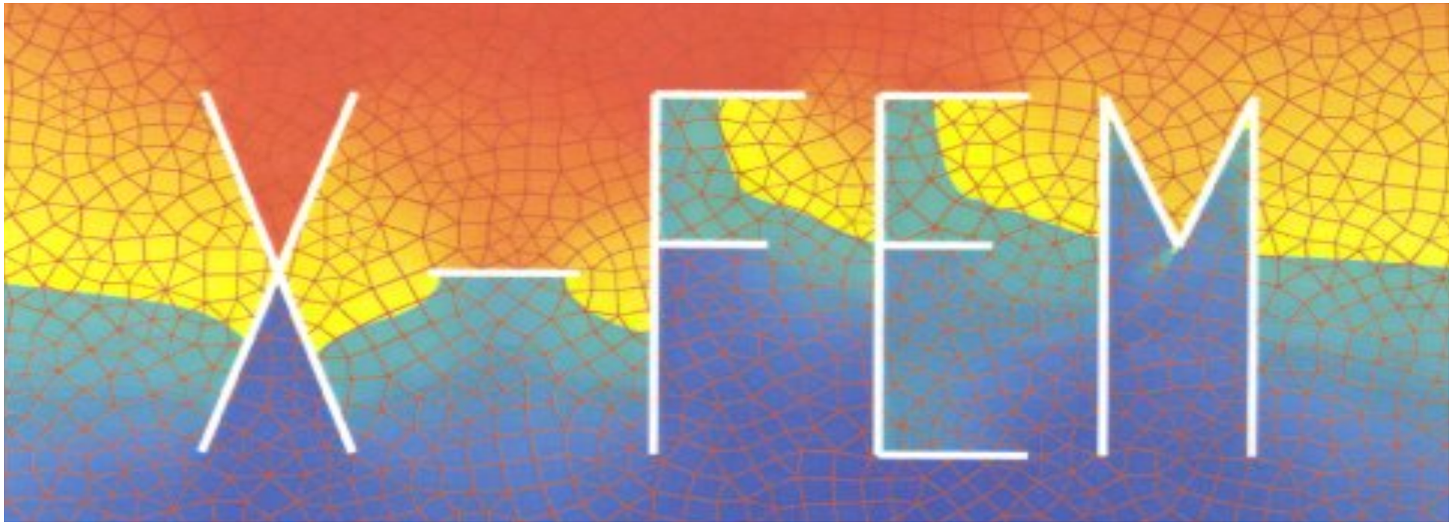
Historical Perspective (XFEM)

- Dolbow et al. (1999)
  - Modeling of cracks growth in plates in the Mindlin–Reissner framework.
- Sukumar et al. (2000)
  - Extended the XFEM for three-dimensional crack modeling
- Stolarska et al. (2001)
  - Coupling the level set method (LSM) with XFEM to model crack growth in two dimensional

Historical Perspective (XFEM)

- Dolbow et al. 2001 and Moes and Belytschko [2002]
  - Comprehensive model for cohesive crack growth in XFEM
- Wagner et al. (2001, 2003)
  - XFEM for the simulation of particulate flows
- Chessa amd Belytschko (2003)
  - XFEM with arbitrary interior discontinuous gradients to two-phase immiscible flow problems

Basic Facts



Basic Facts

FEM procedure

Step 1 Select the Element Type

(Bar element)

Step 2 Select a Displacement Function

Distribution of displacement within the element

$$(\hat{u} = a_1 + a_2\hat{x}) \rightarrow \hat{u} = \begin{bmatrix} 1 & \hat{x} \end{bmatrix} \begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix}$$

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- ✓ most common functions: polynomials

Express "u" as a function of the *nodal displacements*

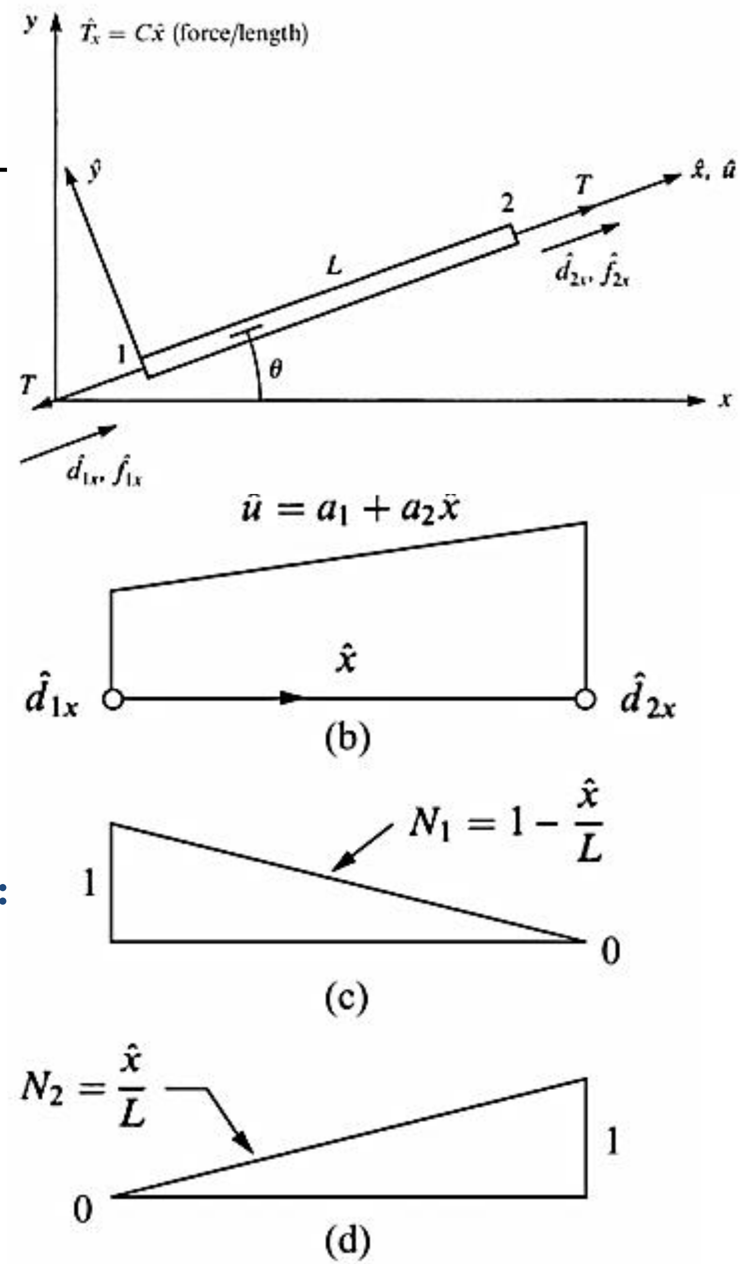
- ✓ Apply the physical boundary conditions:

$$\hat{u} = \left(\frac{\hat{d}_{2x} - \hat{d}_{1x}}{L} \right) \hat{x} + \hat{d}_{1x}$$

$$\hat{u} = \begin{bmatrix} 1 - \frac{\hat{x}}{L} & \frac{\hat{x}}{L} \end{bmatrix} \begin{Bmatrix} \hat{d}_{1x} \\ \hat{d}_{2x} \end{Bmatrix}$$

$$\hat{u} = \begin{bmatrix} N_1 & N_2 \end{bmatrix} \begin{Bmatrix} \hat{d}_{1x} \\ \hat{d}_{2x} \end{Bmatrix}$$

Shape function = Interpolation function



Basic Facts

FEM procedure

Step 3 Define the Strain/Displacement and Stress/Strain Relationships

$\varepsilon_x = d\hat{u}/d\hat{x}$ $\{\varepsilon_x\} = [B]\{\hat{d}\}$ $\{\sigma_x\} = [D]\{\varepsilon_x\}$

Step 4 Derive the Element Stiffness Matrix and Equations

$$\{\sigma_x\} = [D][B]\{\hat{d}\}$$

$$\begin{Bmatrix} \hat{f}_{1x} \\ \hat{f}_{2x} \end{Bmatrix} = \frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} \hat{d}_{1x} \\ \hat{d}_{2x} \end{Bmatrix}$$

Step 5 Assemble the Global matrix and apply the boundary conditions

Step 6 Solve for the Nodal Displacements

Step 7 Solve for the Element Forces

Basic Facts

XFEM is all about:

Step 2

Increasing the accuracy of the displacement function "u"

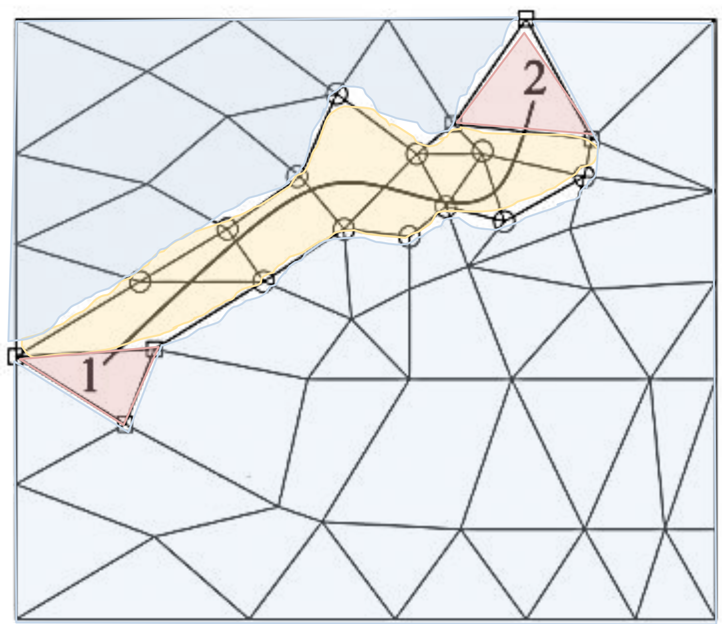
Basic Facts

Applies to all nodes in the model

Applies to nodes whose shape function support is cut by the crack interior

Applies to nodes whose shape function support is cut by the crack tip

$$\mathbf{u} = \sum_{I=1}^{N} N_I(x)[\mathbf{u}_I + H(x)\mathbf{a}_I + \sum_{\alpha=1}^{4} F_\alpha(x)\mathbf{b}_I^\alpha]$$



Basic Facts

This means:

No need to have nodes on the surfaces of the crack/interface

No need for remeshing

Crack/interface is element independent

Basic Facts

- Modeling the crack:

$$\mathbf{u}^h(\mathbf{x}) = \mathbf{u}^{\mathrm{FE}} + \mathbf{u}^{\mathrm{enr}} = \sum_{j=1}^{n} N_j(\mathbf{x})\mathbf{u}_j + \sum_{k=1}^{m} N_k(\mathbf{x})\psi(\mathbf{x})\mathbf{a}_k$$

Rearrangement by Moës et al. (1999)

$$\mathbf{u}^h(\mathbf{x}) = \sum_{j=1}^{n} N_j(\mathbf{x})\mathbf{u}_j + \sum_{h=1}^{m} N_h(\mathbf{x}) H(\xi(x))\mathbf{a}_h + \sum_{k=1}^{mt_1} N_k(\mathbf{x}) \left(\sum_{l=1}^{mf} F_l^1(x)\mathbf{b}_k^{l1} \right) + \sum_{k=1}^{mt_2} N_k(\mathbf{x}) \left(\sum_{l=1}^{mf} F_l^2(x)\mathbf{b}_k^{l2} \right)$$

$\psi(x)$: **the discontinuous enrichment function defined for the set of nodes that the discontinuity has in its influence (support) domain.**

m: **set of nodes that have the crack face (but not the crack tip) in their support domain,**

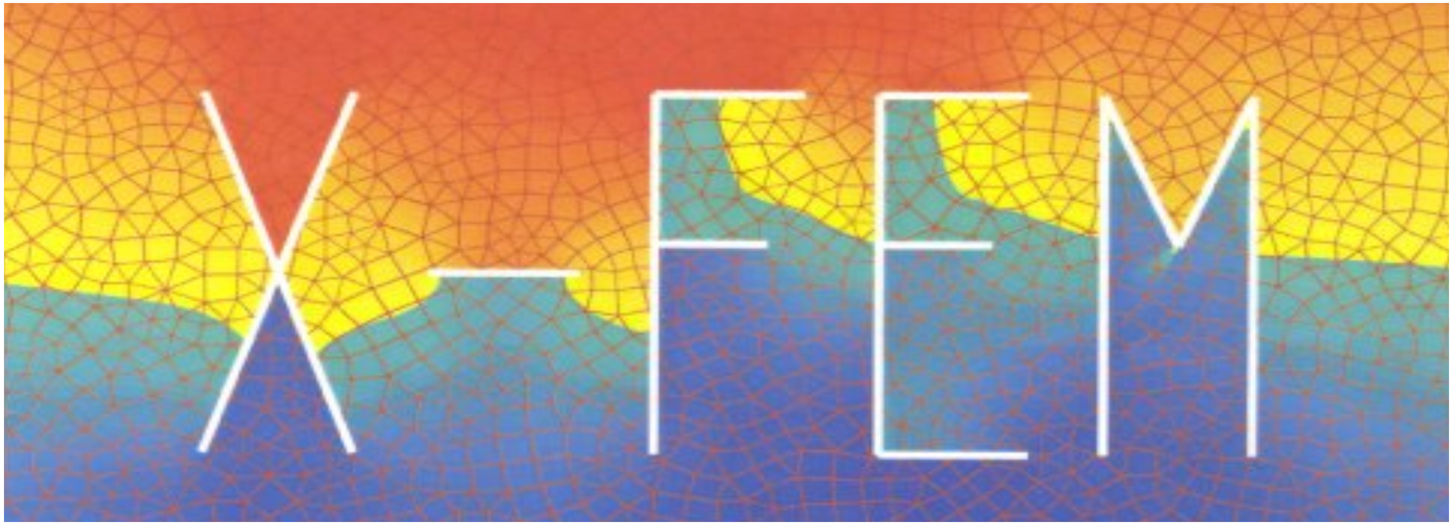
mt_1 & mt_2 : **Set of nodes associated with the crack tip 1 and 2 in their support domain.**

u_j: **the nodal displacements (standard degrees of freedom).**

a_h, b_k^1 and b_k^2: **vectors of additional nodal degrees of freedom for modelling crack faces and the two crack tips.**

$F_l^i(x)$, $i=1,2$: **Crack tip enrichment functions for mf nodes.**

Enrichment Functions



Enrichment Functions

- **Enrichment:**
  - **Simply targets higher accuracy of the approximation by including the information obtained from the analytical solution.**
  - **Example:**
    - **analytical near crack tip solutions for XFEM**

- **Displacement Function in FEM:**

$$\mathbf{u} = \sum_{j=1}^{n} \mathbf{N}_j \bar{\mathbf{u}}_j$$

  - In terms of the m **basis functions p**:

$$\mathbf{u} = \mathbf{p}^T \mathbf{a} = \sum_{k=1}^{m} p_k \mathbf{a}_k$$

$$\begin{cases} 1D: & \begin{cases} \mathbf{p}^T = \{1, x\} & \text{1st order} \\ \mathbf{p}^T = \{1, x, x^2\} & \text{2nd order} \end{cases} \\ 2D: & \begin{cases} \mathbf{p}^T = \{1, x, y\} & \text{1st order} \\ \mathbf{p}^T = \{1, x, y, x^2, xy, y^2\} & \text{2nd order} \end{cases} \end{cases}$$

Enrichment Functions

- **Ways of enriching an approximation:**

  1. **enriching the basis vector (intrinsic enrichment)**

  2. **enriching the approximation (extrinsic enrichment)**

Enrichment Functions

1.Enriching the basis vector (intrinsic enrichment):

- **Idea:** to enhance approximation by including new terms which are derived from a specific problem.

  - **Example:**

$$\mathbf{p} = \{\mathbf{p}^{\mathrm{lin}}, \mathbf{p}^{\mathrm{enr}}\} = \{1, x, y, f_1, f_2\}$$

  - $p^{lin} = \{1, x, y\}$: first-order standard linear basis function
  - $p^{enr} = \{f_1, f_2\}$: new enrichment terms

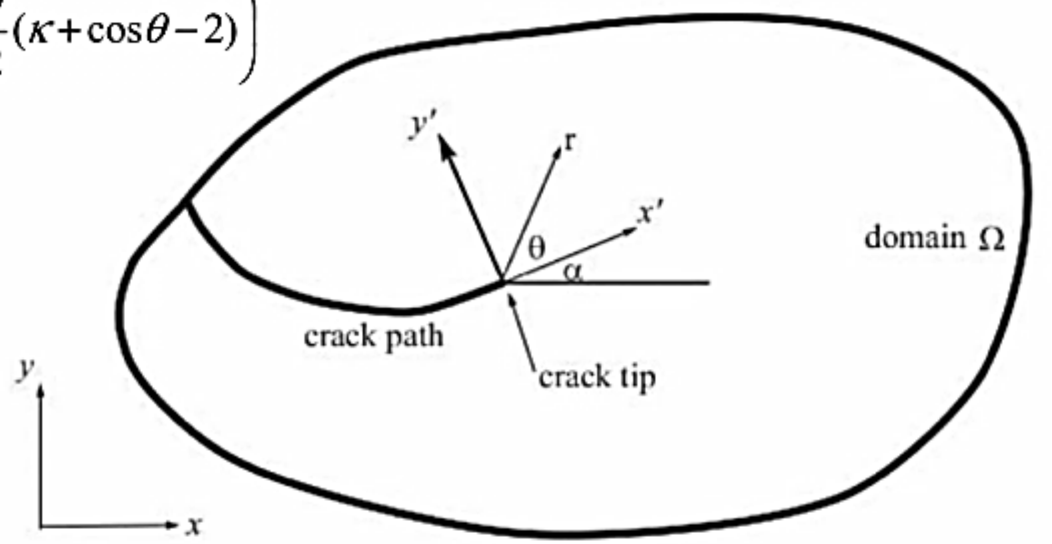
Enrichment Functions

1.enriching the basis vector (intrinsic enrichment):

- **The near tip displacement field can be written as:**

$$u_x = \frac{1}{\mu}\sqrt{\frac{r}{2\pi}}\left(K_{\mathrm{I}}\cos\frac{\theta}{2}(\kappa - \cos\theta) + K_{\mathrm{II}}\sin\frac{\theta}{2}(\kappa + \cos\theta + 2)\right)$$

$$u_y = \frac{1}{\mu}\sqrt{\frac{r}{2\pi}}\left(K_{\mathrm{I}}\sin\frac{\theta}{2}(\kappa - \cos\theta) - K_{\mathrm{II}}\cos\frac{\theta}{2}(\kappa + \cos\theta - 2)\right)$$



Enrichment Functions

1.enriching the basis vector (intrinsic enrichment):

- Near crack tip displacement field ***estimation***:

$$\mathbf{p}^{\mathrm{T}}(\mathbf{x}) = \left[1, x, y, \sqrt{r}\sin\frac{\theta}{2}, \sqrt{r}\cos\frac{\theta}{2}, \sqrt{r}\sin\theta\sin\frac{\theta}{2}, \sqrt{r}\sin\theta\cos\frac{\theta}{2} \right]$$

Enrichment Functions

2. enriching the approximation (extrinsic enrichment):

- Use of extrinsic (outside) bases $p_k(x)$ to increase the order of completeness

$$\mathbf{u}^h(\mathbf{x}) = \mathbf{u}^{FE} + \mathbf{u}^{enr} = \sum_{j=1}^{n} N_j(\mathbf{x})\mathbf{u}_j + \sum_{k=1}^{m} N_k(\mathbf{x})\psi(\mathbf{x})\mathbf{a}_k$$

- **Partition of unity finite element method (PUFEM)**
  - Classical finite element shape functions N_j (x) For all nodes.

$$\sum_{j=1}^{n} N_j(\mathbf{x}) = 1$$

$$\mathbf{u}^h(\mathbf{x}) = \sum_{j=1}^{n} N_j(\mathbf{x})\left(\mathbf{u}_j + \sum_{k=1}^{m} p_k(\mathbf{x})\mathbf{a}_{jk}\right)$$

- **Generalized finite element method**
  - Different shape functions are used.

$$\mathbf{u}^h(\mathbf{x}) = \sum_{j=1}^{n} N_j(\mathbf{x})\mathbf{u}_j + \sum_{j=1}^{n} \overline{N}_j(\mathbf{x})\left(\sum_{k=1}^{m} p_k(\mathbf{x})\mathbf{a}_{jk}\right)$$

Enrichment Functions

2. enriching the approximation (extrinsic enrichment):
   - PUFEM & GFEM: enrichments on global level and entire domain.
   - XFEM: enrichments on a local level
   - XFEM relies also on a number of meshless methods such as EFG and Hp-clouds.

- Now let's see what are XFEM extrinsic enrichment functions for modeling:
  1. Strong discontinuities (crack, fractures,…)
  2. Weak discontinuities (dramatic change in velocity and etc.)

Enrichment Functions

- **Modeling strong discontinuous fields**
  - **one-dimensional problem**
    - **Only nodes 2 and 3 are required to be enriched.**
    - **Nodes 1 and 4 are not influenced by the crack.**

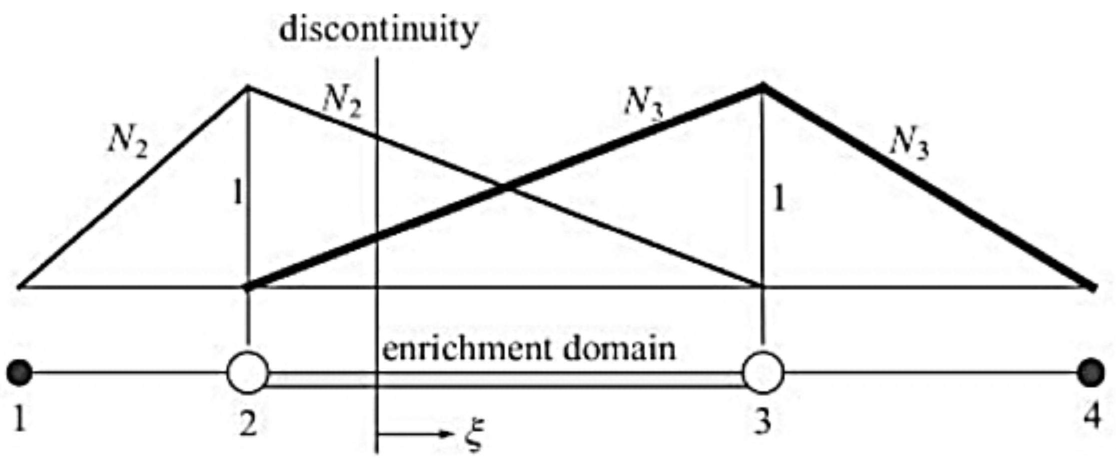


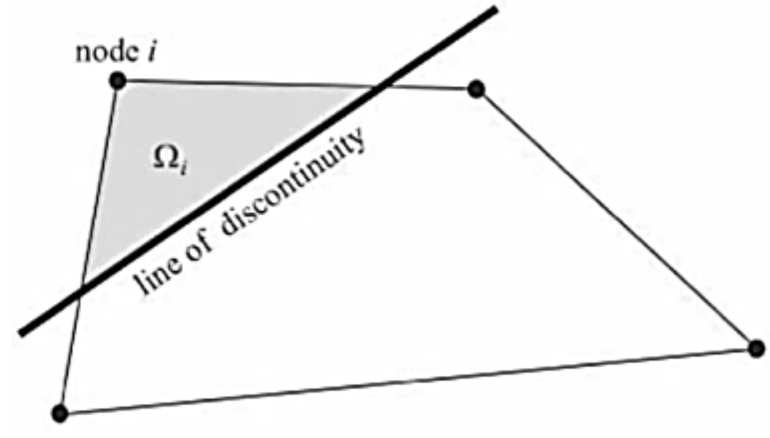
Figure 3.8 Simulation of a crack in a one-dimensional problem using the standard linear finite element shape functions.

Enrichment Functions

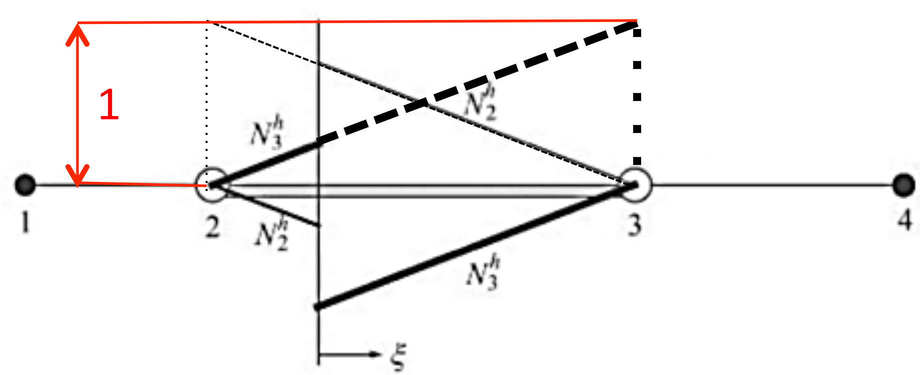
- **Modeling strong discontinuous fields**

1. First Jump (enriched shape) Function:

$$N_i^h = \begin{cases} N_i - 1 & \mathbf{x} \in \Omega_i \\ N_i & \mathbf{x} \notin \Omega_i \end{cases}$$

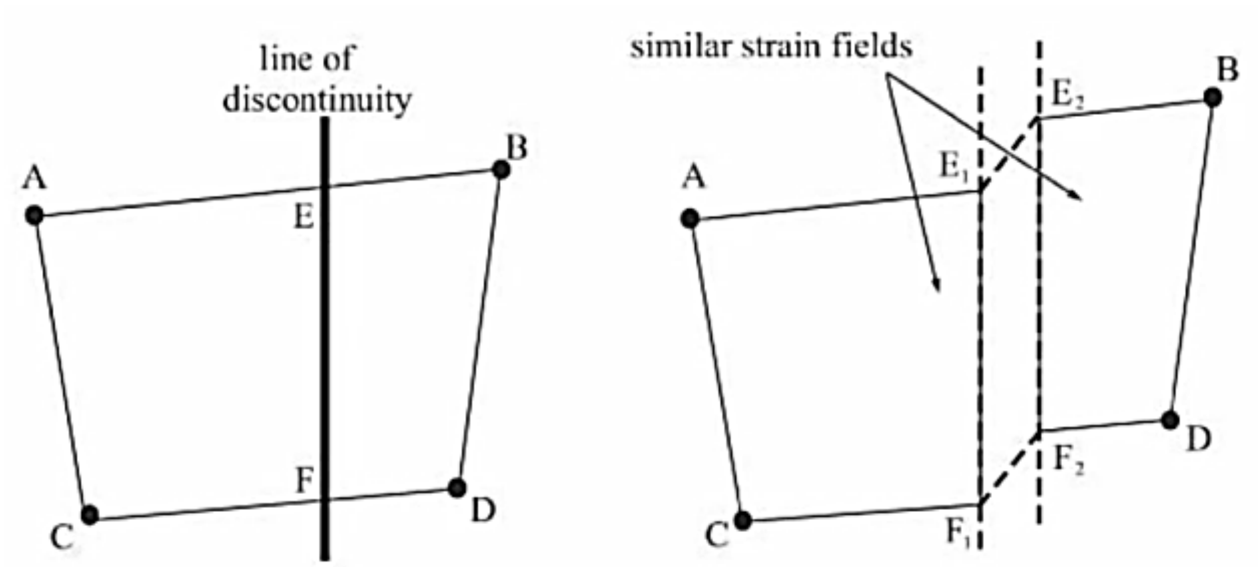


- N_i : the conventional finite element shape function
- Ω_i : part of the element in between the crack and node i



Enrichment Functions

- **Modeling strong discontinuous fields**
  - First Jump Function Problems:
    1. It provides **similar strain fields** in both sides of the discontinuity.
    2. Lower number of degrees of freedom required by approximation than other techniques.



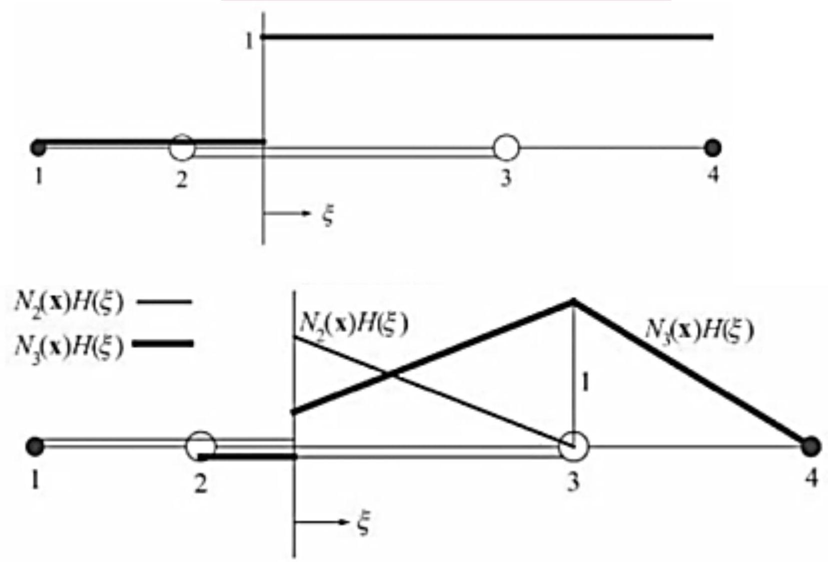
Enrichment Functions

- **Modeling strong discontinuous fields**

1. The Heaviside function

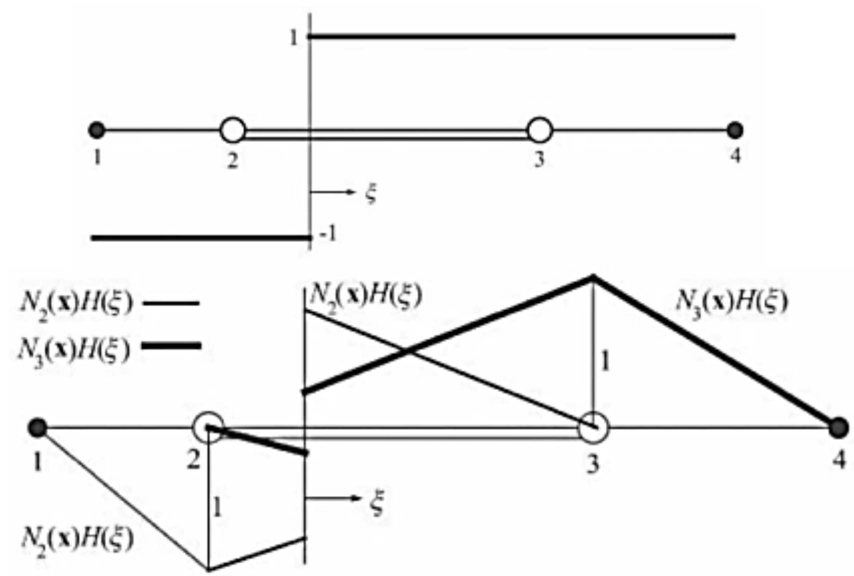
Type I: as a step function

$$H(\xi)=\begin{cases}1 & \forall \xi>0\\ 0 & \forall \xi<0\end{cases}$$



Type II: the signed function

$$H(\xi)=\operatorname{sign}(\xi)=\begin{cases}1 & \forall \xi>0\\ -1 & \forall \xi<0\end{cases}$$



Enrichment Functions

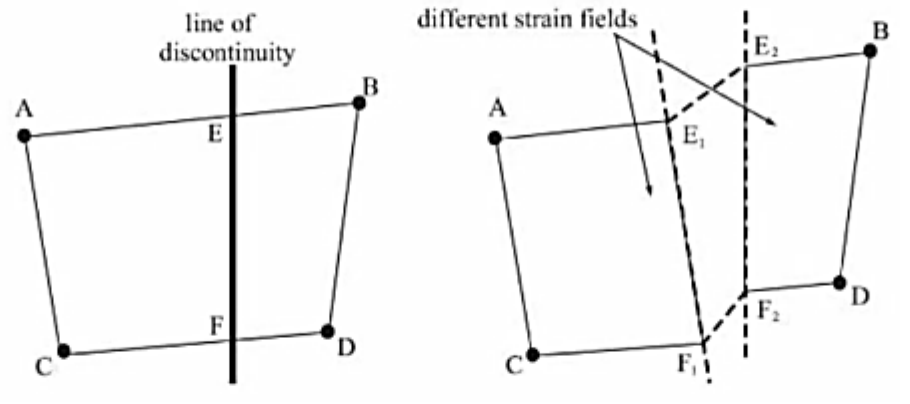
- **Modeling strong discontinuous fields**

2. The Heaviside function

$$\mathbf{u}^h(\mathbf{x}) = \mathbf{u}^{\mathrm{FE}} + \mathbf{u}^{\mathrm{enr}} = \sum_{j=1}^{n} N_j(\mathbf{x})\mathbf{u}_j + \sum_{k=1}^{m} N_k(\mathbf{x})\psi(\mathbf{x})\mathbf{a}_k$$

$$\mathbf{u}^h(\mathbf{x}) = \sum_{j=1}^{n} N_j(\mathbf{x})\mathbf{u}_j + \sum_{k=1}^{m} N_k(\mathbf{x})H(\xi)\mathbf{a}_k$$

- **Independent displacement fields for both sides of the crack.**



Enrichment Functions

- **Modeling weak discontinuous fields**
  - **Replacing the Heaviside function H(ξ) with a signed distance function χ(x) (Bordas and Legay 2005):**

$$\mathbf{u}^h(\mathbf{x}) = \sum_{j=1}^{n} N_j(\mathbf{x})\mathbf{u}_j + \sum_{k=1}^{m} N_k(\mathbf{x})\chi(\mathbf{x})\mathbf{a}_k$$

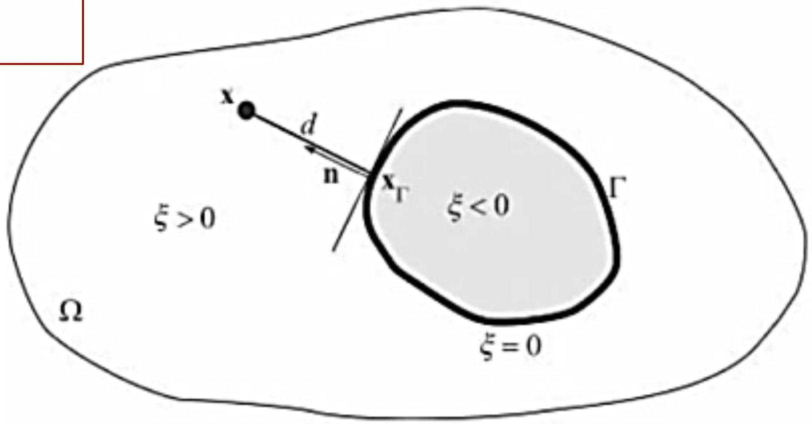
- χ (x) is the weak discontinuous enrichment function.

$$\xi(\mathbf{x}) = \underbrace{\min\|\mathbf{x}-\mathbf{x}_\Gamma\|}_{\mathbf{x}_\Gamma \in \Gamma} \, sign\big(\mathbf{n}\cdot(\mathbf{x}-\mathbf{x}_\Gamma)\big)$$

  - where n is the unit normal vector.
- **The distance d from a point x to an interface Γ:**

$$d = \|\mathbf{x}-\mathbf{x}_\Gamma\|$$

  - where $\mathbf{x}_\Gamma$ is the normal projection of x on Γ.



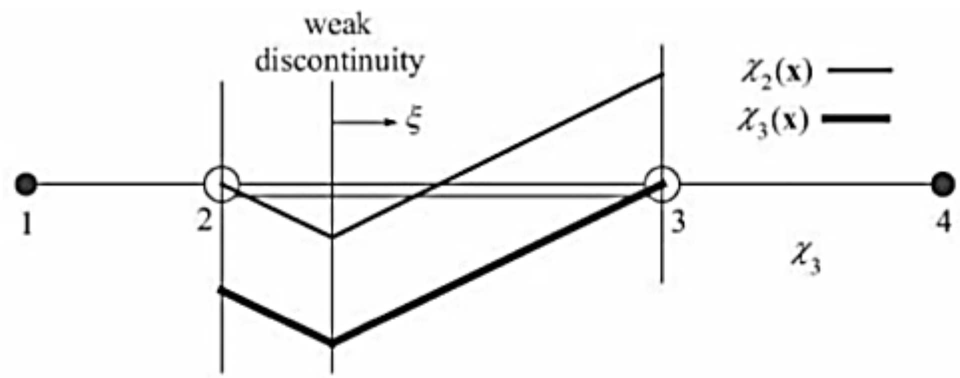
Enrichment Functions

- **Modeling weak discontinuous fields**

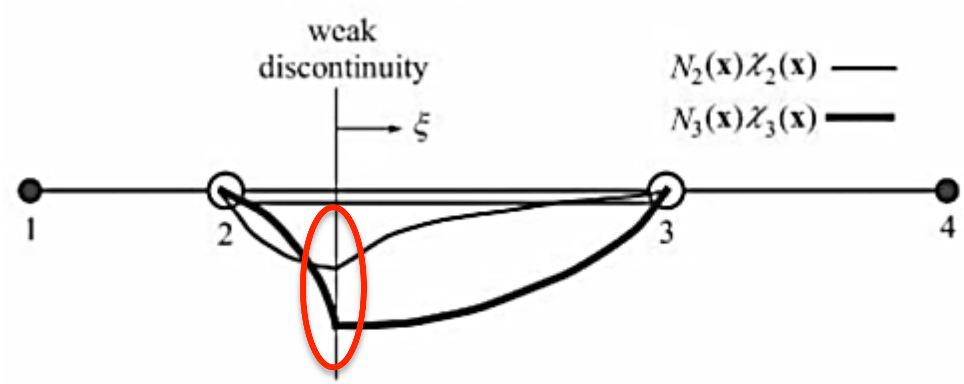
  - A kink in the displacement field is introduced.
  - A jump in its derivative, i.e. a discontinuity in the gradient of the function, is anticipated.

Example:

Velocity

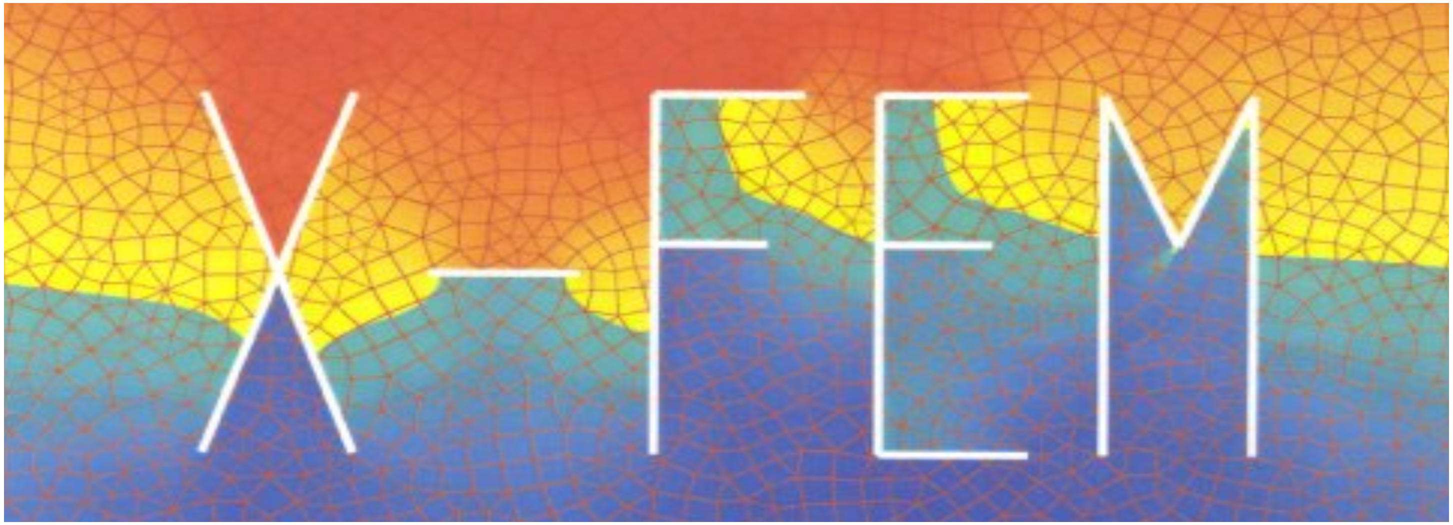


a) enrichment functions



b) enriched shape functions

Governing Equations



Governing Equations

strong form of the equilibrium equation:

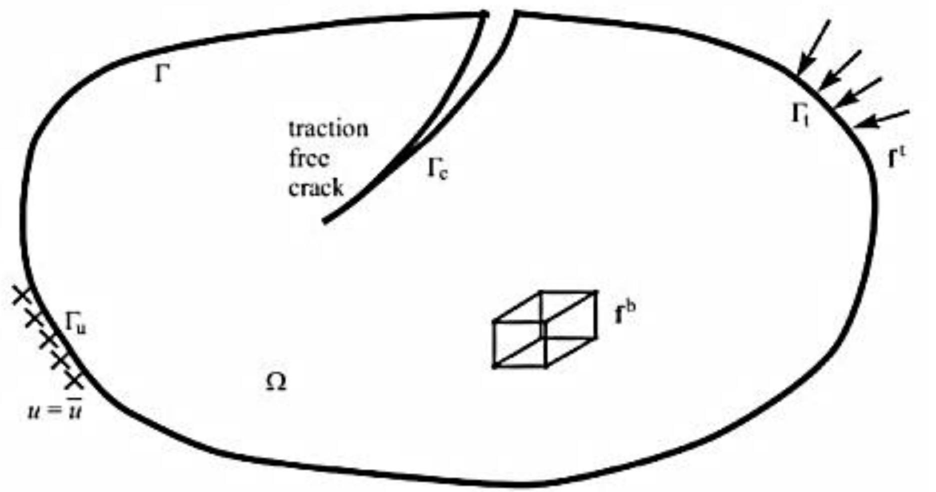
$$\nabla.\boldsymbol{\sigma} + \mathbf{f}^{b} = 0 \quad \text{in } \Omega$$

boundary conditions

$$\boldsymbol{\sigma}\cdot\mathbf{n} = \mathbf{f}^{t} \quad \text{on } \Gamma_t$$

$$\mathbf{u} = \bar{\mathbf{u}} \quad \text{on } \Gamma_u$$

$$\boldsymbol{\sigma}\cdot\mathbf{n} = 0 \quad \text{on } \Gamma_c$$



A body in a state of elastostatic equilibrium

- Γ_t: traction boundary
- Γ_u: displacement boundary
- Γ_c: crack boundary
- σ: stress tensor
- f^b: body force vector
- f^t: external traction vector

Governing Equations

- **Variational formulation of the boundary value problem**

$$\int_{\Omega} \boldsymbol{\sigma} \cdot \delta\boldsymbol{\varepsilon} \; d\Omega = \int_{\Omega} \mathbf{f}^{b} \cdot \delta\mathbf{u} \; d\Omega + \int_{\Gamma} \mathbf{f}^{t} \cdot \delta\mathbf{u} \; d\Gamma$$

Discretization using the XFEM procedure

discrete system of linear equilibrium equations $\mathbf{K}\mathbf{u}^{h} = \mathbf{f}$

K: stiffness matrix
u_h: vector of degrees of nodal freedom
(both classical and enriched)
f: vector of external force

Governing Equations

$$\mathbf{u}^h(\mathbf{x}) = \sum_{j=1}^{n} N_j(\mathbf{x})\mathbf{u}_j + \sum_{h=1}^{m} N_h(\mathbf{x})\big(H(\xi(x)) - H(\xi(x_h))\big)\mathbf{a}_h$$
$$+ \sum_{k=1}^{mt_1} N_k(\mathbf{x})\left(\sum_{l=1}^{mf}\big(F_l^1(x) - F_l^1(x_k)\big)\mathbf{b}_k^{l1}\right)$$
$$+ \sum_{k=1}^{mt_2} N_k(\mathbf{x})\left(\sum_{l=1}^{mf}\big(F_l^2(x) - F_l^2(x_k)\big)\mathbf{b}_k^{l2}\right)$$

$$\mathbf{K}\mathbf{u}^h = \mathbf{f}$$

For Element "e"

$$\mathbf{K}_{ij}^e = \begin{bmatrix} \mathbf{K}_{ij}^{\mathbf{uu}} & \mathbf{K}_{ij}^{\mathbf{ua}} & \mathbf{K}_{ij}^{\mathbf{ub}} \\ \mathbf{K}_{ij}^{\mathbf{au}} & \mathbf{K}_{ij}^{\mathbf{aa}} & \mathbf{K}_{ij}^{\mathbf{ab}} \\ \mathbf{K}_{ij}^{\mathbf{bu}} & \mathbf{K}_{ij}^{\mathbf{ba}} & \mathbf{K}_{ij}^{\mathbf{bb}} \end{bmatrix}$$

$$\mathbf{u}^h = \{\mathbf{u} \quad \mathbf{a} \quad \mathbf{b}_1 \quad \mathbf{b}_2 \quad \mathbf{b}_3 \quad \mathbf{b}_4\}^{\mathrm{T}}$$

$$\mathbf{f}_i^e = \left\{\mathbf{f}_i^u \quad \mathbf{f}_i^{\mathbf{a}} \quad \mathbf{f}_i^{\mathbf{b}1} \quad \mathbf{f}_i^{\mathbf{b}2} \quad \mathbf{f}_i^{\mathbf{b}3} \quad \mathbf{f}_i^{\mathbf{b}4}\right\}^{\mathrm{T}}$$

$$\mathbf{K}_{ij}^{rs} = \int_{\Omega^e} (\mathbf{B}_i^r)^{\mathrm{T}} \mathbf{D} \mathbf{B}_j^s \, \mathrm{d}\Omega \qquad (r, s = \mathbf{u}, \mathbf{a}, \mathbf{b})$$

$$\mathbf{f}_i^{\mathbf{u}} = \int_{\Gamma_t} N_i \mathbf{f}^{\mathrm{t}} \, \mathrm{d}\Gamma + \int_{\Omega^e} N_i \mathbf{f}^{\mathrm{b}} \, \mathrm{d}\Omega$$

$$\mathbf{f}_i^{\mathbf{a}} = \int_{\Gamma_t} N_i H \, \mathbf{f}^{\mathrm{t}} \, \mathrm{d}\Gamma + \int_{\Omega^e} N_i H \, \mathbf{f}^{\mathrm{b}} \, \mathrm{d}\Omega$$

$$\mathbf{f}_i^{\mathbf{b}\alpha} = \int_{\Gamma_t} N_i F_\alpha \, \mathbf{f}^{\mathrm{t}} \, \mathrm{d}\Gamma + \int_{\Omega^e} N_i F_\alpha \, \mathbf{f}^{\mathrm{b}} \, \mathrm{d}\Omega \qquad (\alpha = 1, 2, 3 \text{ and } 4)$$

Governing Equations

$$\mathbf{K}_{ij}^{rs} = \int_{\Omega^e} (\mathbf{B}_i^r)^{\mathrm{T}} \mathbf{D} \mathbf{B}_j^s \, \mathrm{d}\Omega \qquad (r, s = \mathbf{u}, \mathbf{a}, \mathbf{b})$$

$$\mathbf{B}_i^{\mathbf{u}} = \begin{bmatrix} N_{i,x} & 0 \\ 0 & N_{i,y} \\ N_{i,y} & N_{i,x} \end{bmatrix}$$

$$\mathbf{B}_i^{\mathbf{a}} = \begin{bmatrix} (N_i H)_{,x} & 0 \\ 0 & (N_i H)_{,y} \\ (N_i H)_{,y} & (N_i H)_{,x} \end{bmatrix}$$

$$\mathbf{B}_i^{\mathbf{b}} = \begin{bmatrix} \mathbf{B}_i^{\mathbf{b}1} & \mathbf{B}_i^{\mathbf{b}2} & \mathbf{B}_i^{\mathbf{b}3} & \mathbf{B}_i^{\mathbf{b}4} \end{bmatrix}$$

$$\mathbf{B}_i^{\alpha} = \begin{bmatrix} (N_i F_\alpha)_{,x} & 0 \\ 0 & (N_i F_\alpha)_{,y} \\ (N_i F_\alpha)_{,y} & (N_i F_\alpha)_{,x} \end{bmatrix} \qquad (\alpha = 1, 2, 3 \text{ and } 4)$$

Governing Equations

including the effects of interpolation

$$H_{,i}(\xi) = \begin{cases} 1 & \text{at crack tip} \\ 0 & \text{otherwise} \end{cases}$$

$$\mathbf{B}_i^{\mathbf{a}} = \begin{bmatrix} N_{i,x}H & 0 \\ 0 & N_{i,y}H \\ N_{i,y}H & N_{i,x}H \end{bmatrix}$$

$$H(\xi) - H(\xi_i)$$

Dirac delta function

$$\psi(x) = H(\xi)$$

Strong Discontinuity Function

$$\mathbf{B}_i^{\mathbf{a}} = \begin{bmatrix} (N_i H)_{,x} & 0 \\ 0 & (N_i H)_{,y} \\ (N_i H)_{,y} & (N_i H)_{,x} \end{bmatrix}$$

$$\psi(\mathbf{x}) = \chi(\mathbf{x}) = |\xi(\mathbf{x})|$$

Weak Discontinuity Function

$$\xi(\mathbf{x}) = \sum_{j=1}^{4} N_j(\mathbf{x})\, \xi_j$$

$$\psi_{,i}(\mathbf{x}) = sign(\xi)\, \xi_{,i}(\mathbf{x})$$

$$\xi_{,i}(\mathbf{x}) = \sum_{j=1}^{4} N_{j,i}(\mathbf{x})\, \xi_j$$

Governing Equations

$$\mathbf{B}_i^{\alpha} = \begin{bmatrix} (N_i F_\alpha)_{,x} & 0 \\ 0 & (N_i F_\alpha)_{,y} \\ (N_i F_\alpha)_{,y} & (N_i F_\alpha)_{,x} \end{bmatrix} \qquad (\alpha = 1,2,3 \text{ and } 4)$$

$$F_\alpha(r,\theta) = \left\{ \sqrt{r}\sin\frac{\theta}{2}, \sqrt{r}\cos\frac{\theta}{2}, \sqrt{r}\sin\theta\sin\frac{\theta}{2}, \sqrt{r}\sin\theta\cos\frac{\theta}{2} \right\}$$

Local crack tip polar coordinates

$$F_{1,r} = \frac{1}{2\sqrt{r}}\sin\frac{\theta}{2}, \qquad F_{1,\theta} = \frac{\sqrt{r}}{2}\cos\frac{\theta}{2}$$

$$F_{2,r} = \frac{1}{2\sqrt{r}}\cos\frac{\theta}{2}, \qquad F_{2,\theta} = -\frac{\sqrt{r}}{2}\sin\frac{\theta}{2}$$

$$F_{3,r} = \frac{1}{2\sqrt{r}}\sin\frac{\theta}{2}\sin\theta, \quad F_{3,\theta} = \sqrt{r}\left(\frac{1}{2}\cos\frac{\theta}{2}\sin\theta + \sin\frac{\theta}{2}\cos\theta\right)$$

$$F_{4,r} = \frac{1}{2\sqrt{r}}\cos\frac{\theta}{2}\sin\theta, \quad F_{4,\theta} = \sqrt{r}\left(-\frac{1}{2}\sin\frac{\theta}{2}\sin\theta + \cos\frac{\theta}{2}\cos\theta\right)$$

Local crack tip coordinates (x',y')

$$F_{1,x'} = -\frac{1}{2\sqrt{r}}\sin\frac{\theta}{2}, \qquad F_{1,y'} = \frac{1}{2\sqrt{r}}\cos\frac{\theta}{2}$$

$$F_{2,x'} = \frac{1}{2\sqrt{r}}\cos\frac{\theta}{2}, \qquad F_{2,y'} = \frac{1}{2\sqrt{r}}\sin\frac{\theta}{2}$$

$$F_{3,x'} = -\frac{1}{2\sqrt{r}}\sin\frac{3\theta}{2}\sin\theta, \quad F_{3,y'} = \frac{1}{2\sqrt{r}}\left(\sin\frac{\theta}{2} + \sin\frac{3\theta}{2}\cos\theta\right)$$

$$F_{4,x'} = -\frac{1}{2\sqrt{r}}\cos\frac{3\theta}{2}\sin\theta, \quad F_{4,y'} = \frac{1}{2\sqrt{r}}\left(\cos\frac{\theta}{2} + \cos\frac{3\theta}{2}\cos\theta\right)$$

Governing Equations

$$\mathbf{B}_i^{\alpha} = \begin{bmatrix} (N_i F_\alpha)_{,x} & 0 \\ 0 & (N_i F_\alpha)_{,y} \\ (N_i F_\alpha)_{,y} & (N_i F_\alpha)_{,x} \end{bmatrix} \qquad (\alpha = 1,2,3 \text{ and } 4)$$

$$F_\alpha(r,\theta) = \left\{ \sqrt{r}\sin\frac{\theta}{2}, \sqrt{r}\cos\frac{\theta}{2}, \sqrt{r}\sin\theta\sin\frac{\theta}{2}, \sqrt{r}\sin\theta\cos\frac{\theta}{2} \right\}$$

Local crack tip coordinates (x',y')

$$F_{1,x'} = -\frac{1}{2\sqrt{r}}\sin\frac{\theta}{2}, \qquad F_{1,y'} = \frac{1}{2\sqrt{r}}\cos\frac{\theta}{2}$$

$$F_{2,x'} = \frac{1}{2\sqrt{r}}\cos\frac{\theta}{2}, \qquad F_{2,y'} = \frac{1}{2\sqrt{r}}\sin\frac{\theta}{2}$$

$$F_{3,x'} = -\frac{1}{2\sqrt{r}}\sin\frac{3\theta}{2}\sin\theta, \qquad F_{3,y'} = \frac{1}{2\sqrt{r}}\left(\sin\frac{\theta}{2} + \sin\frac{3\theta}{2}\cos\theta\right)$$

$$F_{4,x'} = -\frac{1}{2\sqrt{r}}\cos\frac{3\theta}{2}\sin\theta, \qquad F_{4,y'} = \frac{1}{2\sqrt{r}}\left(\cos\frac{\theta}{2} + \cos\frac{3\theta}{2}\cos\theta\right)$$

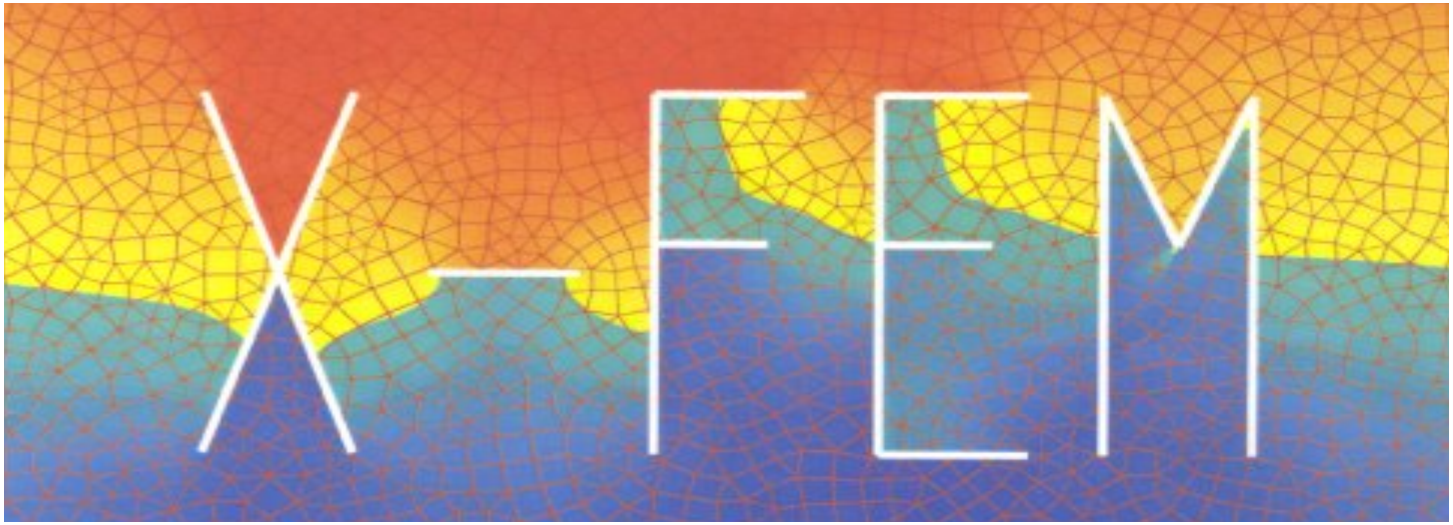
Global coordinate system (x,y)

$$F_{\alpha,x} = F_{\alpha,x'}\cos(\alpha) - F_{\alpha,y'}\sin(\alpha)$$

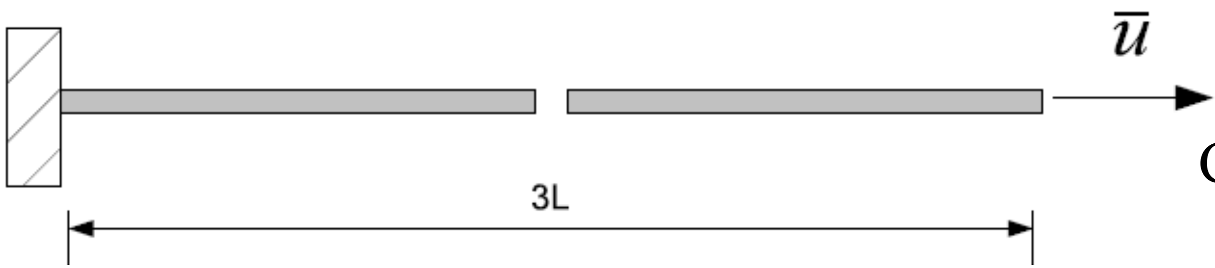
$$F_{\alpha,y} = F_{\alpha,x'}\sin(\alpha) + F_{\alpha,y'}\cos(\alpha)$$

α: angle of crack path with respect to the x axis

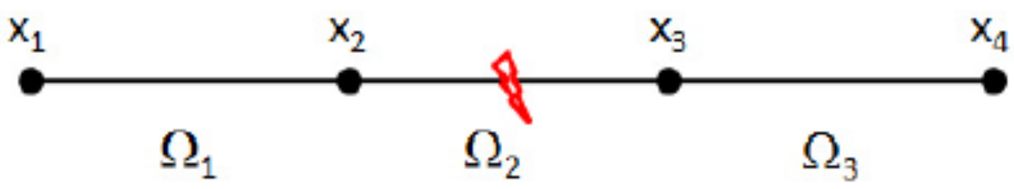
Hand-Calculation Example



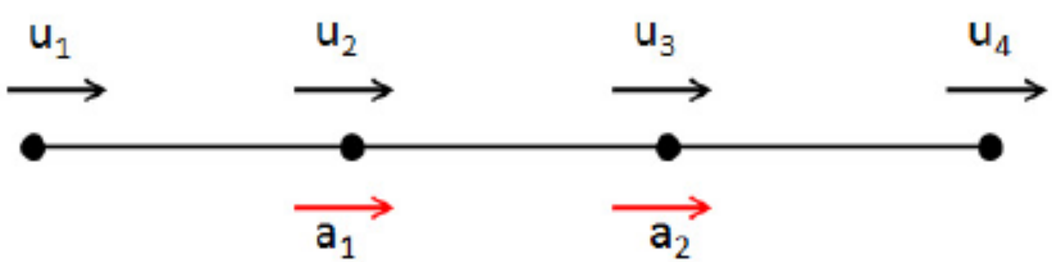
Hand-Calculation Example



Cracked 1D bar element

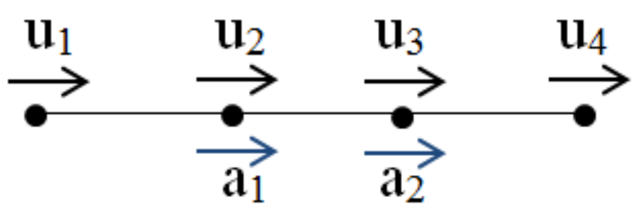


XFEM discretized domain

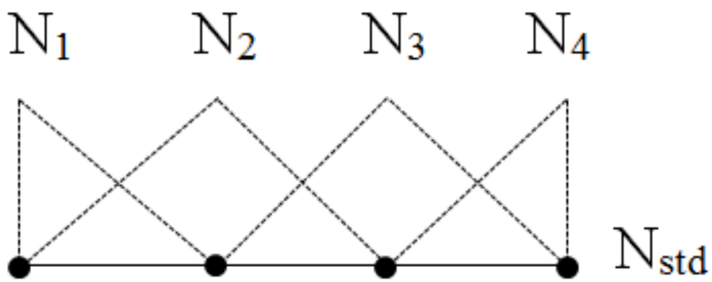


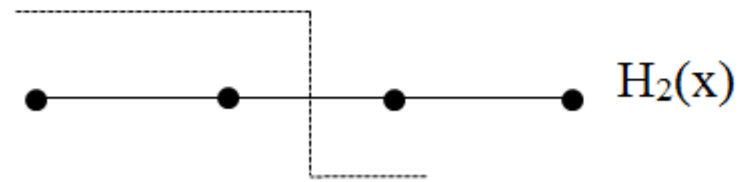
Standard & enriched degrees of freedom associated with each node

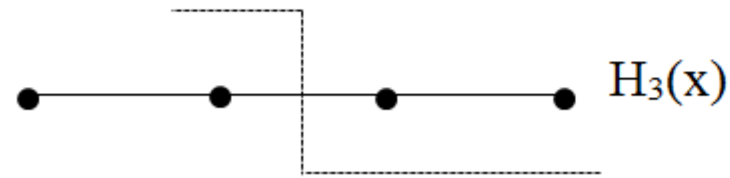
Hand-Calculation Example



$$u(x) = N_i u_i + HN_i a_j$$

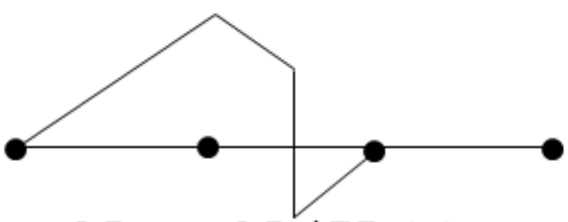




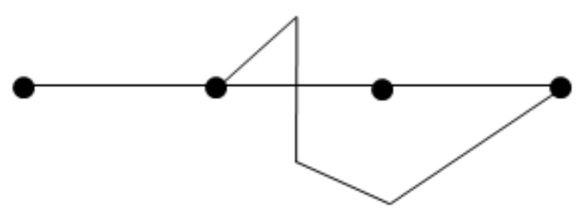


$$N_i = N_{std}^u$$

$$HN_i = N_{enr}^a$$



$N_{enr2} = N_2 * H_2(x)$



$N_{enr3} = N_3 * H_3(x)$

Hand-Calculation Example

- XFEM stiffness matrix for any of elements

$$K_e = \begin{bmatrix} K_{uu} & K_{ua} \\ K_{au} & K_{aa} \end{bmatrix}$$

$$\begin{aligned} K_{uu} &= \int_0^L (B^u_{std})^T D B^u_{std}\, dx \\ K_{ua} &= \int_0^L (B^u_{std})^T D B^a_{enr}\, dx \\ K_{ua} &= \int_0^L (B^a_{enr})^T D B^u_{std}\, dx \\ K_{aa} &= \int_0^L (B^a_{enr})^T D B^a_{enr}\, dx \end{aligned}$$

Hand-Calculation Example

- Element No.1 , H(X) = +1

$$K_{ua} = EA\int_0^L (B^u_{std})^T B^a_{enr}\, dx = \frac{EA}{L}\begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$= K_{ua}^T \left[\frac{x}{L} \right]$$

$$= \left[\frac{1}{L} \right]$$

$$K_{aa} = EA\int_0^L (B^a_{enr})^T B^a_{enr}\, dx = \frac{EA}{L}$$

$$K_{uu} = EA\int_0^L (B^u_{std})^T B^u_{std}\, dx = \frac{EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

Hand-Calculation Example

- Element No.1 , H(X) = +1

$$K_{au} = K_{ua}^T = \frac{EA}{L}\begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$K_e = \begin{bmatrix} K_{uu} & K_{ua} \\ K_{au} & K_{aa} \end{bmatrix}$$

$$K_{aa} = \frac{EA}{L}$$

$$K_{uu} = \frac{EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$K_{e1} = \frac{EA}{L}\begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & 1 \\ -1 & 1 & 1 \end{bmatrix}$$

Hand-Calculation Example

- Element No.3 , H(X) = -1

$$K_{ua} = EA \int^{L} (B^{u}_{\cdot\cdot})^{T} B^{a} \quad N^{a}_{enr} = H \left[1 - \cdots \right] = K_{ua}^{T} \left[\frac{x}{L} - 1 \right]$$

$$K_{aa} = E\cdots \quad K_{e3} = \frac{EA}{L} \begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & 1 \\ -1 & 1 & 1 \end{bmatrix} \quad \left. \frac{-1}{L} \right] = \left[\frac{1}{L} \right]$$

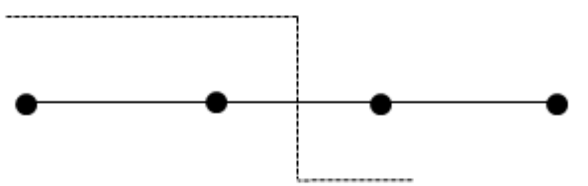
$$K_{uu} = EA \int_{0}^{L} (B^{u}_{std})^{T} B^{u}_{std} \, dx = \frac{EA}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

Hand-Calculation Example

- Element No.2

$$N^u_{std} = \begin{bmatrix} 1 - \frac{x}{L} & \frac{x}{L} \end{bmatrix} \qquad N^a_{enr} = H \begin{bmatrix} 1 - \frac{x}{L} & \frac{x}{L} \end{bmatrix}$$

$$B^u_{std} = \begin{bmatrix} \frac{-1}{L} & \frac{1}{L} \end{bmatrix} \qquad B^a_{enr} = H \begin{bmatrix} \frac{-1}{L} & \frac{1}{L} \end{bmatrix}$$



$$H(X) = \begin{Bmatrix} -1 & for\ x \in \Omega_2^+ \\ +1 & for\ x \in \Omega_2^- \end{Bmatrix}$$

$$K_{ua} = K^+_{ua} + K^-_{ua}$$

$$K_{au} = K^+_{au} + K^-_{aa}$$

$$K_{aa} = K^+_{aa} + K^-_{aa}$$

Hand-Calculation Example

Integrating on Ω_2^+

Use H(X) = -1

$$K_{ua}^{+} = EA\int_{L/2}^{L}(B_{std}^{u})^{T}B_{enr}^{a}\,dx = \frac{EA}{2L}\begin{bmatrix}-1 & 1\\ 1 & -1\end{bmatrix}$$

$$K_{aa}^{+} = EA\int_{L/2}^{L}(B_{enr}^{a})^{T}B_{enr}^{a}\,dx = \frac{EA}{2L}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$$

$$K_{uu} = EA\int_{0}^{L}(B_{std}^{u})^{T}B_{std}^{u}\,dx = \frac{EA}{L}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$$

Hand-Calculation Example

Integrating on Ω_2^-

Use H(X) = +1

$$K_{ua}^- = EA \int_0^{L/2} (B_{std}^u)^T B_{enr}^a \, dx = \frac{EA}{2L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$K_{aa}^- = EA \int_0^{L/2} (B_{enr}^a)^T B_{enr}^a \, dx = \frac{EA}{2L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

Hand-Calculation Example

- Element No.2

$$K_{ua} = K_{ua}^{+} + K_{ua}^{-}$$

$$K_{au} = K_{au}^{+} + K_{aa}^{-}$$

$$K_{aa} = K_{aa}^{+} + K_{aa}^{-}$$

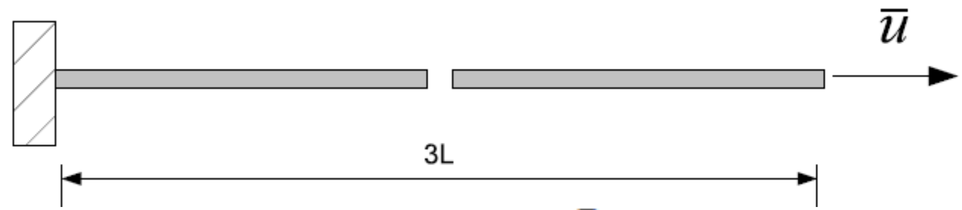
$$K_{ua} = \frac{EA}{2L}\Big[\quad K_{ua}^{T}$$

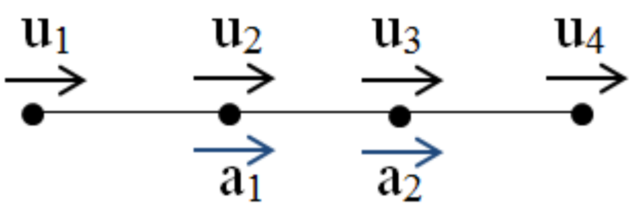
$$K_{e2} = \frac{EA}{L}\begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

$$K_{aa} = \frac{EA}{2L}\begin{bmatrix} \cdots \\ -1-1 \quad 1+1 \end{bmatrix} \begin{bmatrix} \cdots \\ -1 \quad 1 \end{bmatrix}$$

Hand-Calculation Example

- Stiffness matrix





$$\begin{bmatrix} 1 & -1 & 0 & 0 & -1 & 0 \\ -1 & 2 & -1 & 0 & 1 & 0 \\ & & & -1 & 0 & -1 \\ & & & 1 & 0 & 1 \\ & & & 0 & 2 & -1 \\ & & & & & 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 & 1 & 0 \\ -1 & 2 & 0 & -1 \\ 1 & 0 & 2 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} u_2 \\ u_3 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 0 \\ \bar{u} \\ 0 \\ -\bar{u} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{\bar{u}}{2} \\ \frac{\bar{u}}{2} \\ \bar{u} \\ -\frac{\bar{u}}{2} \\ -\frac{\bar{u}}{2} \end{bmatrix}$$

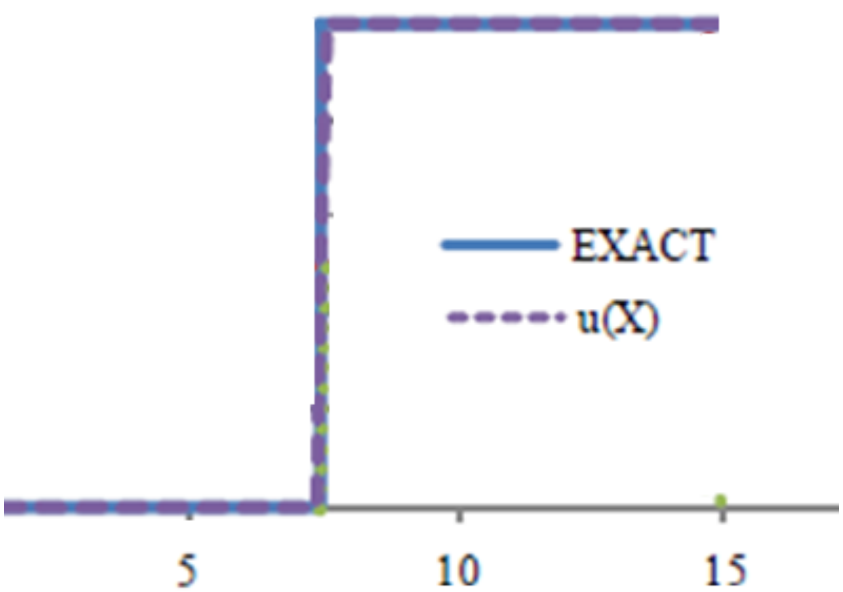
Hand-Calculation Example

- Displacements $u(x) = N_i u_i + HN_i a_j$

$$N_i = N^u_{std}$$

$$HN_i = N^a_{enr}$$

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{\bar{u}}{2} \\ \frac{\bar{u}}{2} \\ \bar{u} \\ -\frac{\bar{u}}{2} \\ -\frac{\bar{u}}{2} \end{bmatrix}$$



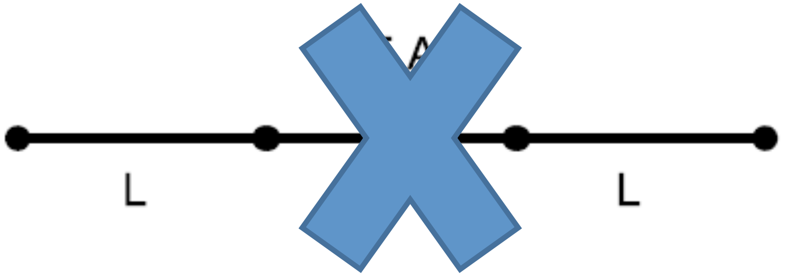
$$\frac{\bar{u}}{2} - \frac{\bar{u}}{2} = 0$$

$$\frac{\bar{u}}{2} + (-1)\left(-\frac{\bar{u}}{2}\right) = \bar{u}$$

Numerical solution of displacement field using XFEM

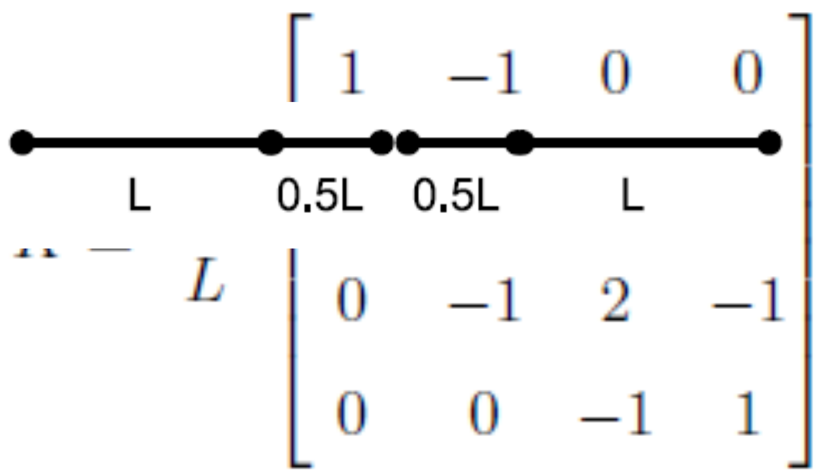
Hand-Calculation Example

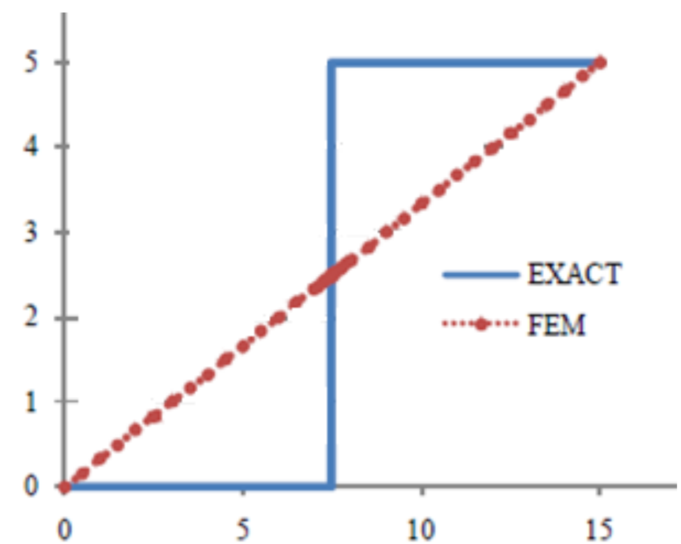
- FEM



FEM mesh discretization

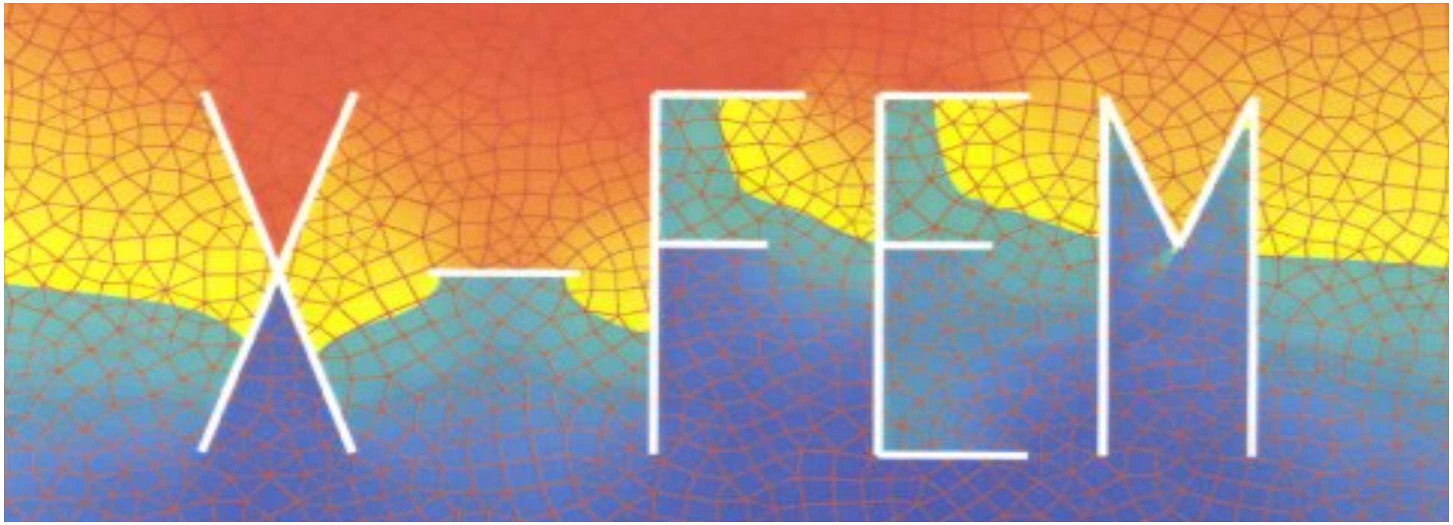
Degrees of freedom for each node





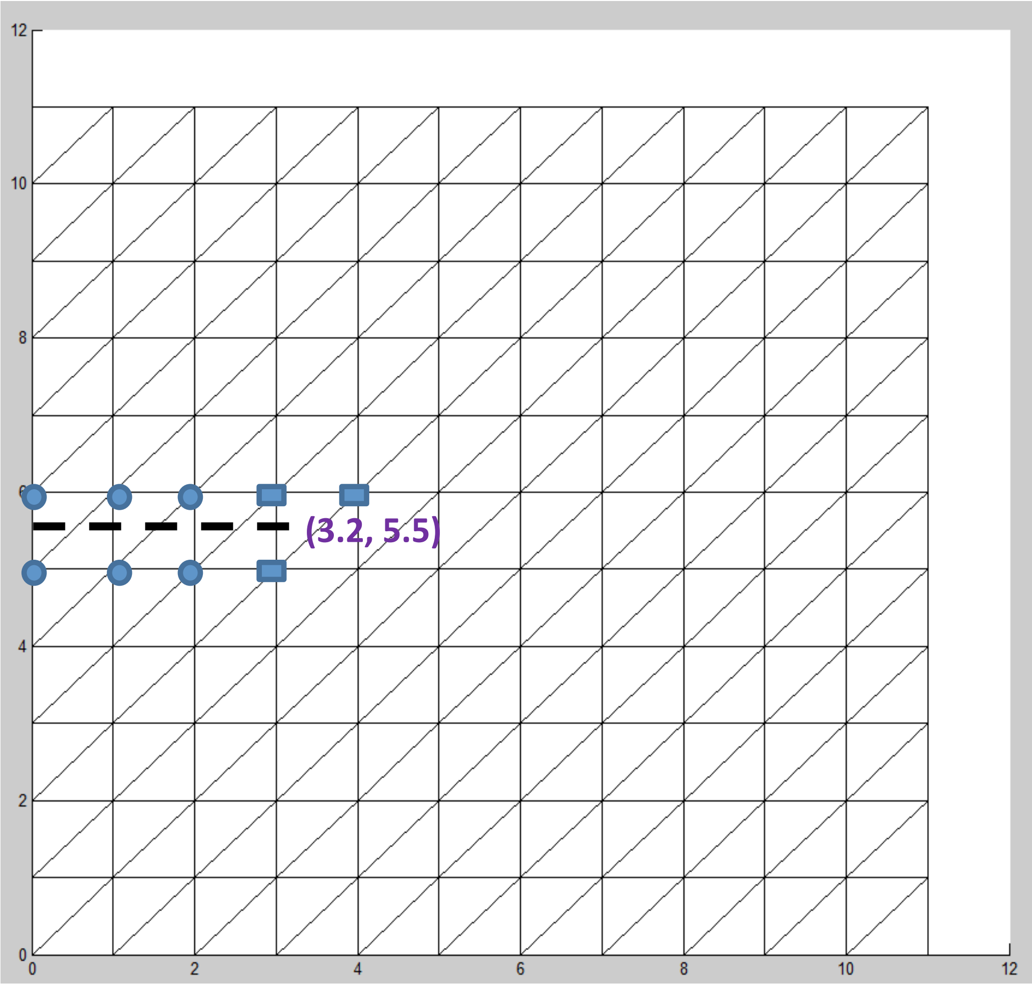
Numerical Example

- 2D Stationary Crack -



Numerical Example

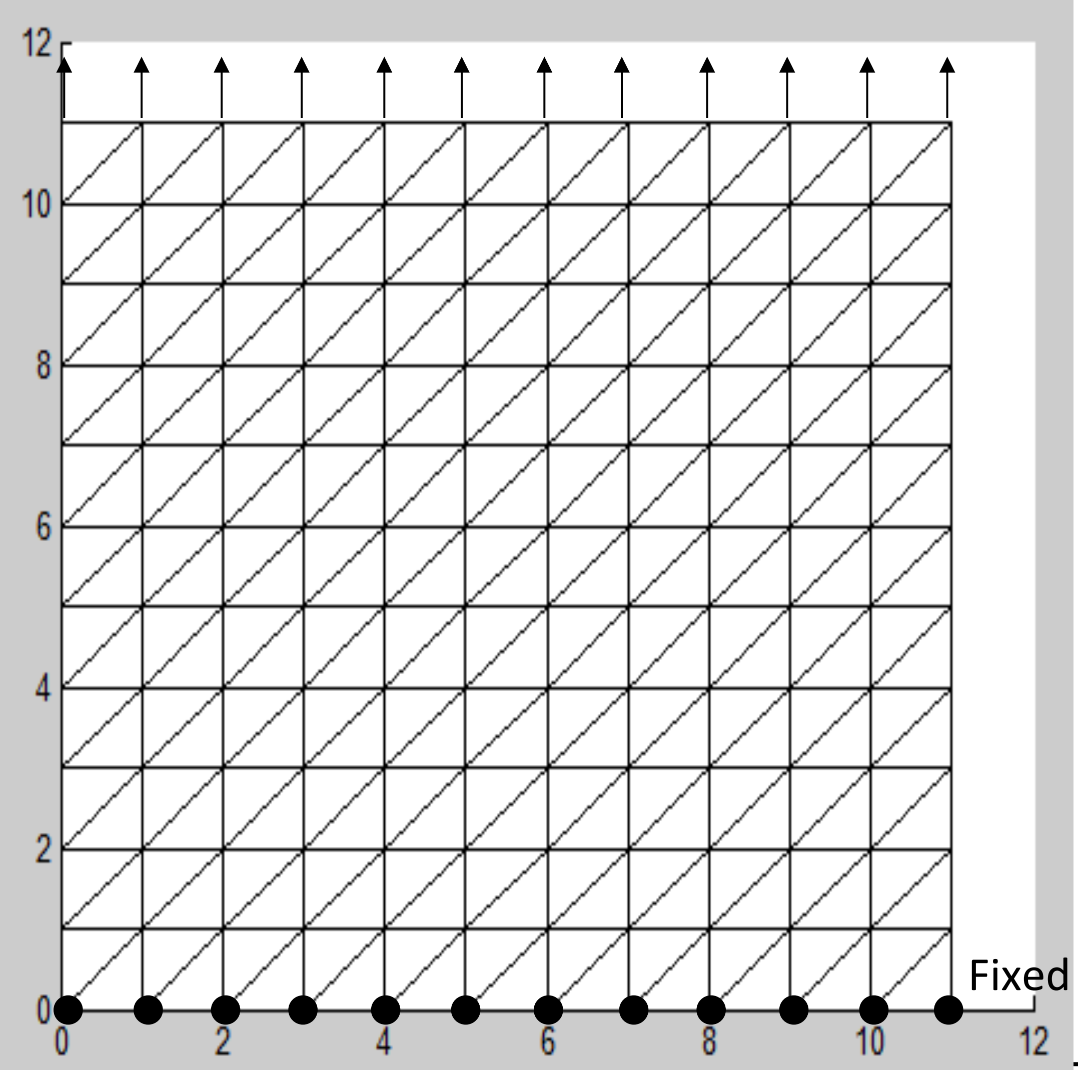
- Numerical Example



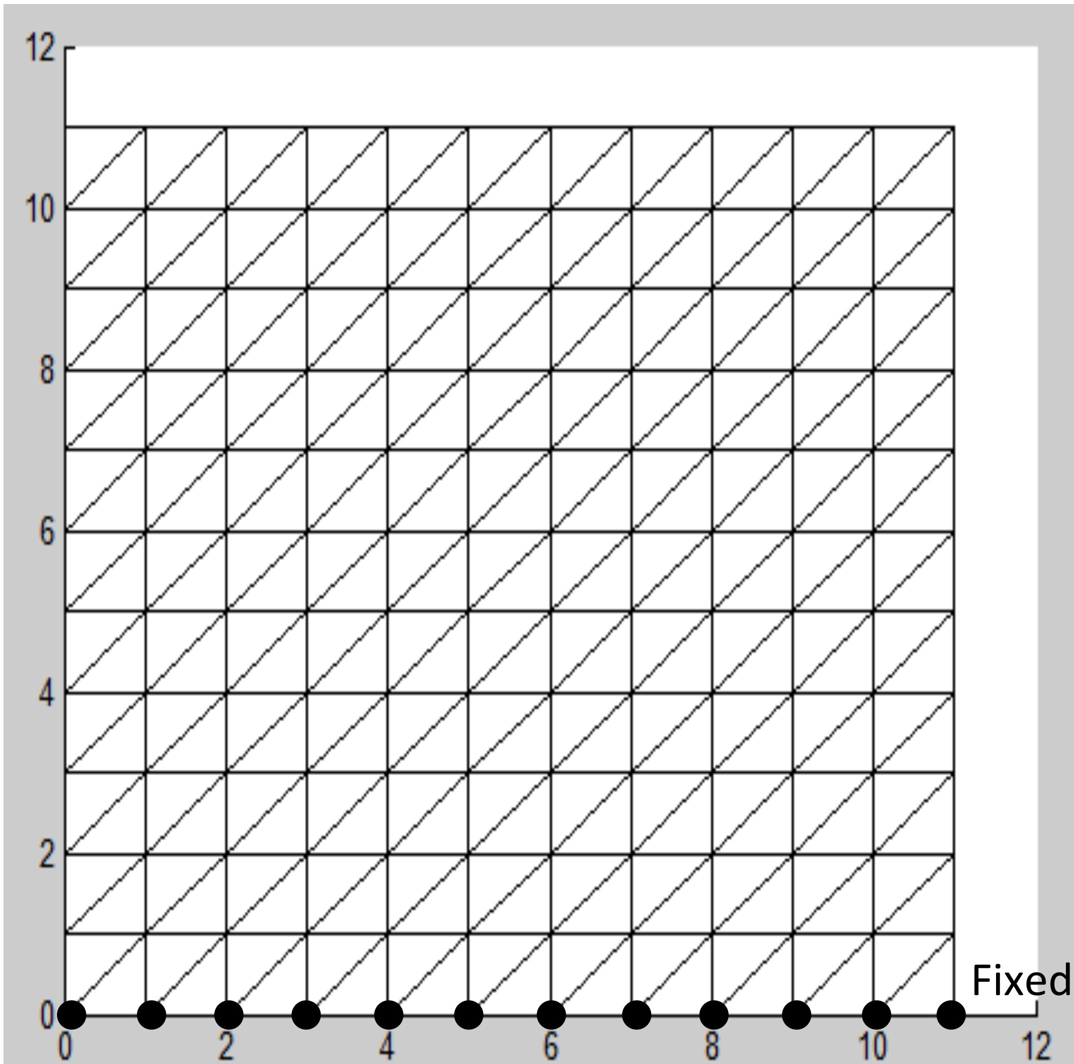
Numerical Example

Boundary Condition / External Force

E=70Gpa / ν=0.3



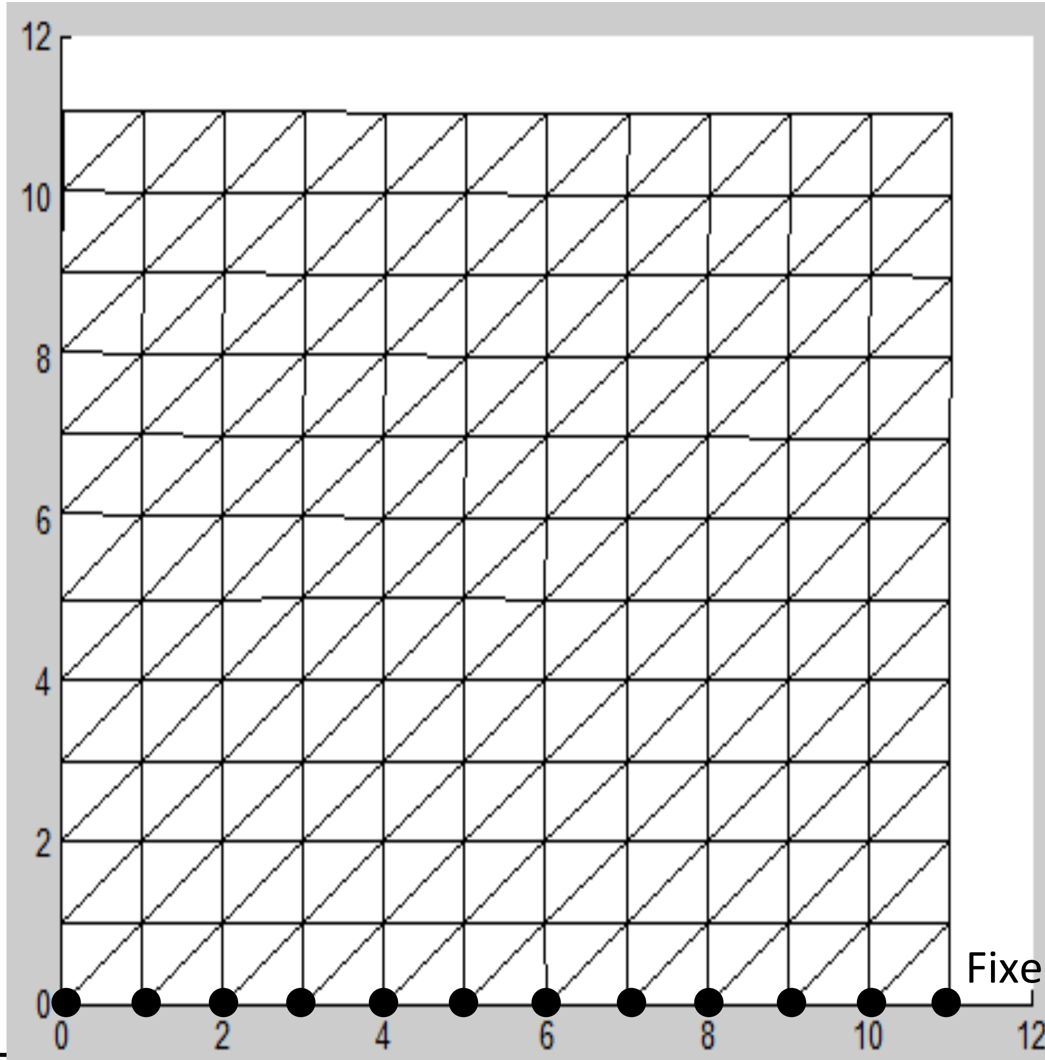
Numerical Example



E=70Gpa / ν=0.3

0Mpa

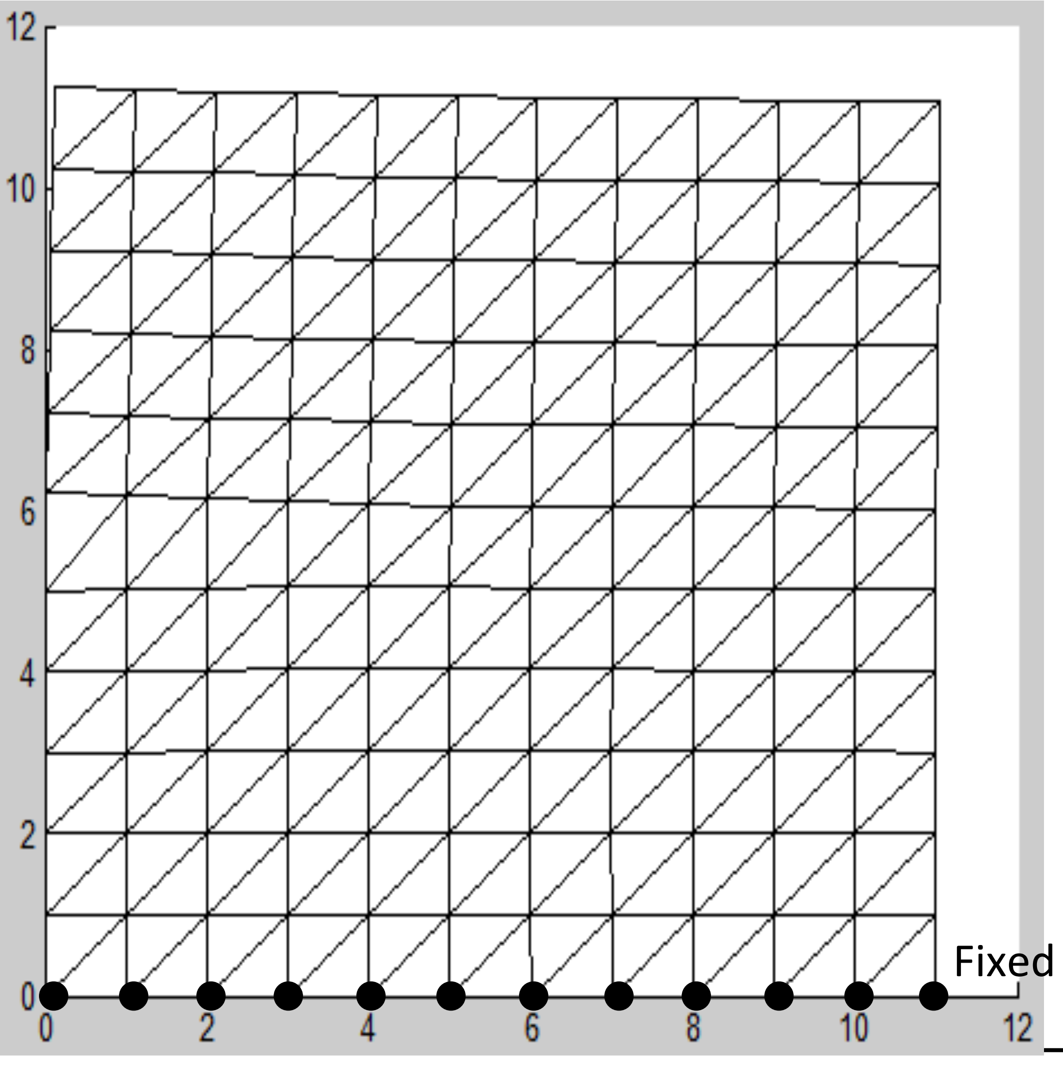
Numerical Example



E=70Gpa / ν=0.3

200Mpa

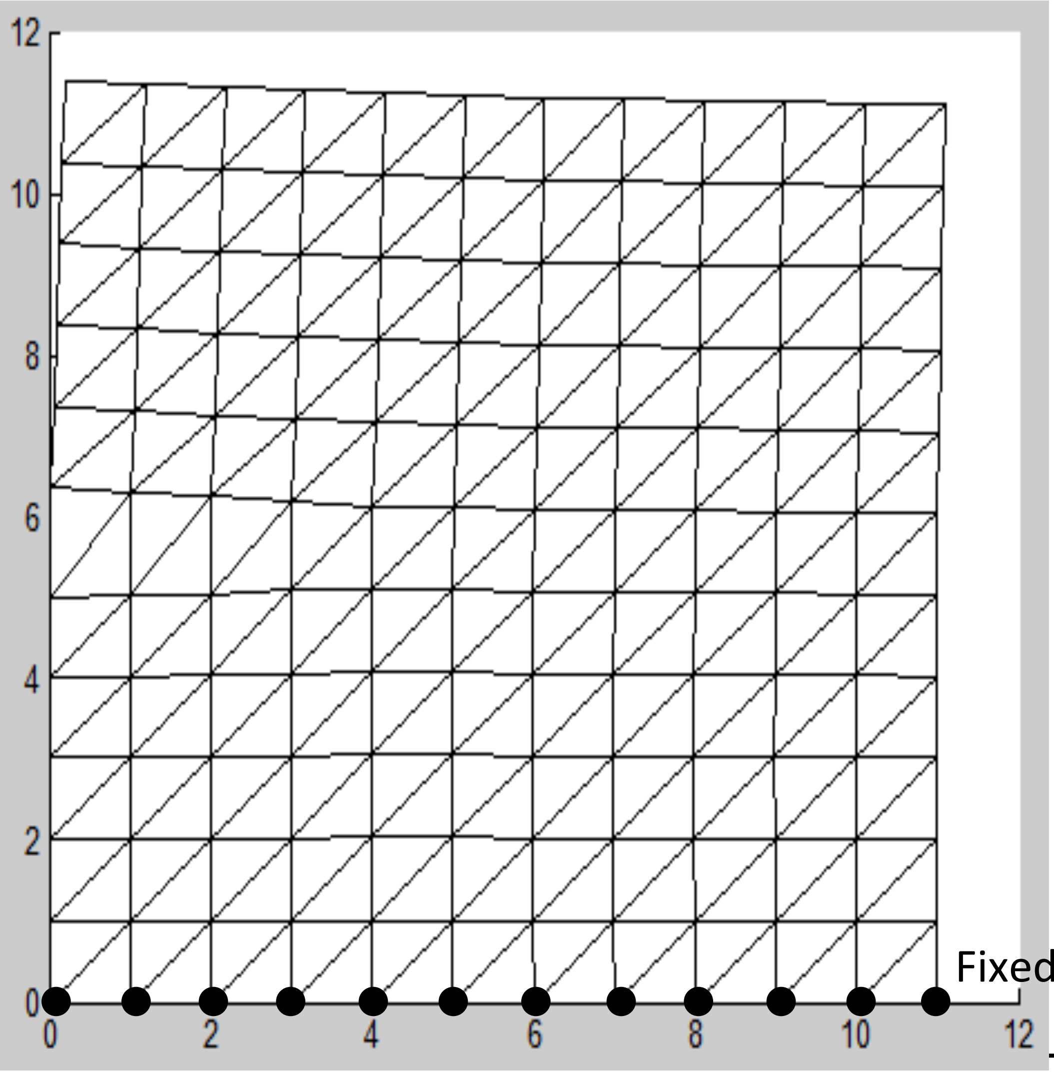
Numerical Example



E=70Gpa / ν=0.3

600Mpa

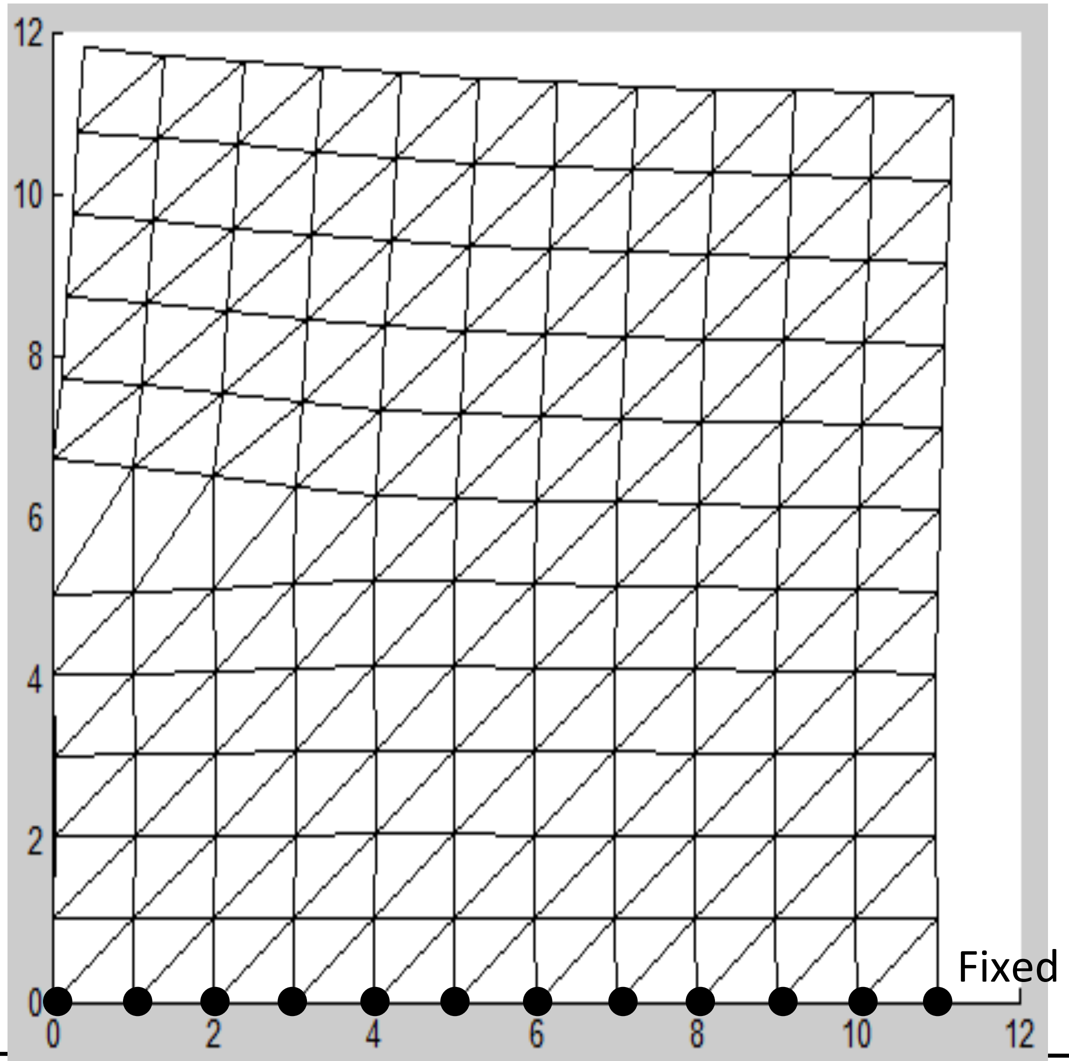
Numerical Example



E=70Gpa / ν=0.3

1Gpa

Numerical Example

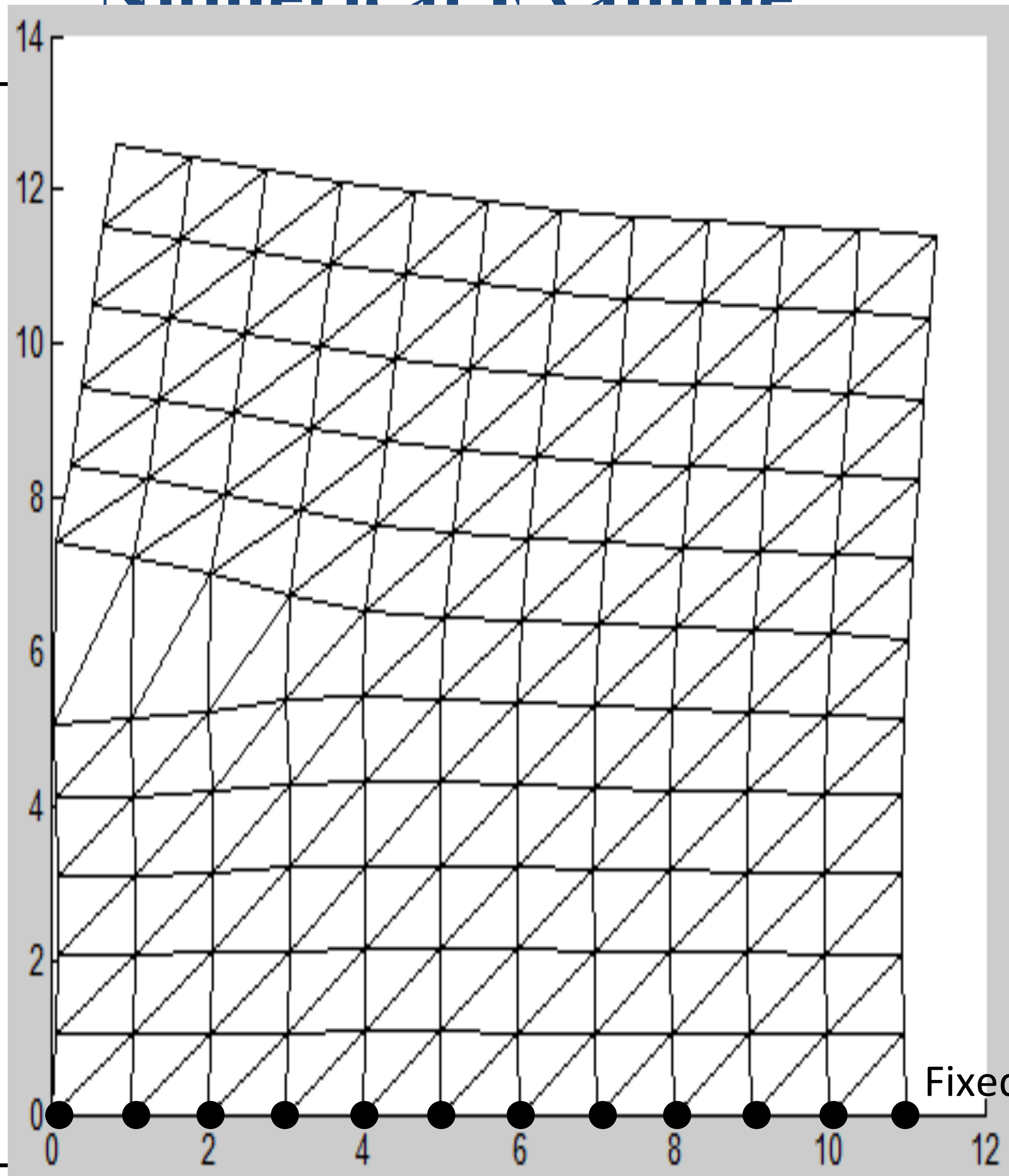


E=70Gpa / ν=0.3

2Gpa

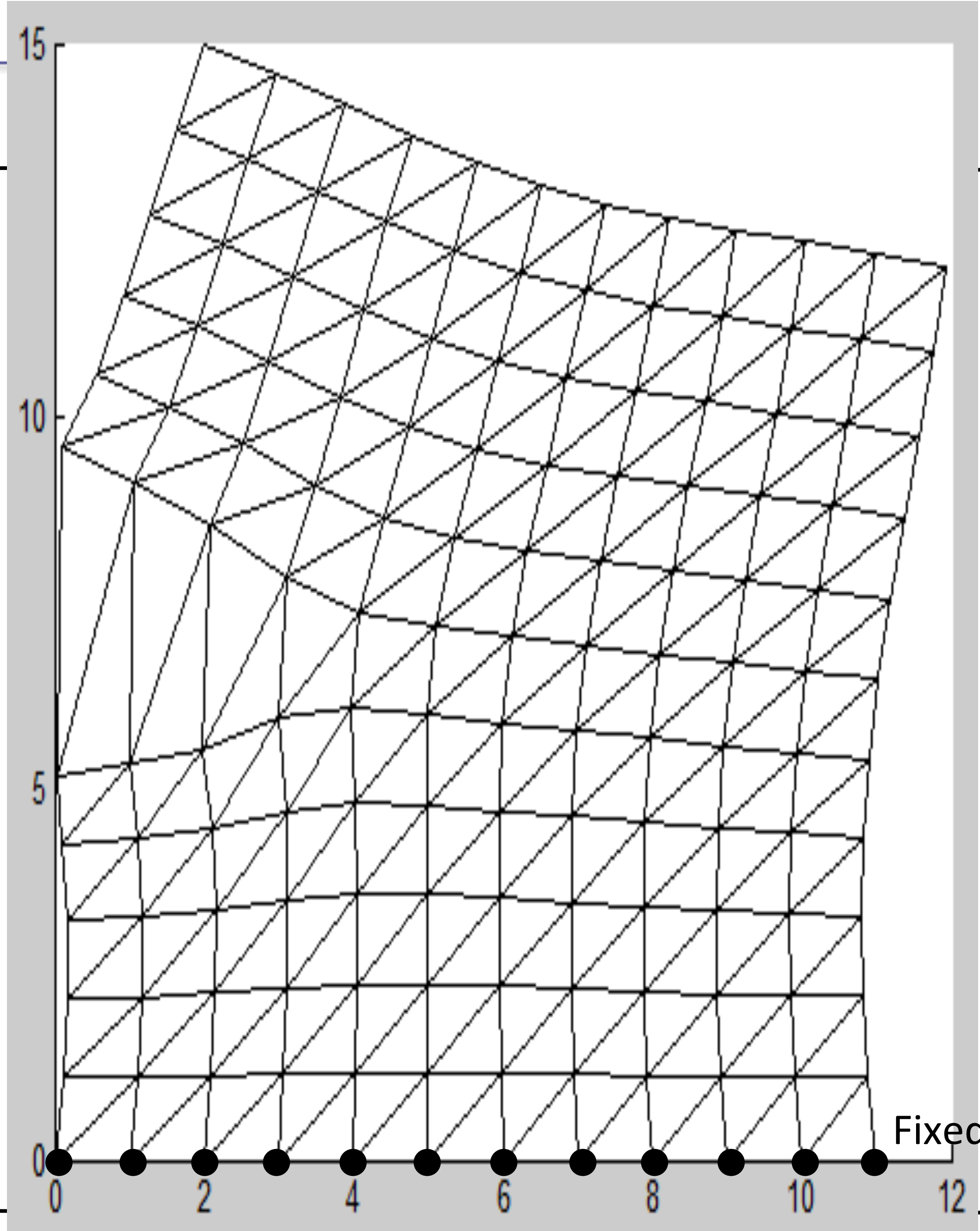
PENNSTATE

Numerical Example



E=70Gpa / ν=0.3

4Gpa



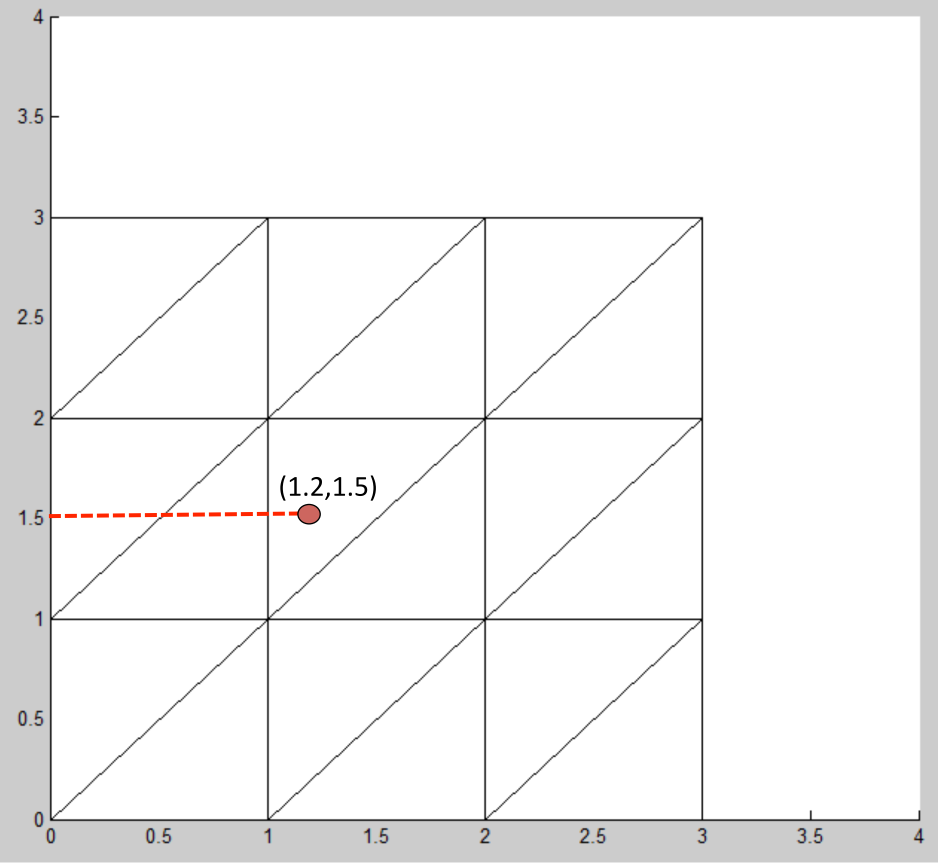
E=70Gpa / ν=0.3

10Gpa

Numerical Example

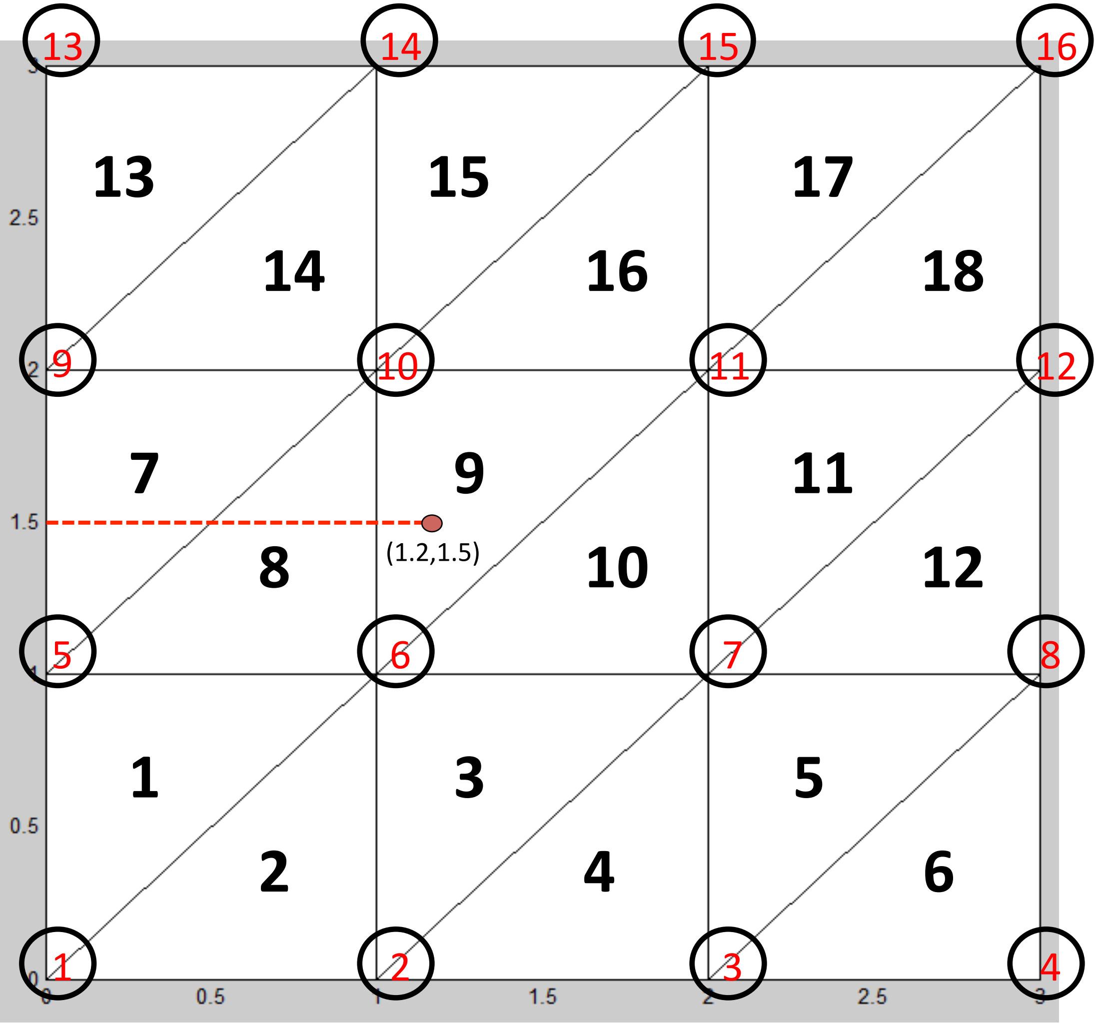
- Example Model

E=1, ν=0.3



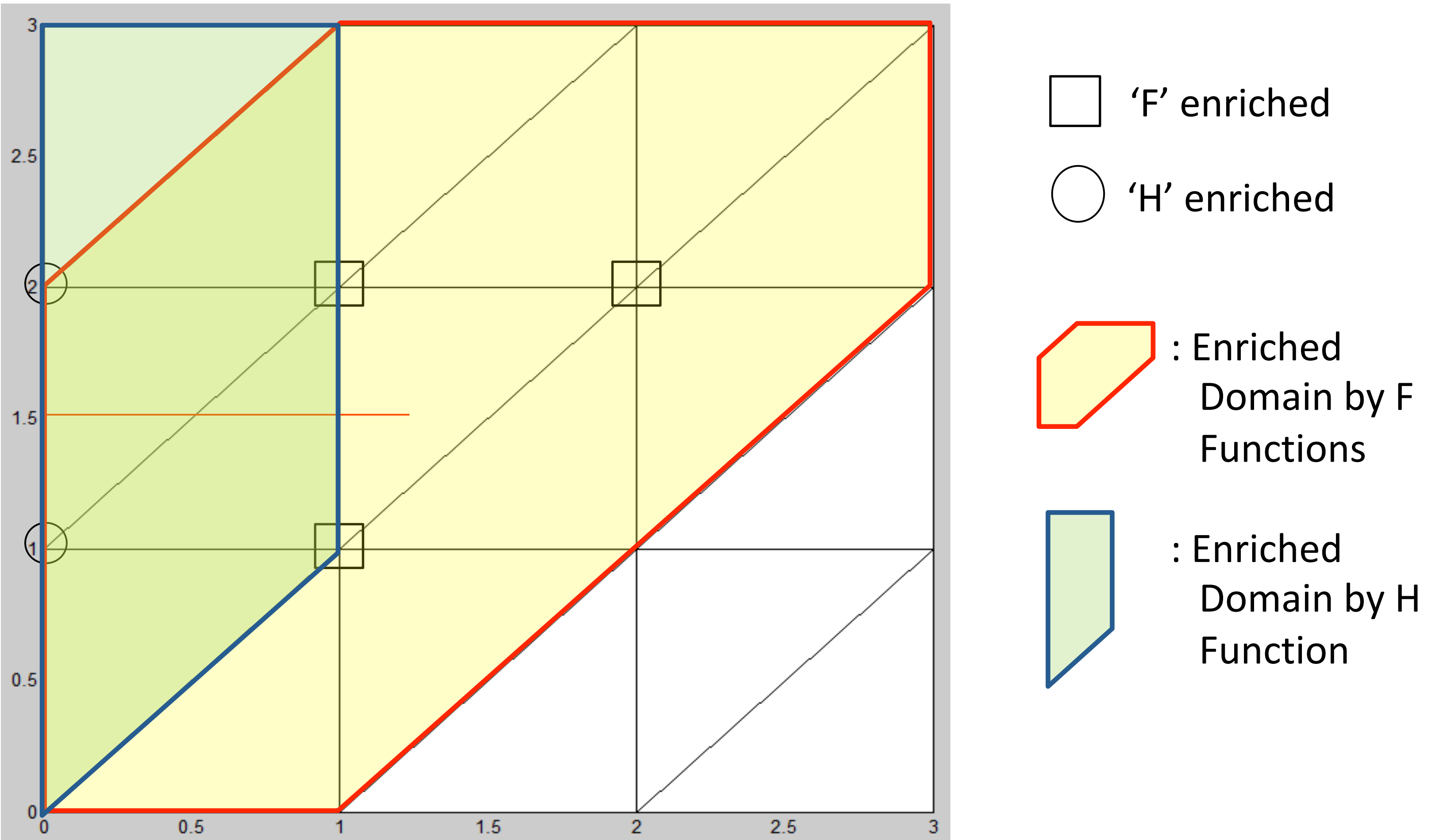
Numerical Example

➢ Node and Element Setting



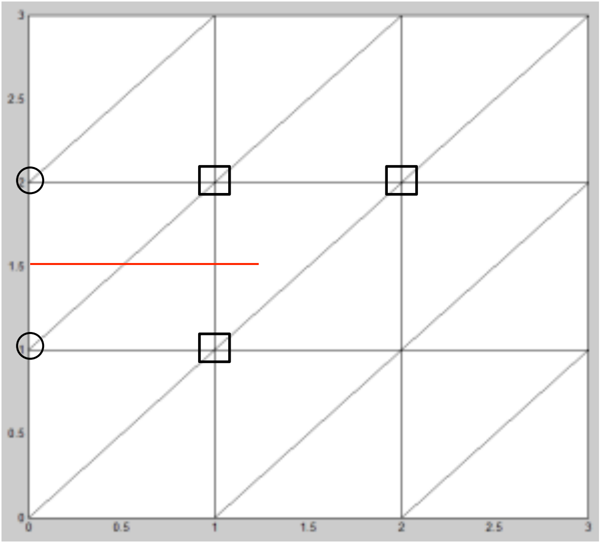
Numerical Example

- Enrichment



Approximation

- Enrichment



□ 'F' enriched (10 unknowns)

○ 'H' enriched (4 Unknowns)

- Formulation

$$\boldsymbol{u}^h = \sum_{i \in I} N_i(\boldsymbol{x}) \boldsymbol{u}_i + \sum_{j \in J} N_j(\boldsymbol{x}) H(\boldsymbol{x}) \boldsymbol{b}_i + \sum_{k \in I} \sum_{l=1}^{4} N_k ($$

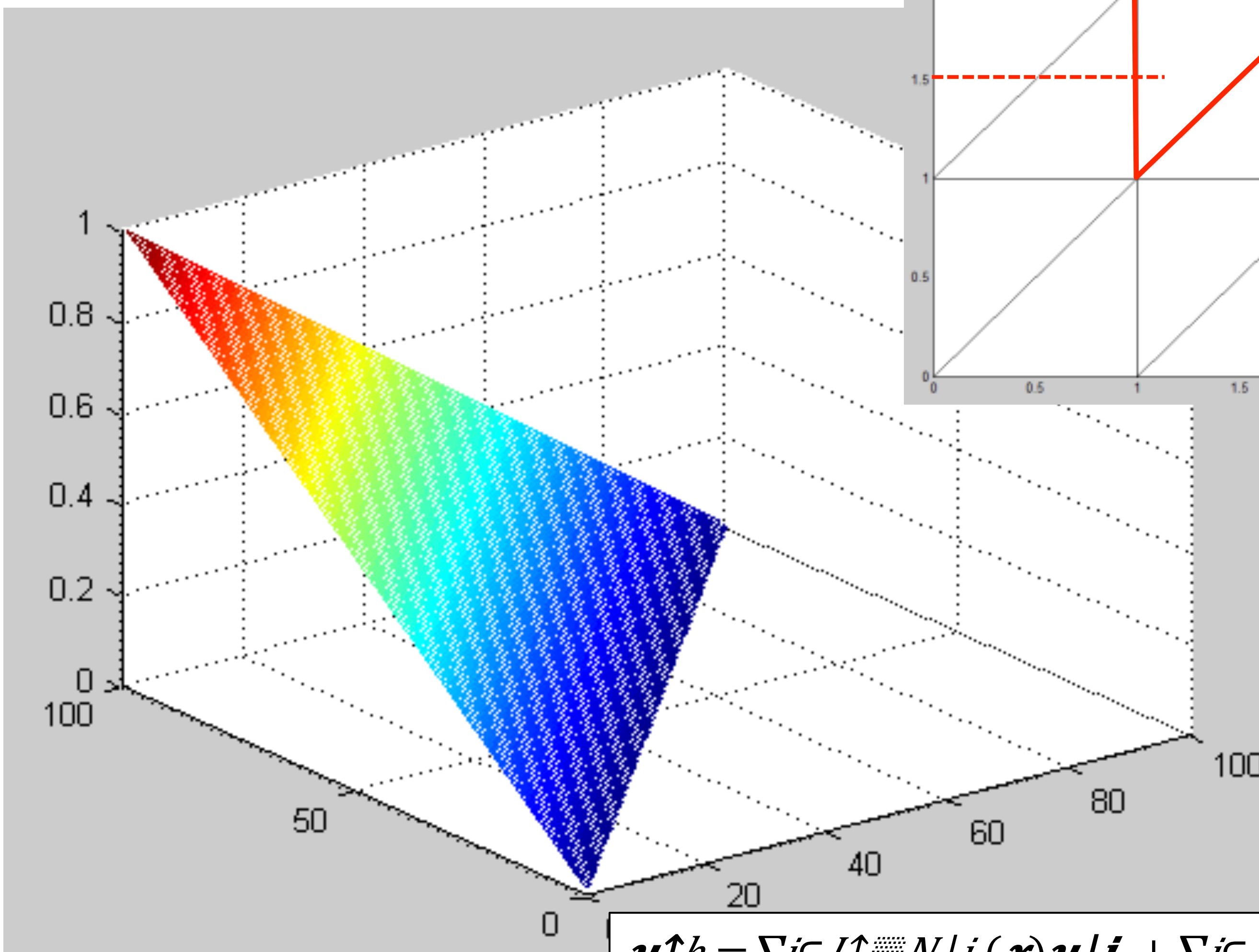
$N_i(\mathbf{x})$=Linear Finite Element Shape Function

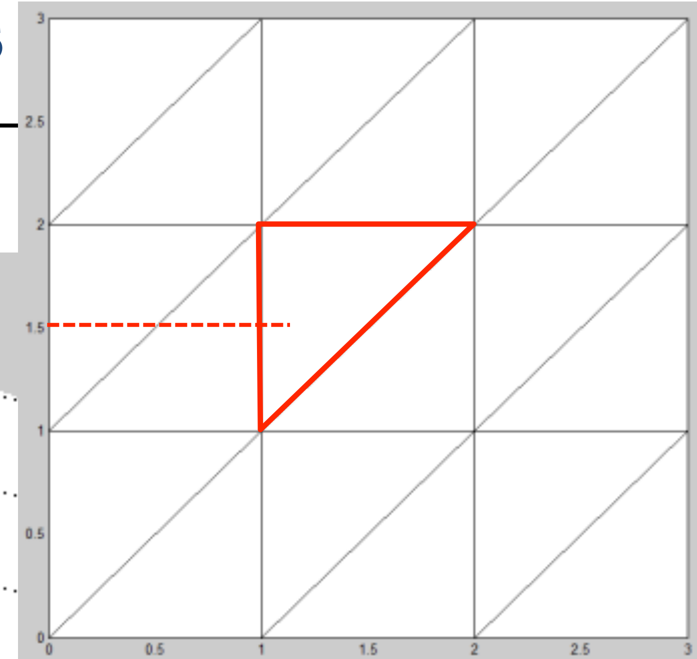
$H_k(\mathbf{x})$= 1 ($y>y_{tip}$) or -1 ($y<y_{tip}$)

$$\{F_l(r,\theta)\} \equiv \left\{\sqrt{r}\sin\left(\frac{\theta}{2}\right), \sqrt{r}\cos\left(\frac{\theta}{2}\right), \sqrt{r}\sin\left(\frac{\theta}{2}\right)\sin(\theta), \sqrt{r}\cos\left(\frac{\theta}{2}\right)\sin(\theta)\right\}$$

Shape of the Functions

N

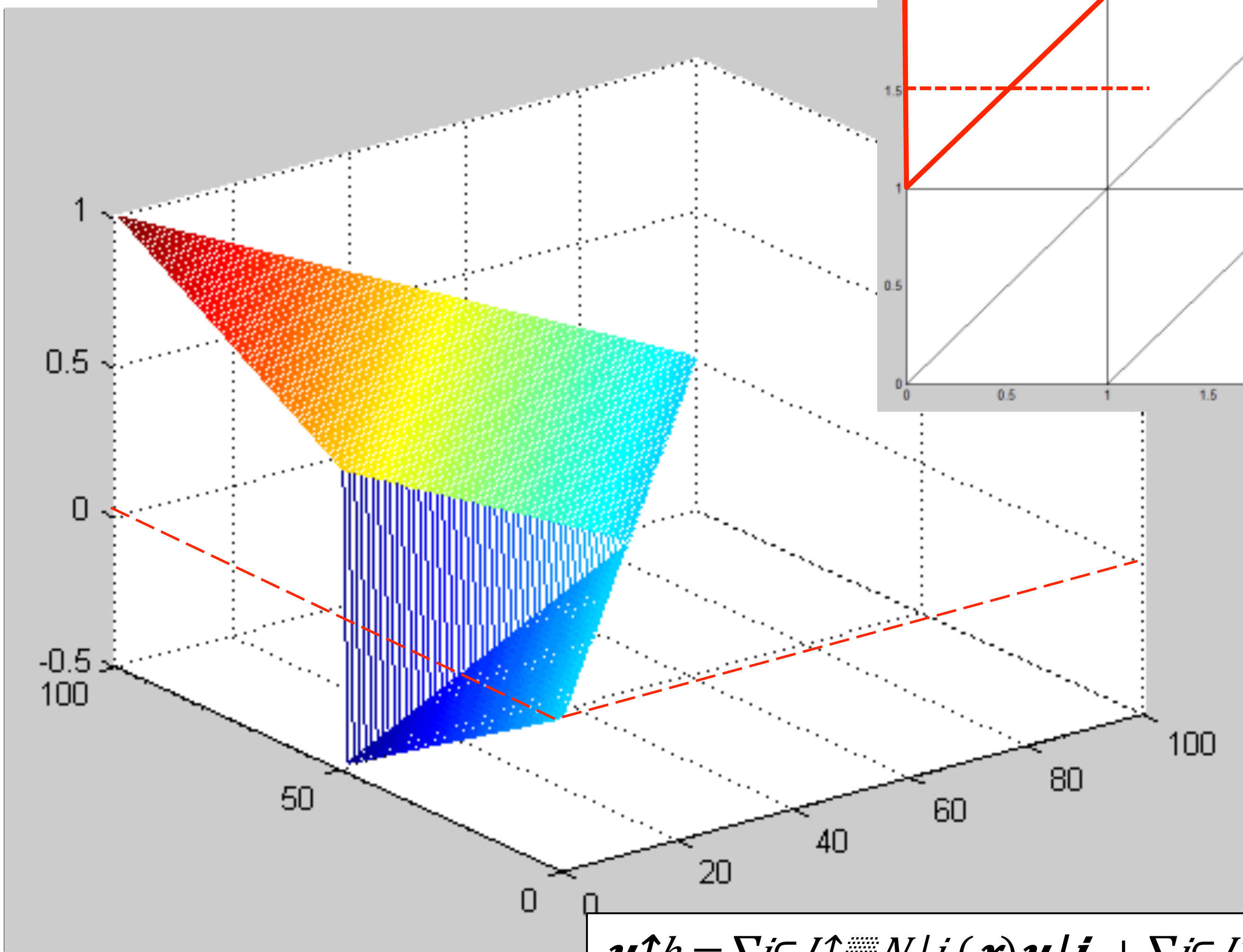


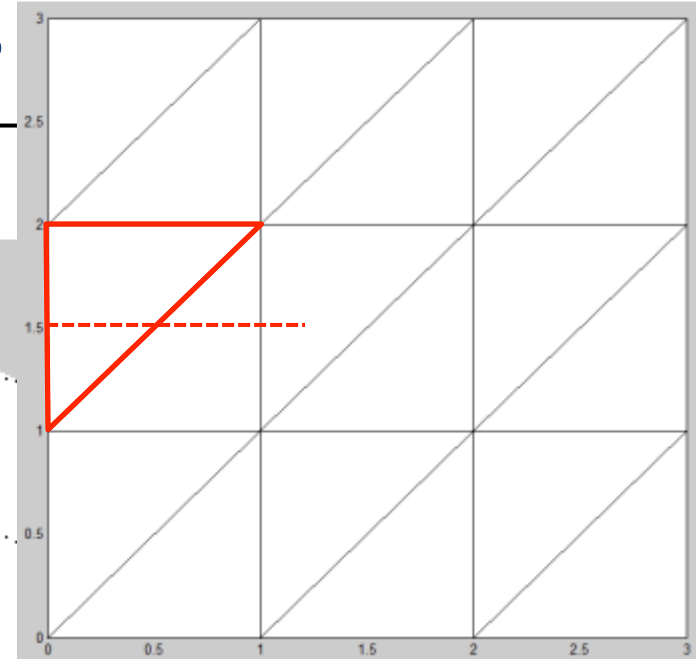


$$u^h = \sum_{i \in I} N_i(\boldsymbol{x}) \boldsymbol{u_i} + \sum_{j \in J} N_j(\boldsymbol{x}) H(\boldsymbol{x})$$

Shape of the Functions

N*H



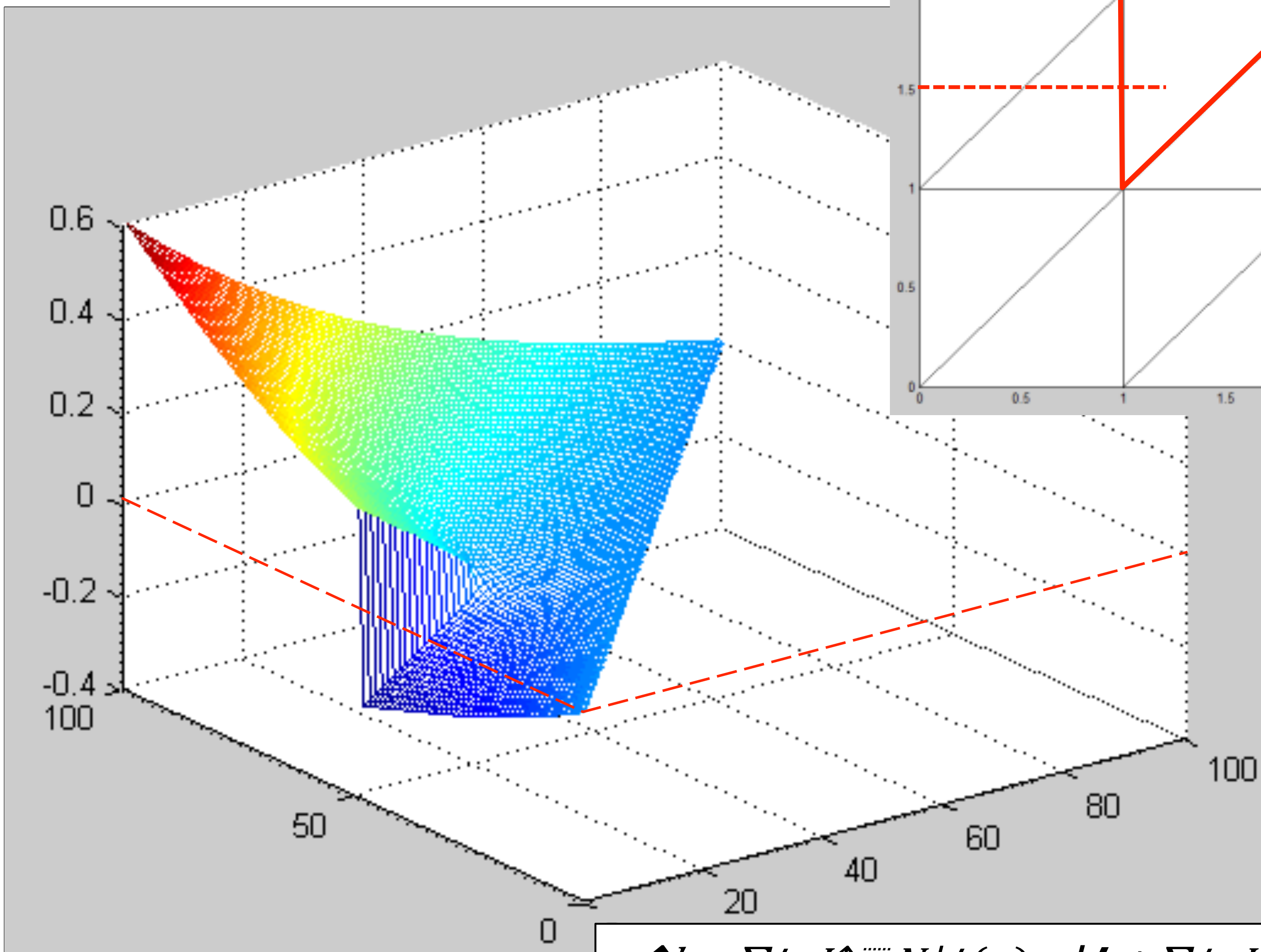


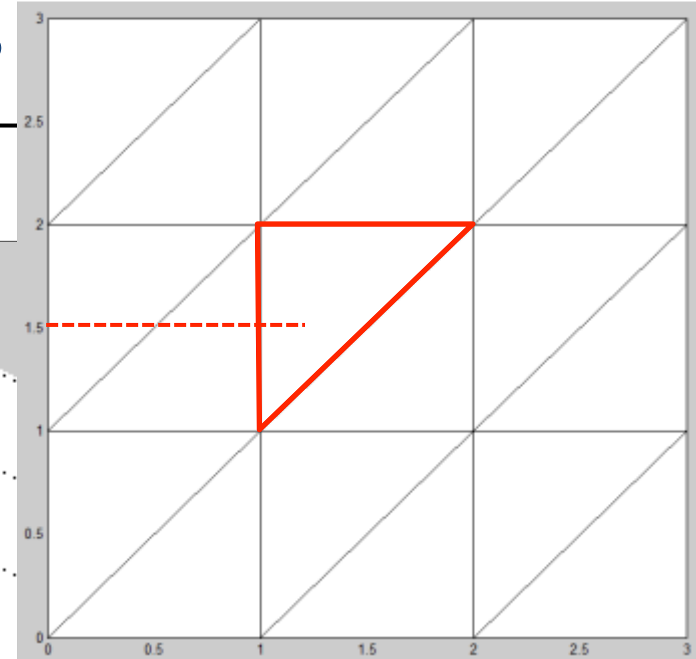
$$u^h = \sum_{i \in I} N_i(\boldsymbol{x})\,\boldsymbol{u}_i + \sum_{j \in J} N_j(\boldsymbol{x}) H(\boldsymbol{x})$$

Shape of the Functions

$F_1 = \sqrt{r}\sin(\theta/2)$

N*F_1



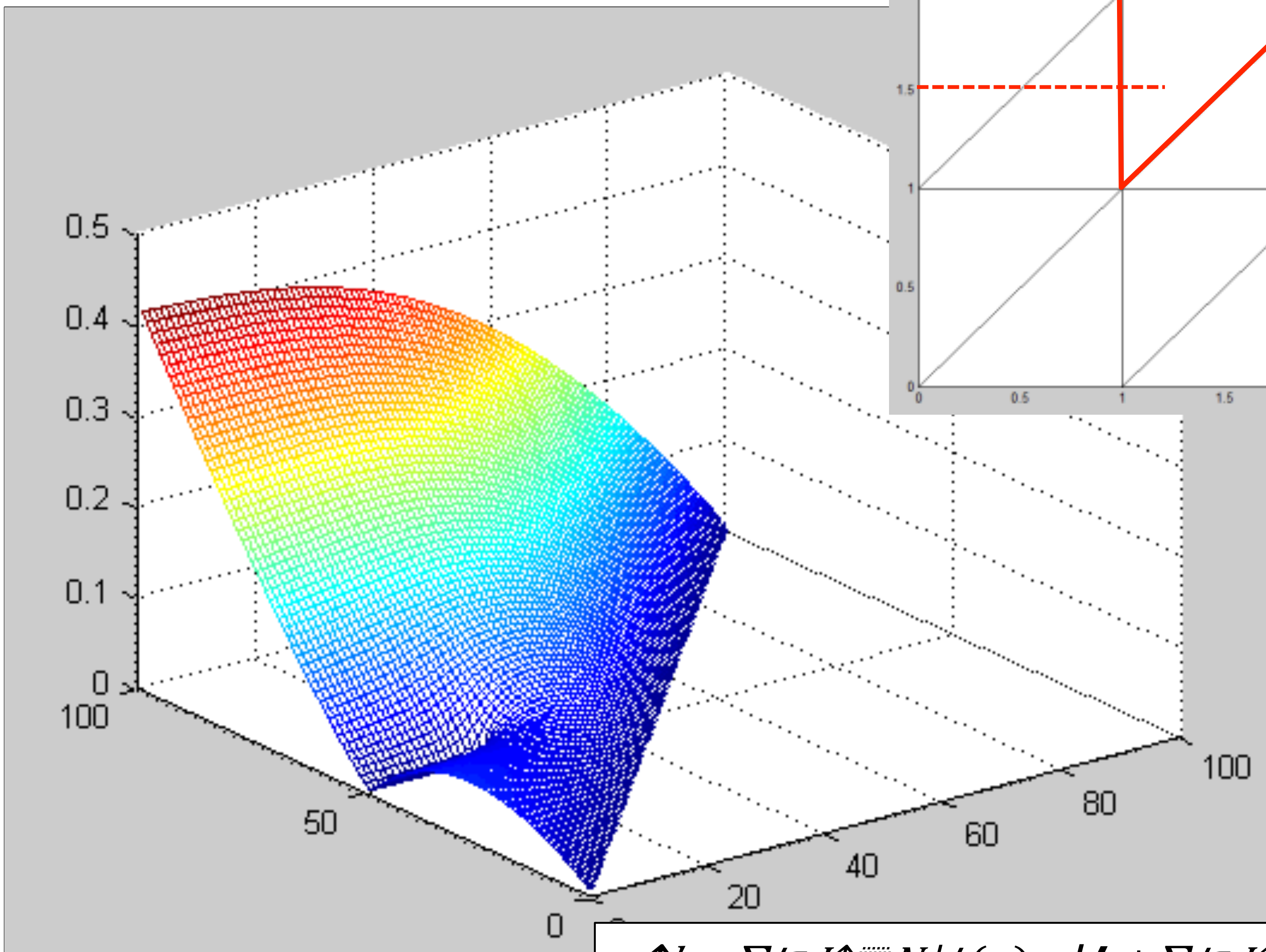


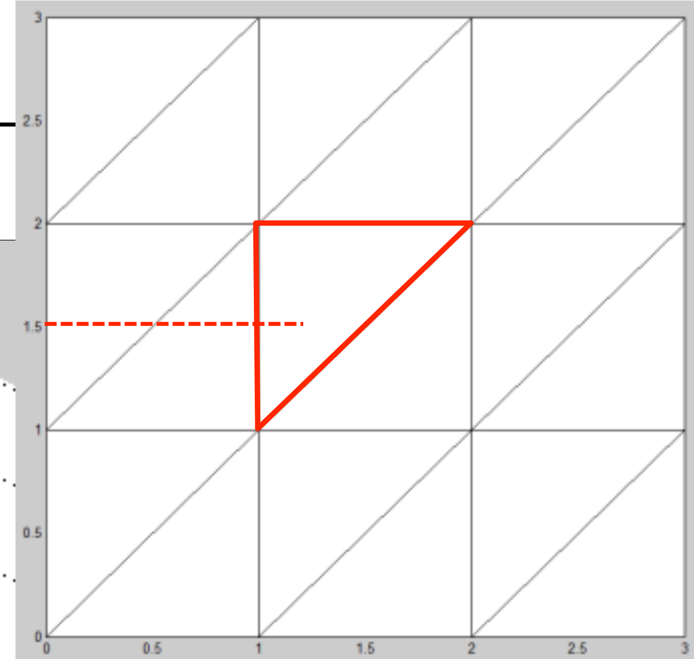
$u^h = \sum_{i \in I} N_i(\boldsymbol{x})\boldsymbol{u}_i + \sum_{j \in J} N_j(\boldsymbol{x}) H(\boldsymbol{x})$

Shape of the Functions

$F_2 = \sqrt{r}\cos(\theta/2)$

N*F_2



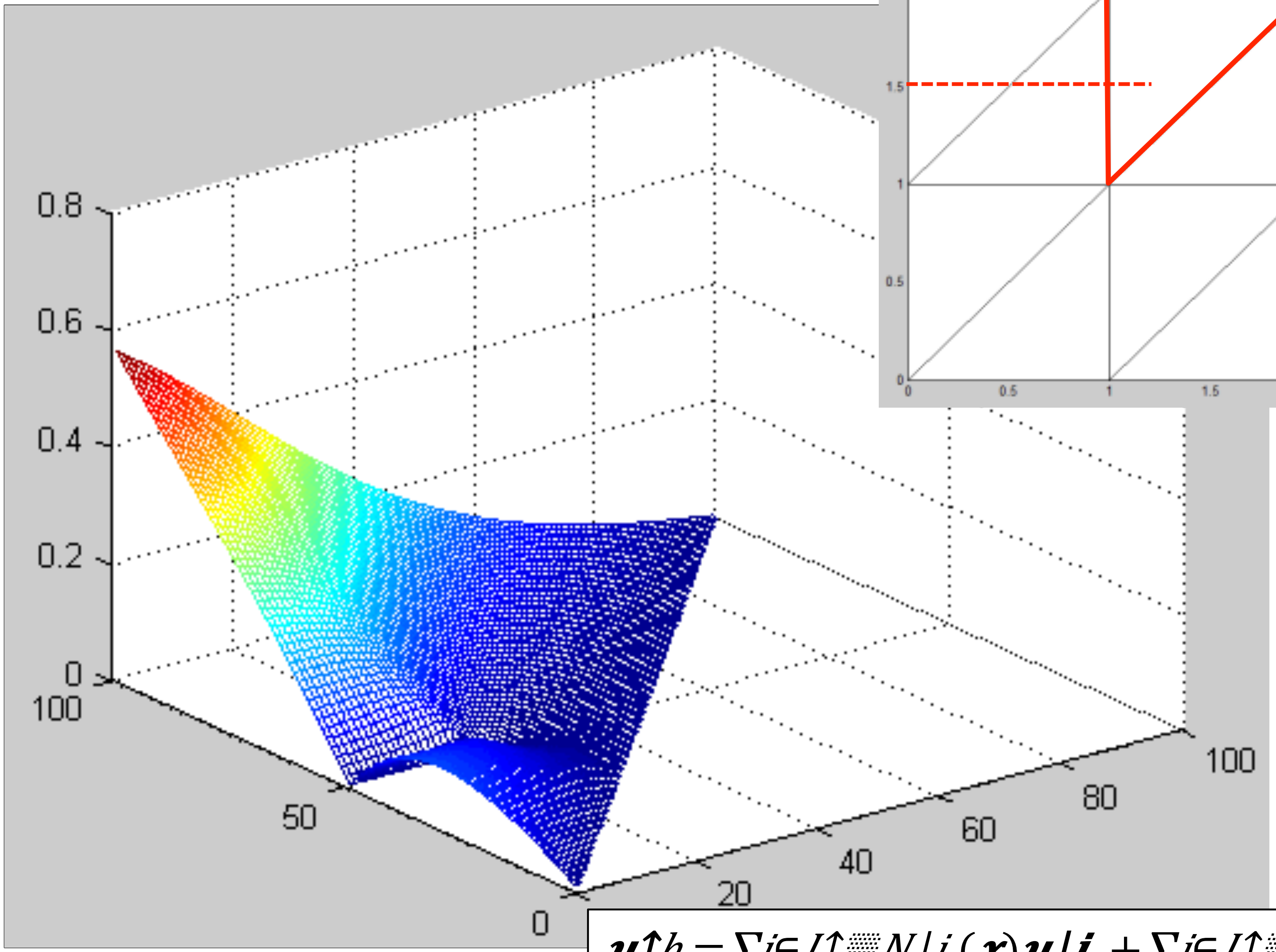


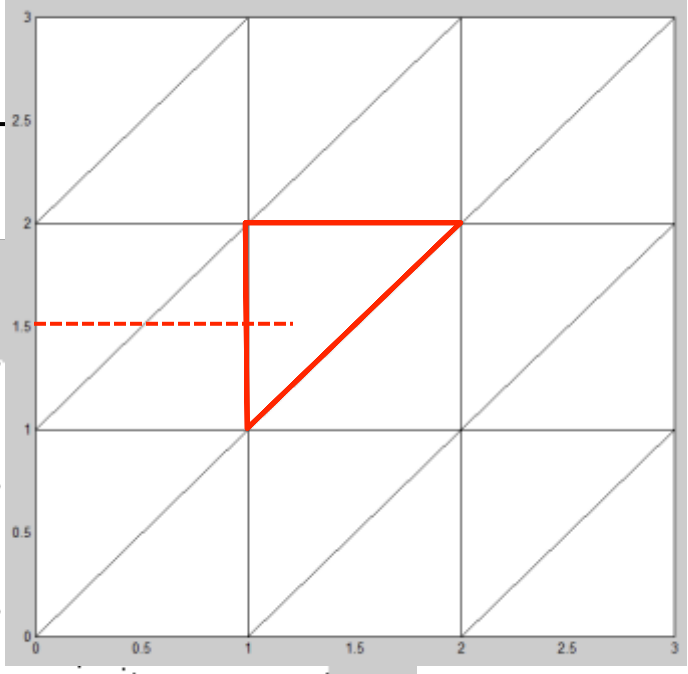
$$\boldsymbol{u}^h = \sum_{i \in I} N_i(\boldsymbol{x})\boldsymbol{u}_i + \sum_{j \in J} N_j(\boldsymbol{x}) H(\boldsymbol{x})$$

Shape of the Functions

$F_3 = \sqrt{r}\sin(\theta/2)\sin(\theta)$

N*F3



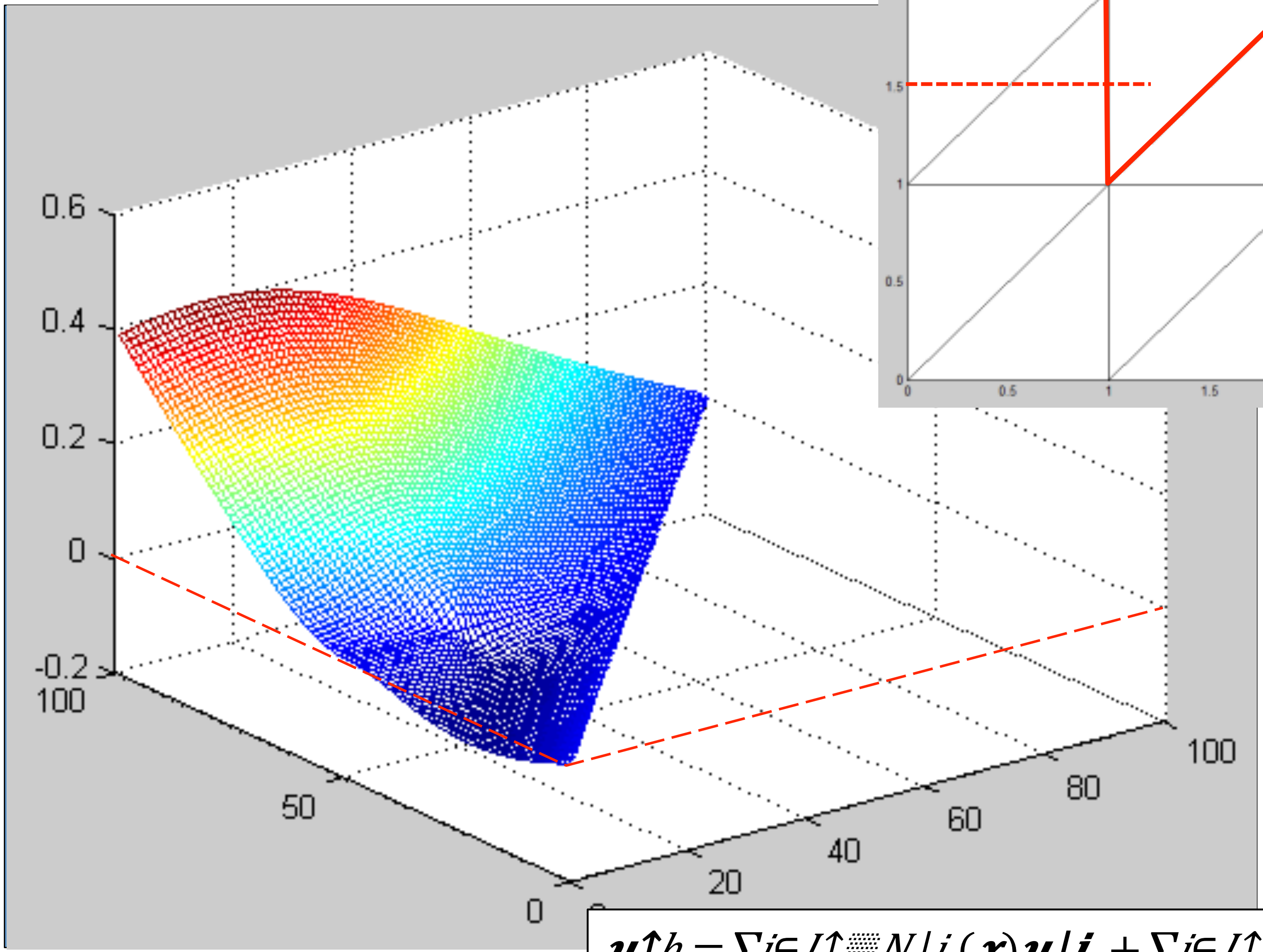


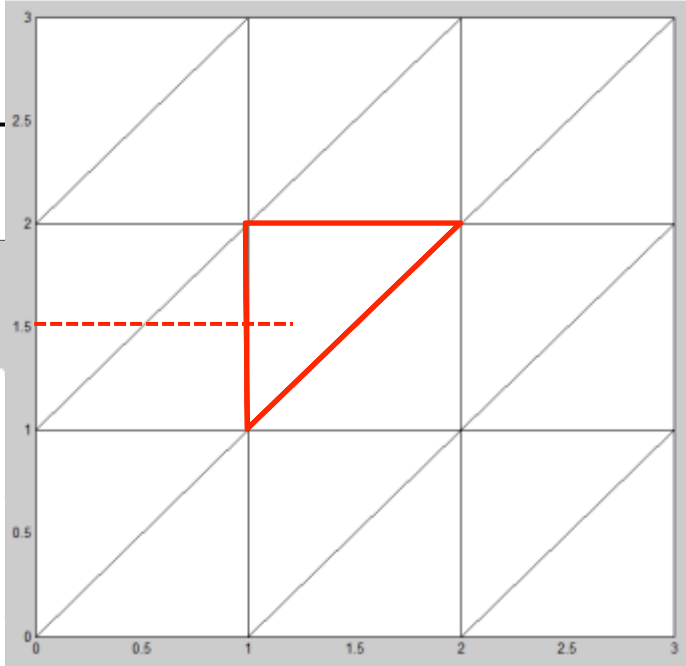
$u^h = \sum_{i \in I} N_i(\boldsymbol{x})\boldsymbol{u}_i + \sum_{j \in J} N_j(\boldsymbol{x})H(\boldsymbol{x})$

Shape of the Functions

$F_4 = \sqrt{r} \cos(\theta/2) \sin(\theta)$

N*F4



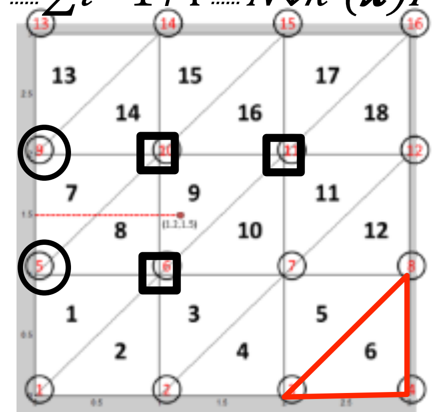


$u^h = \sum_{i \in I} N_i(\boldsymbol{x}) \boldsymbol{u}_i + \sum_{j \in J} N_j(\boldsymbol{x}) H(\boldsymbol{x})$

Formulation

➤ Local Stiffness Matrix for Non-Enriched Element

$$\boldsymbol{u}^h = \sum_{i \in I} N_i(\boldsymbol{x})\boldsymbol{u}_i + \sum_{j \in J} N_j(\boldsymbol{x}) H(\boldsymbol{x}) \boldsymbol{b}_i + \sum_{k \in I} \sum_{l=1}^{4} N_k(\boldsymbol{x}) F_l \ (\ldots$$



- Unknowns
  - Node 3 : 2
  - Node 4 : 2
  - Node 8 : 2

6 Unknowns

- Local Stiffness Matrix

$$K_{Local} = \int [\boldsymbol{B}]^T [\boldsymbol{D}][\boldsymbol{B}] dV$$

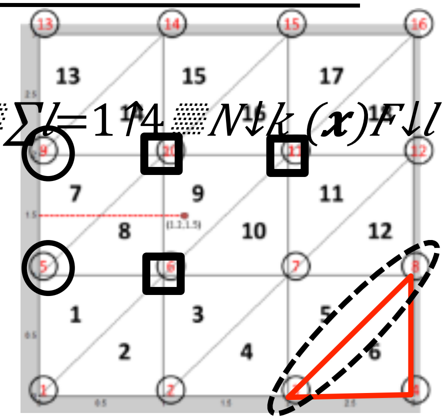
6X6 = 6X3 3X3 3X6

$$\mathbf{B} = \begin{bmatrix} N_{3,x} & 0 \\ 0 & N_{3,y} \\ N_{3,y} & N_{3,x} \end{bmatrix}$$

Formulation

➢ Local Stiffness Matrix for Non-Enriched Element

$$u^h = \sum_{i \in I} N_i(\boldsymbol{x}) \boldsymbol{u_i} + \sum_{j \in J} N_j(\boldsymbol{x}) H(\boldsymbol{x}) \boldsymbol{b_i} + \sum_{k \in I} \sum_{l=1}^{4} N_k(\boldsymbol{x}) F_l(\ldots$$



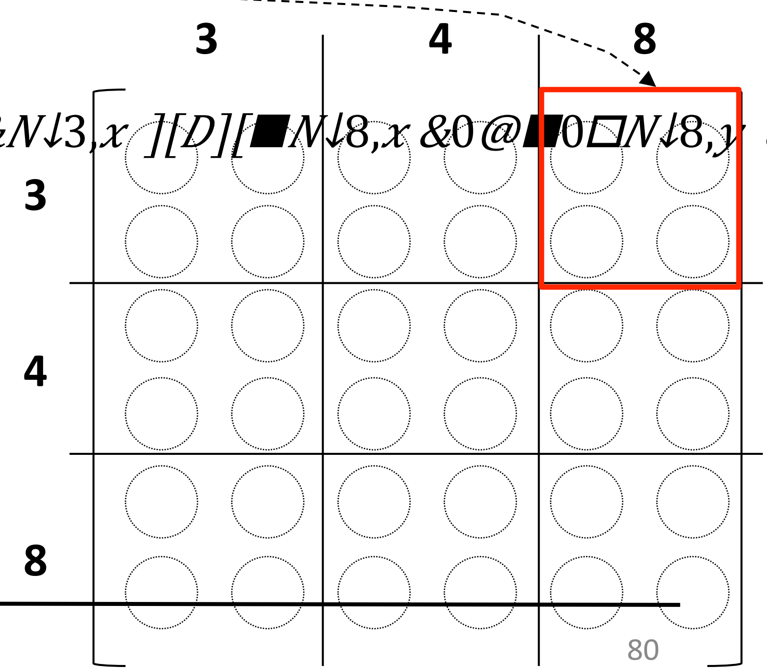
- Unknowns
  - Node 3 : 2
  - Node 4 : 2
  - Node 8 : 2

- For Node 3-8

$$K_{3-8} = \int \begin{bmatrix} N_{3,x} & 0 & N_{3,y} \\ 0 & N_{3,y} & N_{3,x} \end{bmatrix} [D] \begin{bmatrix} N_{8,x} & 0 \\ 0 & N_{8,y} \\ \ldots \end{bmatrix}$$

2X2 = 2X3 3X3 3X2

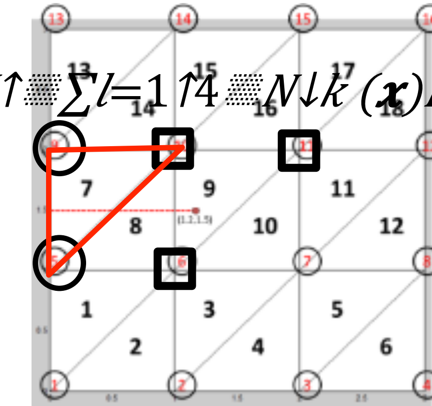
- Local Stiffness Matrix (6X6)



Formulation

➢ Local Stiffness Matrix for Enriched Element

$$u^h = \sum_{i \in I} N_i(x) u_i + \sum_{j \in J} N_j(x) H(x) b_i + \sum_{k \in I} \sum_{l=1}^{4} N_k(x) F_l($$



- Unknowns
  - Node 5 : 4
  - Node 9 : 4
  - Node 10 : 10

18 Unknowns

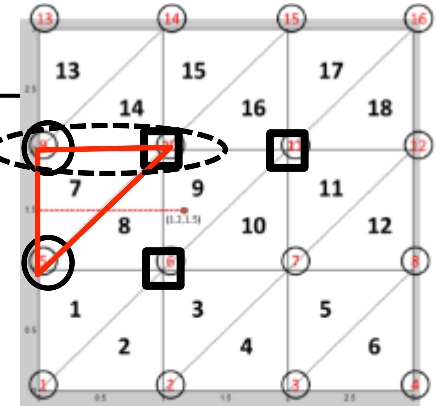
- Local Stiffness Matrix

$$K_{Local} = \int [B]^T [D][B] dV$$

18X18 = 18X3 3X3 3X18

	Node 5 (4 unknowns)	Node 9 (4 unknowns)	Node 10 (10 unknowns)
Node 5 (4 unknowns)			
Node 9 (4 unknowns)			
Node 10 (10 unknowns)			

Formulation



➢ Local Stiffness Matrix for Enriched Element

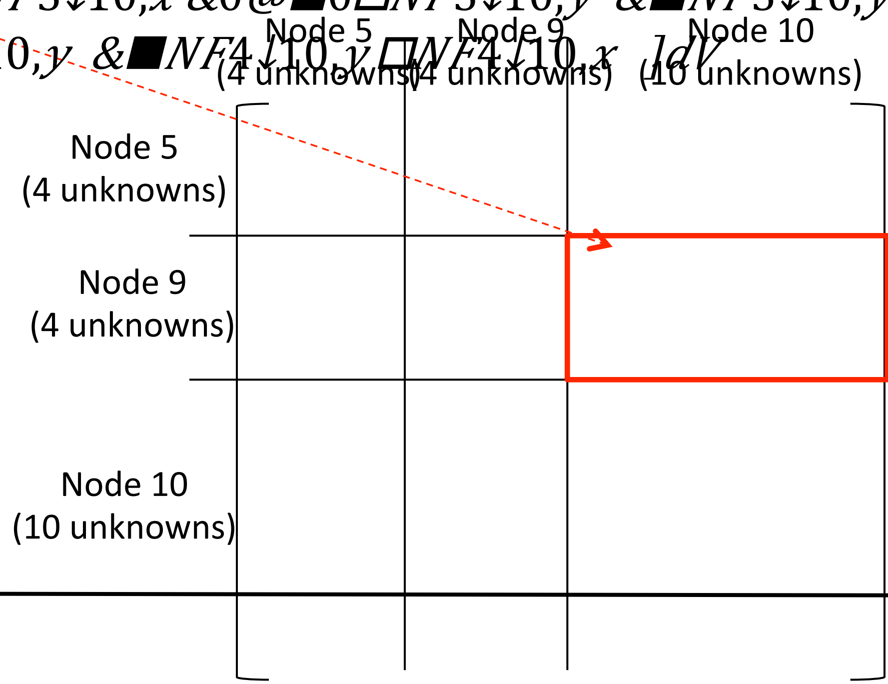
- Local Stiffness Matrix for node 9-10

  - Node 9 : 4 unknowns / H enriched
  - Node 10 : 10 unknowns / F enriched

$$K_{9-10} = \int \begin{bmatrix} \begin{bmatrix} N_{9,x} & 0 \\ 0 & N_{9,y} \\ N_{9,y} & N_{9,x} \end{bmatrix} & \begin{bmatrix} NH_{9,x} & 0 \\ 0 & NH_{9,y} \\ NH_{9,y} & NH_{9,x} \end{bmatrix} \end{bmatrix}^T [\mathbf{D}] \begin{bmatrix} \begin{bmatrix} N_{10,x} & 0 \\ 0 & N_{10,y} \\ N_{10,y} & N_{10,x} \end{bmatrix} & \begin{bmatrix} NF1_{10,x} & 0 \\ 0 & NF1_{10,y} \\ NF1_{10,y} & NF1_{10,x} \end{bmatrix} & \begin{bmatrix} NF2_{10,x} & 0 \\ 0 & NF2_{10,y} \\ NF2_{10,y} & NF2_{10,x} \end{bmatrix} & \begin{bmatrix} NF3_{10,x} & 0 \\ 0 & NF3_{10,y} \\ NF3_{10,y} & NF3_{10,x} \end{bmatrix} & \begin{bmatrix} NF4_{10,x} & 0 \\ 0 & NF4_{10,y} \\ NF4_{10,y} & NF4_{10,x} \end{bmatrix} \end{bmatrix} dV$$

4X3 3X3 3X10

- Local Stiffness Matrix (18x18)

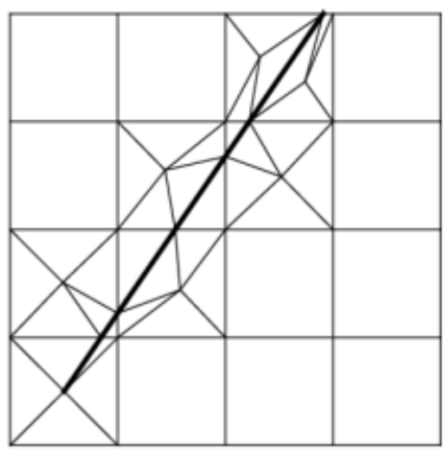


Integration

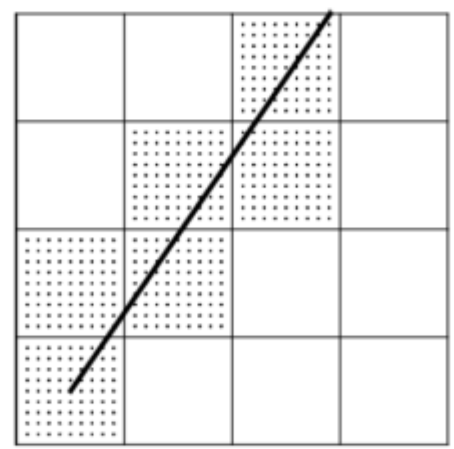
$$K_{Local} = \int [\boldsymbol{B}]^T [\boldsymbol{D}][\boldsymbol{B}]dV$$

Numerical Integration

Gauss quadrature is not apropriate for the numerical integration of the discontinuous enrichment functions, so usually one of the following approaches is employed:



Division into subtriangles



Division into subquads

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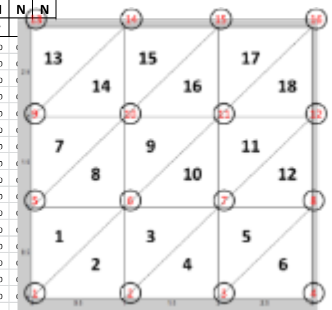
Global Stiffness Matrix

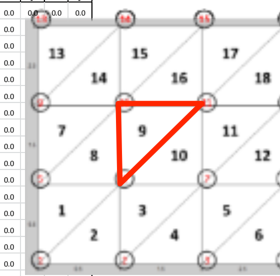


			1		2		3		4		5				6										7		8		9				10										11										12		13		14		15		16	
			N	N	N	N	N	N	N	N	N	N	H	H	N	N	F1	F1	F2	F2	F3	F3	F4	F4	N	N	N	N	N	N	H	H	N	N	F1	F1	F2	F2	F3	F3	F4	F4	N	N	F1	F1	F2	F2	F3	F3	F4	F4	N	N	N	N	N	N	N	N	N	N
			x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y
1	N	x	0.7	0.0	-0.5	0.2	0.0	0.0	0.0	0.0	-0.2	0.2	0.2	-0.2	0.0	-0.4	-0.1	0.3	0.0	-0.2	0.0	-0.3	0.0	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	[illegible]	[illegible]
	N	y	0.0	0.7	0.2	-0.2	0.0	0.0	0.0	0.0	0.2	-0.5	-0.2	0.5	-0.4	0.0	0.3	-0.1	-0.2	0.1	-0.3	0.1	0.1	-0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	[illegible]	[illegible]
2	N	x	-0.5	0.2	1.5	-0.4	-0.5	0.2	0.0	0.0	0.0	0.0	0.0	0.0	-0.4	0.4	0.3	-0.3	-0.2	0.2	-0.2	0.3	0.1	-0.1	0.0	-0.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	[illegible]	[illegible]
	N	y	0.2	-0.2	-0.4	1.5	0.2	-0.2	0.0	0.0	0.0	0.0	0.0	0.0	0.4	-1.1	-0.3	0.7	0.2	-0.6	0.3	-0.6	-0.1	0.5	-0.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	[illegible]	[illegible]
3	N	x	0.0	0.0	-0.5	0.2	1.5	-0.4	-0.5	0.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.4	0.4	0.0	-0.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	[illegible]	[illegible]
	N	y	0.0	0.0	0.2	-0.2	-0.4	1.5	0.2	-0.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.4	-1.1	-0.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	[illegible]	[illegible]
4	N	x	0.0	0.0	0.0	0.0	-0.5	0.2	0.7	-0.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.2	0.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	[illegible]	[illegible]
	N	y	0.0	0.0	0.0	0.0	0.2	-0.2	-0.4	0.7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.2	-0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	[illegible]	[illegible]
5	N	x	-0.2	0.2	0.0	0.0	0.0	0.0	0.0	0.0	1.5	-0.4	-0.9	0.4	-1.1	0.4	0.8	-0.3	-0.4	0.2	-0.6	0.3	0.3	-0.1	0.0	0.0	0.0	0.0	-0.2	0.2	-0.1	0.1	0.0	-0.4	0.0	0.0	-0.1	-0.1	-0.1	-0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	[illegible]	[illegible]
	N	y	0.2	-0.5	0.0	0.0	0.0	0.0	0.0	0.0	-0.4	1.5	0.4	-0.6	0.4	-0.4	-0.3	0.3	0.2	-0.2	0.3	-0.3	-0.1	0.2	0.0	0.0	0.0	0.0	0.2	-0.5	0.1	-0.3	-0.4	0.0	0.0	0.0	-0.1	-0.1	-0.1	-0.1	0.0	-0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	[illegible]	[illegible]
	H	x	0.2	-0.2	0.0	0.0	0.0	0.0	0.0	0.0	-0.9	0.4	1.5	-0.4	0.8	-0.3	-1.1	0.4	0.4	-0.2	0.6	-0.3	-0.4	0.1	0.0	0.0	0.0	0.0	-0.1	0.1	-0.2	0.2	0.0	0.0	0.0	-0.4	0.0	0.0	-0.1	-0.1	-0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	[illegible]	[illegible]
	H	y	-0.2	0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.4	-0.6	-0.4	1.5	-0.3	0.3	0.4	-0.4	-0.2	0.2	-0.3	0.3	0.1	-0.2	0.0	0.0	0.0	0.0	0.1	-0.3	0.2	-0.5	0.0	0.0	-0.4	0.0	0.0	-0.1	-0.1	-0.2	0.0	-0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	[illegible]	[illegible]
6	N	x	0.0	-0.4	-0.4	0.4	0.0	0.0	0.0	0.0	-1.1	0.4	0.9	-0.3	3.0	-0.7	-1.5	0.3	1.3	-0.3	1.5	-0.3	-1.0	0.2	-1.1	0.4	0.0	0.0	0.0	0.0	0.0	0.0	-0.4	0.4	0.0	0.0	-0.1	0.1	-0.1	0.1	-0.1	0.0	0.0	-0.4	-0.1	0.0	-0.1	-0.2	0.0	0.0	-0.1	-0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	[illegible]	[illegible]
	N	y	-0.4	0.0	0.4	-1.1	0.0	0.0	0.0	0.0	0.4	-0.4	-0.3	0.3	-0.7	3.0	0.3	-1.4	-0.3	1.3	-0.3	1.4	0.2	-1.0	0.4	-0.4	0.0	0.0	0.0	0.0	0.0	0.0	0.4	-1.1	0.0	0.0	0.1	-0.4	0.1	-0.4	0.0	-0.3	-0.4	0.0	0.0	-0.1	-0.2	-0.1	0.0	-0.1	-0.1	-0.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	[illegible]	[illegible]
	F1	x	-0.1	0.3	0.3	-0.3	0.0	0.0	0.0	0.0	0.6	-0.2	-0.9	0.3	-1.5	0.3	1.5	-0.3	-0.7	0.2	-1.1	0.2	0.6	-0.1	0.5	-0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.1	-0.1	-0.1	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	[illegible]	[illegible]
	F1	y	0.3	-0.1	-0.3	0.7	0.0	0.0	0.0	0.0	-0.3	0.3	0.3	-0.3	0.3	-1.4	-0.3	1.5	0.2	-0.8	0.2	-1.1	-0.1	0.8	-0.1	0.1	0.0	0.0	0.0	0.0	0.0	0.0	-0.1	0.3	0.1	-0.4	0.0	-0.1	0.0	-0.1	0.0	-0.1	0.1	0.1	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	[illegible]	[illegible]
	F2	x	0.0	-0.2	-0.2	0.2	0.0	0.0	0.0	0.0	-0.4	0.2	0.4	-0.2	1.3	-0.3	-0.7	0.2	0.9	-0.2	0.7	-0.2	-0.6	0.2	-0.6	0.2	0.0	0.0	0.0	0.0	0.0	0.0	-0.3	0.2	0.0	0.0	-0.1	0.1	-0.1	0.1	0.0	0.0	0.1	-0.2	0.0	0.0	0.0	-0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	F2	y	-0.2	0.1	0.2	-0.6	0.0	0.0	0.0	0.0	0.2	-0.2	-0.2	0.2	-0.3	1.3	0.2	-0.8	-0.2	0.9	-0.2	0.9	0.2	-0.6	0.2	-0.2	0.0	0.0	0.0	0.0	0.0	0.0	0.2	-0.5	0.0	0.1	0.1	-0.1	0.1	-0.1	0.0	-0.1	-0.2	0.0	0.0	-0.1	-0.1	0.0	0.0	0.0	0.0	-0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	F3	x	0.0	-0.3	-0.2	0.3	0.0	0.0	0.0	0.0	-0.6	0.3	0.6	-0.2	1.5	-0.3	-1.1	0.2	0.7	-0.2	1.1	-0.2	-0.6	0.2	-0.5	0.1	0.0	0.0	0.0	0.0	0.0	0.0	-0.1	0.1	0.1	-0.1	0.0	0.0	0.0	0.0	0.0	0.0	-0.1	-0.1	0.0	0.0	-0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	F3	y	-0.3	0.1	0.3	-0.6	0.0	0.0	0.0	0.0	0.3	-0.3	-0.2	0.3	-0.3	1.4	0.2	-1.1	-0.2	0.9	-0.2	1.2	0.2	-0.7	0.1	-0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.1	-0.4	-0.1	0.2	0.0	0.0	0.0	0.0	0.0	0.0	-0.1	-0.1	0.0	0.0	0.0	-0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	F4	x	0.0	0.1	0.1	-0.1	0.0	0.0	0.0	0.0	0.3	-0.1	-0.4	0.1	-1.0	0.2	0.6	-0.1	-0.6	0.2	-0.6	0.2	0.7	-0.2	0.3	-0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.1	-0.1	-0.1	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	F4	y	0.1	-0.1	-0.1	0.5	0.0	0.0	0.0	0.0	-0.1	0.2	0.1	-0.2	0.2	-1.0	-0.1	0.8	0.2	-0.6	0.2	-0.7	-0.2	0.8	-0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.1	0.2	0.1	-0.2	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.2	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7	N	x	0.0	0.0	0.0	-0.4	-0.4	0.4	0.0	0.0	0.0	0.0	0.0	0.0	-1.1	0.4	0.5	-0.1	-0.6	0.2	-0.5	0.1	0.3	-0.1	3.0	-0.7	-1.1	0.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.4	0.4	-0.1	0.0	-0.2	0.2	-0.1	0.0	-0.1	0.1	0.0	-0.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	N	y	0.0	0.0	-0.4	0.0	0.4	-1.1	0.0	0.0	0.0	0.0	0.0	0.0	0.4	-0.4	-0.1	0.1	0.2	-0.2	0.1	-0.1	-0.1	0.0	-0.7	3.0	0.4	-0.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.4	-1.1	0.0	-0.2	0.2	-0.9	0.0	-0.1	0.1	-0.3	-0.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
8	N	x	0.0	0.0	0.0	0.0	0.0	-0.4	-0.2	0.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-1.1	0.4	1.5	-0.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.2	0.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	N	y	0.0	0.0	0.0	0.0	-0.4	0.0	0.2	-0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.4	-0.4	-0.4	1.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.2	-0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
9	N	x	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.2	0.2	-0.1	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.5	-0.4	1.1	-0.2	-1.1	0.4	-0.8	0.3	-0.4	0.1	-0.5	0.1	-0.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.2	0.2	0.0	-0.4	0.0	0.0	0.0	0.0
	N	y	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.2	-0.5	0.1	-0.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.4	1.5	-0.2	1.1	0.4	-0.4	0.3	-0.3	0.1	-0.1	0.1	-0.1	0.0	-0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.2	-0.5	-0.4	0.0	0.0	0.0	0.0	0.0
	H	x	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.1	0.1	-0.2	0.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.1	-0.2	1.5	-0.4	-0.8	0.3	-1.1	0.4	-0.3	0.0	-0.4	0.1	-0.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.2	0.2	0.0	-0.4	0.0	0.0	0.0	0.0
	H	y	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	-0.3	0.2	-0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.2	1.1	-0.4	1.5	0.3	-0.3	0.4	-0.4	0.0	0.0	0.1	0.0	0.0	-0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.2	-0.5	-0.4	0.0	0.0	0.0	0.0	0.0
10	N	x	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.4	0.0	0.0	-0.4	0.4	0.1	-0.1	-0.3	0.2	-0.1	0.1	0.1	-0.1	0.0	0.0	0.0	0.0	-1.1	0.4	-0.9	0.3	3.0	-0.7	1.3	-0.4	1.2	-0.3	1.4	-0.3	1.1	-0.3	-1.1	0.4	-0.2	0.0	-0.9	0.2	-0.2	0.0	-0.4	0.1	0.0	0.0	0.0	0.0	-0.4	0.4	0.0	-0.4	0.0	0.0
	N	y	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.4	0.0	0.0	0.0	0.4	-1.1	-0.1	0.3	0.2	-0.5	0.1	-0.4	-0.1	0.2	0.0	0.0	0.0	0.0	0.4	-0.4	0.3	-0.3	-0.7	3.0	-0.4	1.1	-0.3	1.3	-0.3	1.4	-0.3	1.1	0.4	-0.4	0.0	0.0	0.2	-0.3	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.0	0.4	-1.1	-0.4	0.0	0.0	0.0
	F1	x	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.3	0.0	0.0	-0.1	0.1	0.0	0.0	0.1	-0.1	-0.1	0.1	0.0	0.0	0.0	0.0	-0.5	0.2	-0.9	0.3	1.3	-0.4	1.5	-0.4	0.7	-0.2	0.9	-0.3	0.6	-0.2	-0.5	0.2	-0.2	0.1	-0.4	0.1	-0.1	0.0	-0.3	0.1	0.0	0.0	0.0	0.0	-0.2	0.2	0.0	-0.2	0.0	0.0
	F1	y	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.3	0.0	0.0	0.0	0.1	-0.4	0.0	0.1	-0.1	0.2	0.1	-0.2	0.0	0.0	0.0	0.0	0.2	-0.2	0.3	-0.3	-0.4	1.1	-0.4	1.5	-0.2	0.8	-0.3	0.9	-0.2	0.7	0.2	-0.3	0.1	-0.1	0.1	-0.2	0.0	0.0	0.1	-0.1	0.0	0.0	0.0	0.0	0.2	-0.7	-0.2	0.1	0.0	0.0
	F2	x	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.1	-0.1	0.0	0.0	-0.1	0.1	0.0	0.0	-0.1	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.4	0.1	-0.4	0.1	1.2	-0.3	0.7	-0.2	0.8	-0.1	0.7	-0.1	0.8	-0.1	-0.5	0.2	-0.1	0.1	-0.5	0.2	-0.1	0.0	-0.3	0.1	0.0	0.0	0.0	0.0	-0.3	0.2	0.1	-0.2	0.0	0.0
	F2	y	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.1	-0.1	0.0	-0.1	0.1	-0.4	0.0	-0.1	0.1	-0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	-0.1	0.1	-0.1	-0.3	1.3	-0.2	0.8	-0.1	0.9	-0.1	0.9	-0.1	0.8	0.2	-0.2	0.1	0.0	0.2	-0.2	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.0	0.2	-0.6	-0.2	0.1	0.0	0.0
	F3	x	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.1	-0.1	0.0	-0.1	-0.1	0.1	0.0	0.0	-0.1	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.5	0.1	-0.5	0.1	1.4	-0.3	0.9	-0.3	0.7	-0.1	1.0	-0.2	0.7	-0.1	-0.5	0.2	-0.2	0.1	-0.4	0.1	-0.1	0.0	-0.3	0.1	0.0	0.0	0.0	0.0	-0.2	0.2	0.0	-0.2	0.0	0.0
	F3	y	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.1	-0.1	-0.1	-0.1	0.1	-0.4	0.0	-0.1	0.1	-0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	-0.1	0.1	-0.1	-0.3	1.4	-0.3	0.9	-0.1	0.9	-0.2	1.1	-0.1	0.8	0.2	-0.3	0.1	-0.1	0.1	-0.2	0.0	0.0	0.1	-0.1	0.0	0.0	0.0	0.0	0.2	-0.6	-0.2	0.1	0.0	0.0
	F4	x	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.3	0.0	-0.3	0.0	1.1	-0.3	0.6	-0.2	0.8	-0.1	0.7	-0.1	0.8	-0.1	-0.5	0.2	-0.1	0.1	-0.5	0.2	-0.1	0.0	-0.3	0.1	0.0	0.0	0.0	0.0	-0.2	0.2	0.1	-0.2	0.0	0.0
	F4	y	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.1	0.0	-0.1	0.0	-0.3	0.0	-0.1	0.0	-0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.1	0.0	-0.1	-0.3	1.1	-0.2	0.7	-0.1	0.8	-0.1	0.8	-0.1	0.8	0.2	-0.2	0.1	0.0	0.2	-0.2	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.0	0.2	-0.5	-0.2	0.1	0.0	0.0
11	N	x	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.4	0.1	0.1	0.1	-0.2	-0.1	-0.1	0.0	0.1	-0.4	0.4	0.0	0.0	0.0	0.0	0.0	0.0	-1.1	0.4	-0.5	0.2	-0.5	0.2	-0.5	0.2	-0.5	0.2	3.0	-0.7	0.7	-0.2	2.7	-0.6	0.5	-0.1	1.3	-0.3	-1.1	0.4	0.0	0.0	0.0	0.0	-0.4	0.4	0.0	-0.4
	N	y	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.4	0.0	0.1	0.1	-0.2	0.0	-0.1	-0.1	0.1	0.2	0.4	-1.1	0.0	0.0	0.0	0.0	0.0	0.0	0.4	-0.4	0.2	-0.3	0.2	-0.2	0.2	-0.3	0.2	-0.2	-0.7	3.0	-0.2	0.7	-0.6	2.7	-0.1	0.5	-0.3	1.3	0.4	-0.4	0.0	0.0	0.0	0.0	0.4	-1.1	-0.4	0.0
	F1	x	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.2	0.0	-0.2	0.1	-0.1	0.1	-0.2	0.1	-0.1	0.1	0.7	-0.2	0.4	-0.1	0.7	-0.1	0.2	-0.1	0.6	-0.1	-0.3	0.1	0.0	0.0	0.0	0.0	-0.1	0.1	0.0	-0.1
	F1	y	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.1	0.0	0.0	0.0	-0.1	0.0	0.0	0.0	0.0	0.0	-0.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	-0.1	0.1	0.0	0.1	-0.1	0.1	0.0	-0.2	0.7	-0.1	0.4	-0.1	0.8	-0.1	0.3	-0.1	0.7	0.1	-0.2	0.0	0.0	0.0	0.0	0.1	-0.4	-0.1	0.1
	F2	x	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.1	-0.2	0.0	0.0	0.0	-0.1	-0.1	0.0	0.0	0.0	-0.2	0.2	0.0	0.0	0.0	0.0	0.0	0.0	-0.9	0.2	-0.4	0.1	-0.5	0.2	-0.4	0.1	-0.5	0.2	2.7	-0.6	0.7	-0.1	2.8	-0.6	0.4	-0.1	1.3	-0.2	-1.1	0.4	0.0	0.0	0.0	0.0	-0.4	0.4	0.1	-0.4
	F2	y	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.2	-0.1	0.0	0.1	-0.1	0.0	0.0	-0.1	0.0	0.1	0.2	-0.9	0.0	0.0	0.0	0.0	0.0	0.0	0.2	-0.3	0.1	-0.2	0.2	-0.2	0.1	-0.2	0.2	-0.2	-0.6	2.7	-0.1	0.8	-0.6	2.7	-0.1	0.5	-0.2	1.4	0.4	-0.4	0.0	0.0	0.0	0.0	0.4	-1.0	-0.4	0.0
	F3	x	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.2	0.0	-0.1	0.0	-0.1	0.0	-0.1	0.0	-0.1	0.0	0.5	-0.1	0.2	-0.1	0.4	-0.1	0.2	-0.1	0.3	-0.1	-0.2	0.1	0.0	0.0	0.0	0.0	0.0	0.1	0.0	-0.1
	F3	y	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.1	0.5	-0.1	0.3	-0.1	0.5	-0.1	0.3	-0.1	0.4	0.1	-0.1	0.0	0.0	0.0	0.0	0.1	-0.2	-0.1	0.0
	F4	x	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.1	-0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.0	-0.1	0.1	0.0	0.0	0.0	0.0	0.0	0.0	-0.4	0.1	-0.3	0.1	-0.3	0.1	-0.3	0.1	-0.3	0.1	1.3	-0.3	0.6	-0.1	1.3	-0.2	0.3	-0.1	1.0	-0.2	-0.5	0.2	0.0	0.0	0.0	0.0	-0.2	0.2	0.0	-0.2
	F4	y	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.1	-0.2	0.0	0.0	0.0	-0.1	0.0	0.0	0.0	0.1	0.1	-0.3	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.1	-0.1	0.1	0.0	0.1	-0.1	0.1	0.0	-0.3	1.3	-0.1	0.7	-0.2	1.4	-0.1	0.4	-0.2	1.2	0.2	-0.3	0.0	0.0	0.0	0.0	0.2	-0.6	-0.2	0.1
12	N	x	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.4	-0.2	0.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-1.1	0.4	-0.3	0.1	-1.1	0.4	-0.2	0.1	-0.5	0.2	1.5	-0.4	0.0	0.0	0.0	0.0	0.0	0.0	-0.2	0.2
	N	y	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.4	0.0	0.2	-0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.4	-0.4	0.1	-0.2	0.4	-0.4	0.1	-0.1	0.2	-0.3	-0.4	1.5	0.0	0.0	0.0	0.0	0.0	0.0	0.2	-0.5
13	N	x	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.2	0.2	-0.1	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.7	-0.4	-0.5	0.2	0.0	0.0	0.0	0.0
	N	y	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.2	-0.5	0.2	-0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.4	0.7	0.2	-0.2	0.0	0.0	0.0	0.0
14	N	x	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.4	-0.1	-0.4	-0.4	0.4	-0.2	0.2	-0.3	0.2	-0.2	0.2	-0.2	0.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.5	0.2	1.5	-0.4	-0.5	0.2	0.0	0.0
	N	y	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.4	0.0	-0.4	0.0	0.4	-1.1	0.2	-0.7	0.2	-0.6	0.2	-0.6	0.2	-0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.2	-0.2	-0.4	1.5	0.2	-0.2	0.0	0.0
15	N	x	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.4	0.0	-0.2	0.1	-0.2	0.0	-0.2	0.1	-0.2	-0.4	0.4	-0.1	0.1	-0.4	0.4	0.0	0.1	-0.2	0.2	0.0	0.0	0.0	0.0	-0.5	0.2	1.5	-0.4	-0.5	0.2
	N	y	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.4	0.0	-0.2	0.1	-0.2	0.1	-0.2	0.1	-0.2	0.1	0.4	-1.1	0.1	-0.4	0.4	-1.0	0.1	-0.2	0.2	-0.6	0.0	0.0	0.0	0.0	0.2	-0.2	-0.4	1.5	0.2	-0.2
16	N	x	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.4	0.0	-0.1	0.1	-0.4	0.0	-0.1	0.0	-0.2	-0.2	0.2	0.0	0.0	0.0	0.0	-0.5	0.2	0.7	0.0
	N	y	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.4	0.0	-0.1	0.1	-0.4	0.0	-0.1	0.0	-0.2	0.1	0.2	-0.5	0.0	0.0	0.0	0.0	0.2	-0.2	0.0	0.7

*) Cells with Values(non-zero) are highlighted

Global Stiffness Matrix

			1		2		3		4		5				6										7		8		9				10										11										12		13		14		15		16	
			N	N	N	N	N	N	N	N	N	N	H	H	N	N	F1	F1	F2	F2	F3	F3	F4	F4	N	N	N	N	N	N	H	H	N	N	F1	F1	F2	F2	F3	F3	F4	F4	N	N	F1	F1	F2	F2	F3	F3	F4	F4	N	N	N	N	N	N	N	N	N	N
			x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y
1	N	x	0.7	0.0	-0.5	0.2	0.0	0.0	0.0	0.0	-0.2	0.2	0.2	-0.2	0.0	-0.4	-0.1	0.3	0.0	-0.2	0.0	-0.3	0.0	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	N	y	0.0	0.7	0.2	-0.2	0.0	0.0	0.0	0.0	0.2	-0.5	-0.2	0.5	-0.4	0.0	0.3	-0.1	-0.2	0.1	-0.3	0.1	0.1	-0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	[illegible]	[illegible]	[illegible]
2	N	x	-0.5	0.2	1.5	-0.4	-0.5	0.2	0.0	0.0	0.0	0.0	0.0	0.0	-0.4	0.4	0.3	-0.3	-0.2	0.2	-0.2	0.3	0.1	-0.1	0.0	-0.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	[illegible]	[illegible]	[illegible]
	N	y	0.2	-0.2	-0.4	1.5	0.2	-0.2	0.0	0.0	0.0	0.0	0.0	0.0	0.4	-1.1	-0.3	0.7	0.2	-0.6	0.3	-0.6	-0.1	0.5	-0.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	[illegible]	[illegible]	[illegible]
3	N	x	0.0	0.0	-0.5	0.2	1.5	-0.4	-0.5	0.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.4	0.4	0.0	-0.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	[illegible]	[illegible]	[illegible]
	N	y	0.0	0.0	0.2	-0.2	-0.4	1.5	0.2	-0.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.4	-1.1	-0.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	[illegible]	[illegible]	[illegible]
4	N	x	0.0	0.0	0.0	0.0	-0.5	0.2	0.7	-0.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.2	0.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	[illegible]	[illegible]	[illegible]
	N	y	0.0	0.0	0.0	0.0	0.2	-0.2	-0.4	0.7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.2	-0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	[illegible]	[illegible]	[illegible]
5	N	x	-0.2	0.2	0.0	0.0	0.0	0.0	0.0	0.0	1.5	-0.4	-0.9	0.4	-1.1	0.4	0.8	-0.3	-0.4	0.2	-0.6	0.3	0.3	-0.1	0.0	0.0	0.0	0.0	-0.2	0.2	-0.1	0.1	0.0	-0.4	0.0	0.0	-0.1	-0.1	-0.1	-0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	[illegible]	[illegible]	[illegible]
	N	y	0.2	-0.5	0.0	0.0	0.0	0.0	0.0	0.0	-0.4	1.5	0.4	-0.6	0.4	-0.4	-0.3	0.3	0.2	-0.2	0.3	-0.3	-0.1	0.2	0.0	0.0	0.0	0.0	0.2	-0.5	0.1	-0.3	-0.4	0.0	0.0	0.0	-0.1	-0.1	-0.1	-0.1	0.0	-0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	[illegible]	[illegible]	[illegible]
	H	x	0.2	-0.2	0.0	0.0	0.0	0.0	0.0	0.0	-0.9	0.4	1.5	-0.4	0.8	-0.3	-1.1	0.4	0.4	-0.2	0.6	-0.3	-0.4	0.1	0.0	0.0	0.0	0.0	-0.1	0.1	-0.2	0.2	0.0	0.0	0.0	-0.4	0.0	0.0	-0.1	-0.1	-0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	[illegible]	[illegible]	[illegible]
	H	y	-0.2	0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.4	-0.6	-0.4	1.5	-0.3	0.3	0.4	-0.4	-0.2	0.2	-0.3	0.3	0.1	-0.2	0.0	0.0	0.0	0.0	0.1	-0.3	0.2	-0.5	0.0	0.0	-0.4	0.0	0.0	-0.1	-0.1	-0.2	0.0	-0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	[illegible]	[illegible]	[illegible]
6	N	x	0.0	-0.4	-0.4	0.4	0.0	0.0	0.0	0.0	-1.1	0.4	0.9	-0.3	3.0	-0.7	-1.5	0.3	1.3	-0.3	1.5	-0.3	-1.0	0.2	-1.1	0.4	0.0	0.0	0.0	0.0	0.0	0.0	-0.4	0.4	0.0	0.0	-0.1	0.1	-0.1	0.1	-0.1	0.0	0.0	-0.4	-0.1	0.0	-0.1	-0.2	0.0	0.0	-0.1	-0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	[illegible]	[illegible]	[illegible]
	N	y	-0.4	0.0	0.4	-1.1	0.0	0.0	0.0	0.0	0.4	-0.4	-0.3	0.3	-0.7	3.0	0.3	-1.4	-0.3	1.3	-0.3	1.4	0.2	-1.0	0.4	-0.4	0.0	0.0	0.0	0.0	0.0	0.0	0.4	-1.1	0.0	0.0	0.1	-0.4	0.1	-0.4	0.0	-0.3	-0.4	0.0	0.0	-0.1	-0.2	-0.1	0.0	-0.1	-0.1	-0.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	[illegible]	[illegible]	[illegible]
	F1	x	-0.1	0.3	0.3	-0.3	0.0	0.0	0.0	0.0	0.6	-0.2	-0.9	0.3	-1.5	0.3	1.5	-0.3	-0.7	0.2	-1.1	0.2	0.6	-0.1	0.5	-0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.1	-0.1	-0.1	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	[illegible]	[illegible]	[illegible]
	F1	y	0.3	-0.1	-0.3	0.7	0.0	0.0	0.0	0.0	-0.3	0.3	0.3	-0.3	0.3	-1.4	-0.3	1.5	0.2	-0.8	0.2	-1.1	-0.1	0.8	-0.1	0.1	0.0	0.0	0.0	0.0	0.0	0.0	-0.1	0.3	0.1	-0.4	0.0	-0.1	0.0	-0.1	0.0	-0.1	0.1	0.1	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	[illegible]	[illegible]	[illegible]
	F2	x	0.0	-0.2	-0.2	0.2	0.0	0.0	0.0	0.0	-0.4	0.2	0.4	-0.2	1.3	-0.3	-0.7	0.2	0.9	-0.2	0.7	-0.2	-0.6	0.2	-0.6	0.2	0.0	0.0	0.0	0.0	0.0	0.0	-0.3	0.2	0.0	0.0	-0.1	0.1	-0.1	0.1	0.0	0.0	0.1	-0.2	0.0	0.0	0.0	-0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	F2	y	-0.2	0.1	0.2	-0.6	0.0	0.0	0.0	0.0	0.2	-0.2	-0.2	0.2	-0.3	1.3	0.2	-0.8	-0.2	0.9	-0.2	0.9	0.2	-0.6	0.2	-0.2	0.0	0.0	0.0	0.0	0.0	0.0	0.2	-0.5	0.0	0.1	0.1	-0.1	0.1	-0.1	0.0	-0.1	-0.2	0.0	0.0	-0.1	-0.1	0.0	0.0	0.0	0.0	-0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	F3	x	0.0	-0.3	-0.2	0.3	0.0	0.0	0.0	0.0	-0.6	0.3	0.6	-0.2	1.5	-0.3	-1.1	0.2	0.7	-0.2	1.1	-0.2	-0.6	0.2	-0.5	0.1	0.0	0.0	0.0	0.0	0.0	0.0	-0.1	0.1	0.1	-0.1	0.0	0.0	0.0	0.0	0.0	0.0	-0.1	-0.1	0.0	0.0	-0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	F3	y	-0.3	0.1	0.3	-0.6	0.0	0.0	0.0	0.0	0.3	-0.3	-0.2	0.3	-0.3	1.4	0.2	-1.1	-0.2	0.9	-0.2	1.2	0.2	-0.7	0.1	-0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.1	-0.4	-0.1	0.2	0.0	0.0	0.0	0.0	0.0	0.0	-0.1	-0.1	0.0	0.0	0.0	-0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	F4	x	0.0	0.1	0.1	-0.1	0.0	0.0	0.0	0.0	0.3	-0.1	-0.4	0.1	-1.0	0.2	0.6	-0.1	-0.6	0.2	-0.6	0.2	0.7	-0.2	0.3	-0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.1	-0.1	-0.1	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	F4	y	0.1	-0.1	-0.1	0.5	0.0	0.0	0.0	0.0	-0.1	0.2	0.1	-0.2	0.2	-1.0	-0.1	0.8	0.2	-0.6	0.2	-0.7	-0.2	0.8	-0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.1	0.2	0.1	-0.2	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.2	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7	N	x	0.0	0.0	0.0	-0.4	-0.4	0.4	0.0	0.0	0.0	0.0	0.0	0.0	-1.1	0.4	0.5	-0.1	-0.6	0.2	-0.5	0.1	0.3	-0.1	3.0	-0.7	-1.1	0.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.4	0.4	-0.1	0.0	-0.2	0.2	-0.1	0.0	-0.1	0.1	0.0	-0.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	N	y	0.0	0.0	-0.4	0.0	0.4	-1.1	0.0	0.0	0.0	0.0	0.0	0.0	0.4	-0.4	-0.1	0.1	0.2	-0.2	0.1	-0.1	-0.1	0.0	-0.7	3.0	0.4	-0.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.4	-1.1	0.0	-0.2	0.2	-0.9	0.0	-0.1	0.1	-0.3	-0.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
8	N	x	0.0	0.0	0.0	0.0	0.0	-0.4	-0.2	0.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-1.1	0.4	1.5	-0.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.2	0.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	N	y	0.0	0.0	0.0	0.0	-0.4	0.0	0.2	-0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.4	-0.4	-0.4	1.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.2	-0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
9	N	x	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.2	0.2	-0.1	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.5	-0.4	1.1	-0.2	-1.1	0.4	-0.8	0.3	-0.4	0.1	-0.5	0.1	-0.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.2	0.2	0.0	-0.4	0.0	0.0	0.0	0.0
	N	y	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.2	-0.5	0.1	-0.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.4	1.5	-0.2	1.1	0.4	-0.4	0.3	-0.3	0.1	-0.1	0.1	-0.1	0.0	-0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.2	-0.5	-0.4	0.0	0.0	0.0	0.0	0.0
	H	x	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.1	0.1	-0.2	0.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.1	-0.2	1.5	-0.4	-0.8	0.3	-1.1	0.4	-0.3	0.0	-0.4	0.1	-0.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.2	0.2	0.0	-0.4	0.0	0.0	0.0	0.0
	H	y	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	-0.3	0.2	-0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.2	1.1	-0.4	1.5	0.3	-0.3	0.4	-0.4	0.0	0.0	0.1	0.0	0.0	-0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.2	-0.5	-0.4	0.0	0.0	0.0	0.0	0.0
10	N	x	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.4	0.0	0.0	-0.4	0.4	0.1	-0.1	-0.3	0.2	-0.1	0.1	0.1	-0.1	0.0	0.0	0.0	0.0	-1.1	0.4	-0.9	0.3	3.0	-0.7	1.3	-0.4	1.2	-0.3	1.4	-0.3	1.1	-0.3	-1.1	0.4	-0.2	0.0	-0.9	0.2	-0.2	0.0	-0.4	0.1	0.0	0.0	0.0	0.0	-0.4	0.4	0.0	-0.4	0.0	0.0
	N	y	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.4	0.0	0.0	0.0	0.4	-1.1	-0.1	0.3	0.2	-0.5	0.1	-0.4	-0.1	0.2	0.0	0.0	0.0	0.0	0.4	-0.4	0.3	-0.3	-0.7	3.0	-0.4	1.1	-0.3	1.3	-0.3	1.4	-0.3	1.1	0.4	-0.4	0.0	0.0	0.2	-0.3	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.0	0.4	-1.1	-0.4	0.0	0.0	0.0
	F1	x	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.3	0.0	0.0	-0.1	0.1	0.0	0.0	0.1	-0.1	-0.1	0.1	0.0	0.0	0.0	0.0	-0.5	0.2	-0.9	0.3	1.3	-0.4	1.5	-0.4	0.7	-0.2	0.9	-0.3	0.6	-0.2	-0.5	0.2	-0.2	0.1	-0.4	0.1	-0.1	0.0	-0.3	0.1	0.0	0.0	0.0	0.0	-0.2	0.2	0.0	-0.2	0.0	0.0
	F1	y	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.3	0.0	0.0	0.0	0.1	-0.4	0.0	0.1	-0.1	0.2	0.1	-0.2	0.0	0.0	0.0	0.0	0.2	-0.2	0.3	-0.3	-0.4	1.1	-0.4	1.5	-0.2	0.8	-0.3	0.9	-0.2	0.7	0.2	-0.3	0.1	-0.1	0.1	-0.2	0.0	0.0	0.1	-0.1	0.0	0.0	0.0	0.0	0.2	-0.7	-0.2	0.1	0.0	0.0
	F2	x	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.1	-0.1	0.0	0.0	-0.1	0.1	0.0	0.0	-0.1	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.4	0.1	-0.4	0.1	1.2	-0.3	0.7	-0.2	0.8	-0.1	0.7	-0.1	0.8	-0.1	-0.5	0.2	-0.1	0.1	-0.5	0.2	-0.1	0.0	-0.3	0.1	0.0	0.0	0.0	0.0	-0.3	0.2	0.1	-0.2	0.0	0.0
	F2	y	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.1	-0.1	0.0	-0.1	0.1	-0.4	0.0	-0.1	0.1	-0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	-0.1	0.1	-0.1	-0.3	1.3	-0.2	0.8	-0.1	0.9	-0.1	0.9	-0.1	0.8	0.2	-0.2	0.1	0.0	0.2	-0.2	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.0	0.2	-0.6	-0.2	0.1	0.0	0.0
	F3	x	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.1	-0.1	0.0	-0.1	-0.1	0.1	0.0	0.0	-0.1	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.5	0.1	-0.5	0.1	1.4	-0.3	0.9	-0.3	0.7	-0.1	1.0	-0.2	0.7	-0.1	-0.5	0.2	-0.2	0.1	-0.4	0.1	-0.1	0.0	-0.3	0.1	0.0	0.0	0.0	0.0	-0.2	0.2	0.0	-0.2	0.0	0.0
	F3	y	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.1	-0.1	-0.1	-0.1	0.1	-0.4	0.0	-0.1	0.1	-0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	-0.1	0.1	-0.1	-0.3	1.4	-0.3	0.9	-0.1	0.9	-0.2	1.1	-0.1	0.8	0.2	-0.3	0.1	-0.1	0.1	-0.2	0.0	0.0	0.1	-0.1	0.0	0.0	0.0	0.0	0.2	-0.6	-0.2	0.1	0.0	0.0
	F4	x	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.3	0.0	-0.3	0.0	1.1	-0.3	0.6	-0.2	0.8	-0.1	0.7	-0.1	0.8	-0.1	-0.5	0.2	-0.1	0.1	-0.5	0.2	-0.1	0.0	-0.3	0.1	0.0	0.0	0.0	0.0	-0.2	0.2	0.1	-0.2	0.0	0.0
	F4	y	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.1	0.0	-0.1	0.0	-0.3	0.0	-0.1	0.0	-0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.1	0.0	-0.1	-0.3	1.1	-0.2	0.7	-0.1	0.8	-0.1	0.8	-0.1	0.8	0.2	-0.2	0.1	0.0	0.2	-0.2	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.0	0.2	-0.5	-0.2	0.1	0.0	0.0
11	N	x	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.4	0.1	0.1	0.1	-0.2	-0.1	-0.1	0.0	0.1	-0.4	0.4	0.0	0.0	0.0	0.0	0.0	0.0	-1.1	0.4	-0.5	0.2	-0.5	0.2	-0.5	0.2	-0.5	0.2	3.0	-0.7	0.7	-0.2	2.7	-0.6	0.5	-0.1	1.3	-0.3	-1.1	0.4	0.0	0.0	0.0	0.0	-0.4	0.4	0.0	-0.4
	N	y	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.4	0.0	0.1	0.1	-0.2	0.0	-0.1	-0.1	0.1	0.2	0.4	-1.1	0.0	0.0	0.0	0.0	0.0	0.0	0.4	-0.4	0.2	-0.3	0.2	-0.2	0.2	-0.3	0.2	-0.2	-0.7	3.0	-0.2	0.7	-0.6	2.7	-0.1	0.5	-0.3	1.3	0.4	-0.4	0.0	0.0	0.0	0.0	0.4	-1.1	-0.4	0.0
	F1	x	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.2	0.0	-0.2	0.1	-0.1	0.1	-0.2	0.1	-0.1	0.1	0.7	-0.2	0.4	-0.1	0.7	-0.1	0.2	-0.1	0.6	-0.1	-0.3	0.1	0.0	0.0	0.0	0.0	-0.1	0.1	0.0	-0.1
	F1	y	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.1	0.0	0.0	0.0	-0.1	0.0	0.0	0.0	0.0	0.0	-0.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	-0.1	0.1	0.0	0.1	-0.1	0.1	0.0	-0.2	0.7	-0.1	0.4	-0.1	0.8	-0.1	0.3	-0.1	0.7	0.1	-0.2	0.0	0.0	0.0	0.0	0.1	-0.4	-0.1	0.1
	F2	x	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.1	-0.2	0.0	0.0	0.0	-0.1	-0.1	0.0	0.0	0.0	-0.2	0.2	0.0	0.0	0.0	0.0	0.0	0.0	-0.9	0.2	-0.4	0.1	-0.5	0.2	-0.4	0.1	-0.5	0.2	2.7	-0.6	0.7	-0.1	2.8	-0.6	0.4	-0.1	1.3	-0.2	-1.1	0.4	0.0	0.0	0.0	0.0	-0.4	0.4	0.1	-0.4
	F2	y	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.2	-0.1	0.0	0.1	-0.1	0.0	0.0	-0.1	0.0	0.1	0.2	-0.9	0.0	0.0	0.0	0.0	0.0	0.0	0.2	-0.3	0.1	-0.2	0.2	-0.2	0.1	-0.2	0.2	-0.2	-0.6	2.7	-0.1	0.8	-0.6	2.7	-0.1	0.5	-0.2	1.4	0.4	-0.4	0.0	0.0	0.0	0.0	0.4	-1.0	-0.4	0.0
	F3	x	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.2	0.0	-0.1	0.0	-0.1	0.0	-0.1	0.0	-0.1	0.0	0.5	-0.1	0.2	-0.1	0.4	-0.1	0.2	-0.1	0.3	-0.1	-0.2	0.1	0.0	0.0	0.0	0.0	0.0	0.1	0.0	-0.1
	F3	y	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.1	0.5	-0.1	0.3	-0.1	0.5	-0.1	0.3	-0.1	0.4	0.1	-0.1	0.0	0.0	0.0	0.0	0.1	-0.2	-0.1	0.0
	F4	x	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.1	-0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.0	-0.1	0.1	0.0	0.0	0.0	0.0	0.0	0.0	-0.4	0.1	-0.3	0.1	-0.3	0.1	-0.3	0.1	-0.3	0.1	1.3	-0.3	0.6	-0.1	1.3	-0.2	0.3	-0.1	1.0	-0.2	-0.5	0.2	0.0	0.0	0.0	0.0	-0.2	0.2	0.0	-0.2
	F4	y	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.1	-0.2	0.0	0.0	0.0	-0.1	0.0	0.0	0.0	0.1	0.1	-0.3	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.1	-0.1	0.1	0.0	0.1	-0.1	0.1	0.0	-0.3	1.3	-0.1	0.7	-0.2	1.4	-0.1	0.4	-0.2	1.2	0.2	-0.3	0.0	0.0	0.0	0.0	0.2	-0.6	-0.2	0.1
12	N	x	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.4	-0.2	0.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-1.1	0.4	-0.3	0.1	-1.1	0.4	-0.2	0.1	-0.5	0.2	1.5	-0.4	0.0	0.0	0.0	0.0	0.0	0.0	-0.2	0.2
	N	y	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.4	0.0	0.2	-0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.4	-0.4	0.1	-0.2	0.4	-0.4	0.1	-0.1	0.2	-0.3	-0.4	1.5	0.0	0.0	0.0	0.0	0.0	0.0	0.2	-0.5
13	N	x	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.2	0.2	-0.1	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.7	-0.4	-0.5	0.2	0.0	0.0	0.0	0.0
	N	y	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.2	-0.5	0.2	-0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.4	0.7	0.2	-0.2	0.0	0.0	0.0	0.0
14	N	x	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.4	-0.1	-0.4	-0.4	0.4	-0.2	0.2	-0.3	0.2	-0.2	0.2	-0.2	0.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.5	0.2	1.5	-0.4	-0.5	0.2	0.0	0.0
	N	y	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.4	0.0	-0.4	0.0	0.4	-1.1	0.2	-0.7	0.2	-0.6	0.2	-0.6	0.2	-0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.2	-0.2	-0.4	1.5	0.2	-0.2	0.0	0.0
15	N	x	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.4	0.0	-0.2	0.1	-0.2	0.0	-0.2	0.1	-0.2	-0.4	0.4	-0.1	0.1	-0.4	0.4	0.0	0.1	-0.2	0.2	0.0	0.0	0.0	0.0	-0.5	0.2	1.5	-0.4	-0.5	0.2
	N	y	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.4	0.0	-0.2	0.1	-0.2	0.1	-0.2	0.1	-0.2	0.1	0.4	-1.1	0.1	-0.4	0.4	-1.0	0.1	-0.2	0.2	-0.6	0.0	0.0	0.0	0.0	0.2	-0.2	-0.4	1.5	0.2	-0.2
16	N	x	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.4	0.0	-0.1	0.1	-0.4	0.0	-0.1	0.0	-0.2	-0.2	0.2	0.0	0.0	0.0	0.0	-0.5	0.2	0.7	0.0
	N	y	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.4	0.0	-0.1	0.1	-0.4	0.0	-0.1	0.0	-0.2	0.1	0.2	-0.5	0.0	0.0	0.0	0.0	0.2	-0.2	0.0	0.7



*) Cells with Values(non-zero) are highlighted

Next Steps

- ❑ **Apply Boundary Conditions and Solve!**

  → You got all the unknowns (**u**, **b**, **c**)

- ❑ **Calculate Displacement**

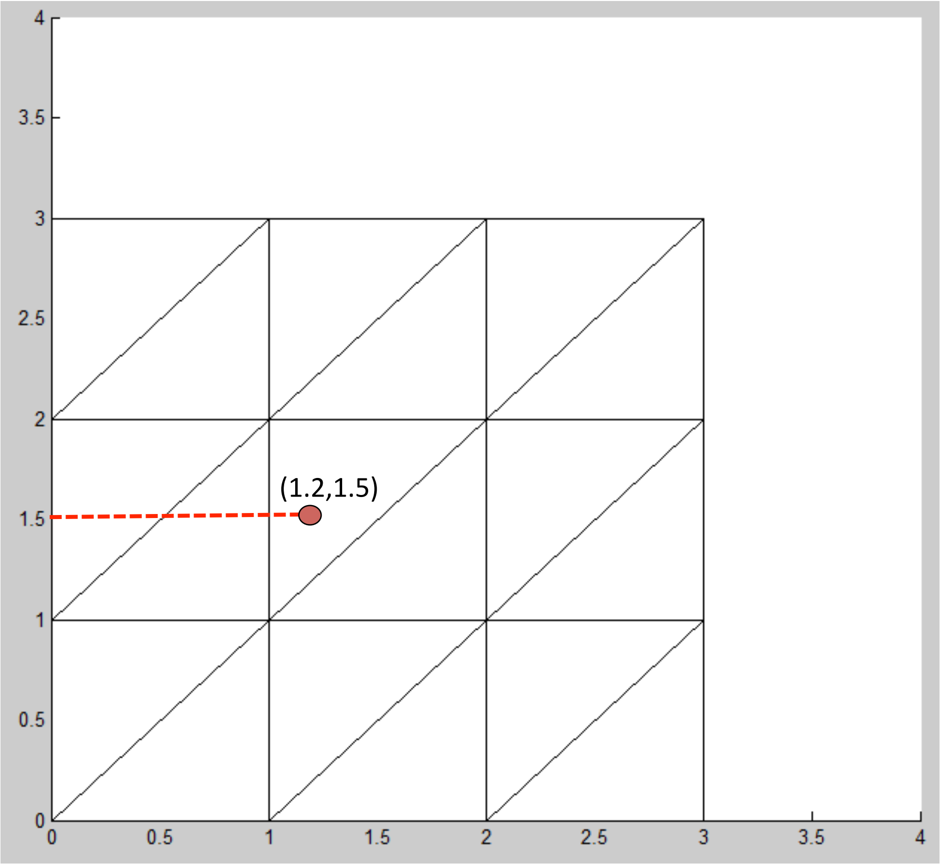
$$\boldsymbol{u}^{h} = \sum_{i \in I} N_i(\boldsymbol{x})\boldsymbol{u}_{\boldsymbol{i}} + \sum_{j \in J} N_j(\boldsymbol{x}) H(\boldsymbol{x})\boldsymbol{b}_{\boldsymbol{i}} + \sum_{k \in I} \sum_{l=1}^{4} N_k \,($$

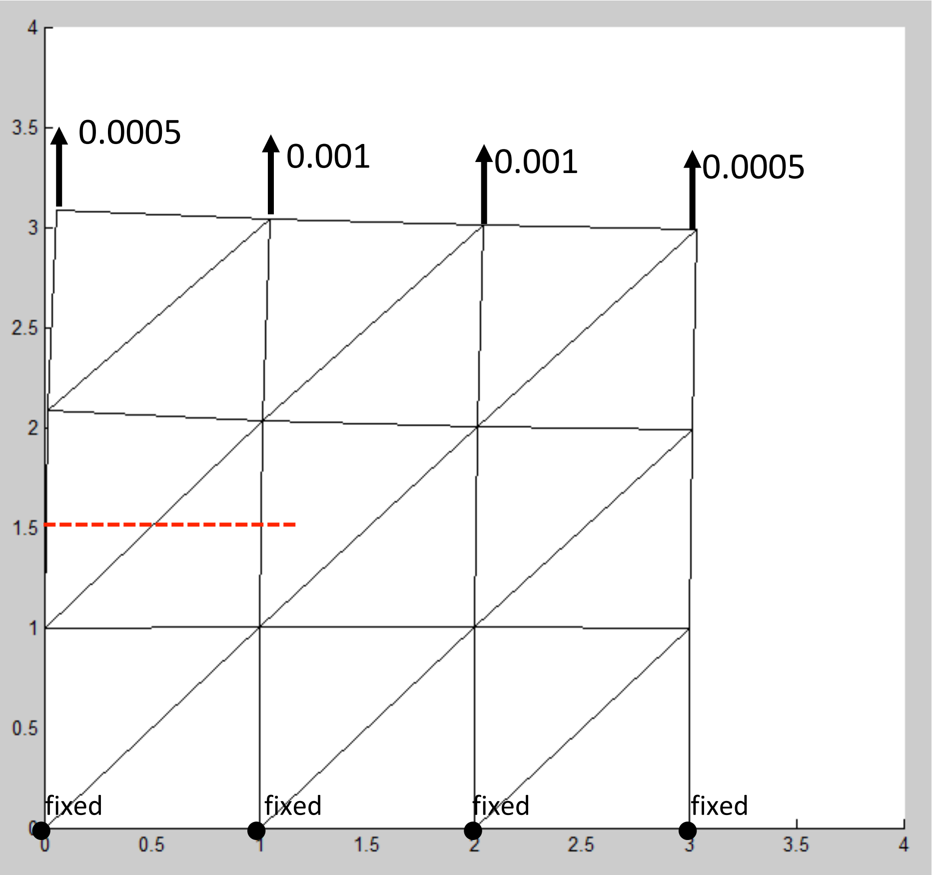
**) For nodal displacement consideration only, following formulation is more convenient.*

$$\boldsymbol{u}^{h} = \sum_{i \in I} N_i(\boldsymbol{x})\boldsymbol{u}_{\boldsymbol{i}} + \sum_{j \in J} N_j(\boldsymbol{x})[H(\boldsymbol{x}) - H(\boldsymbol{x}_j)]\boldsymbol{b}_{\boldsymbol{i}} + \sum_{k \in I} \sum_{l=1}^{4} N_k(\boldsymbol{x})[F_l(\boldsymbol{x}) - F_l(\boldsymbol{x}_j)]\boldsymbol{c}_{\boldsymbol{k}}^{\boldsymbol{l}}$$

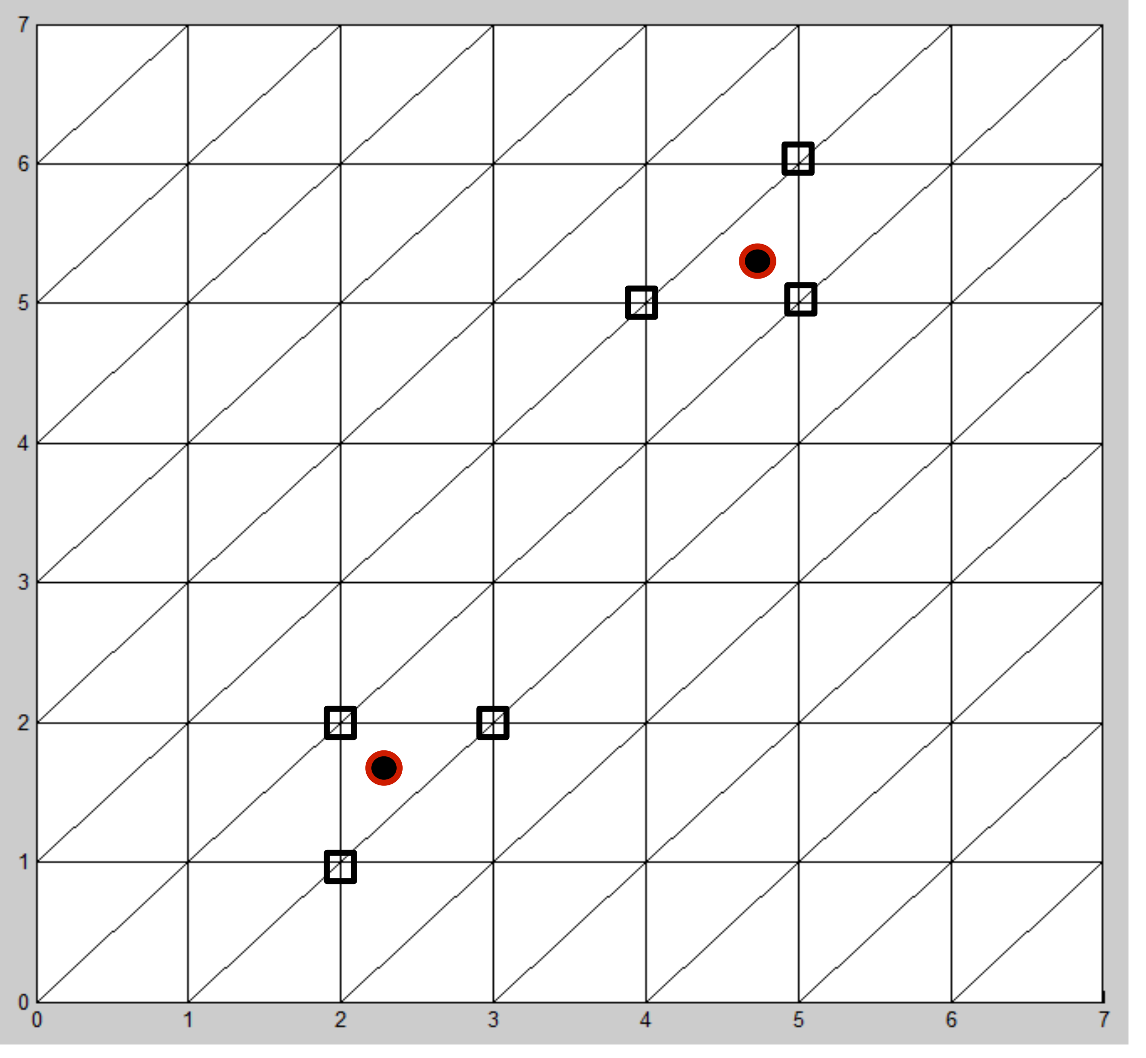
Result

E=1, ν=0.3





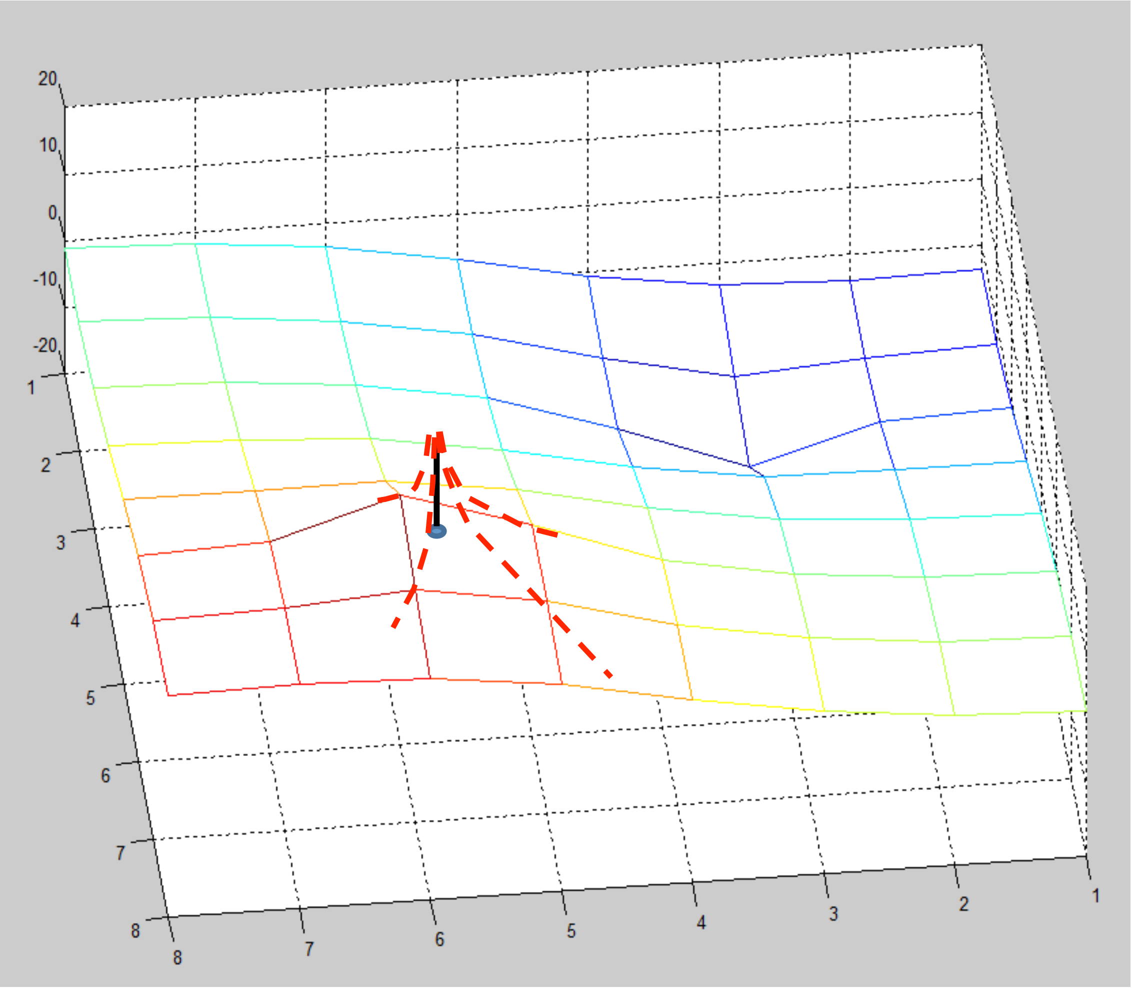
Well Enrichment in Darcy Flow



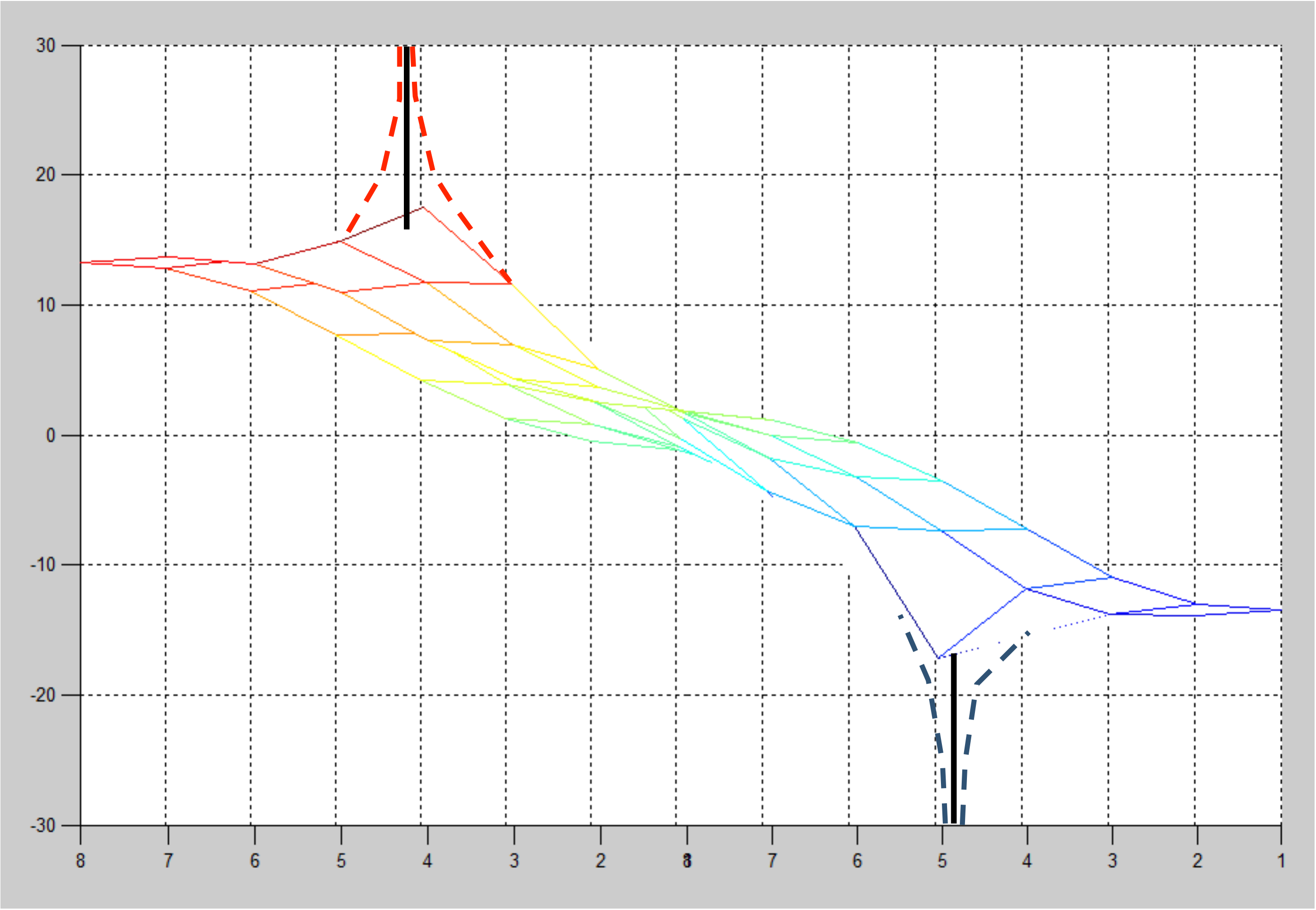
Well Enrichment in Darcy Flow

➢ Result

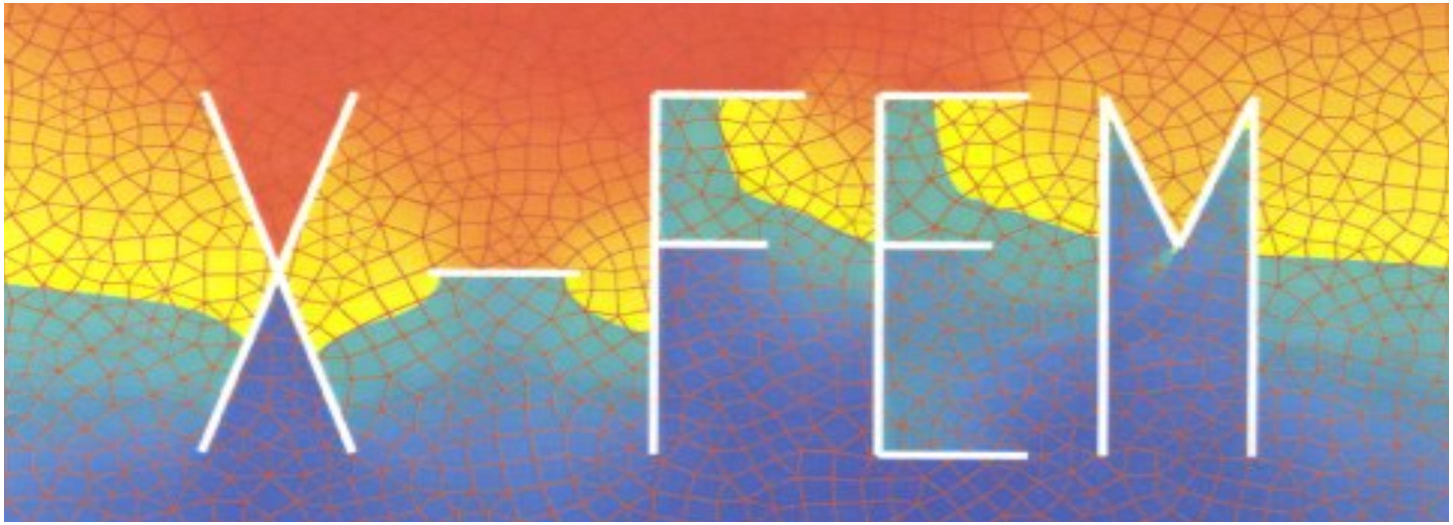
Solution for P_{well} 30 / -30 Rw=0.025



Well Enrichment in Darcy Flow

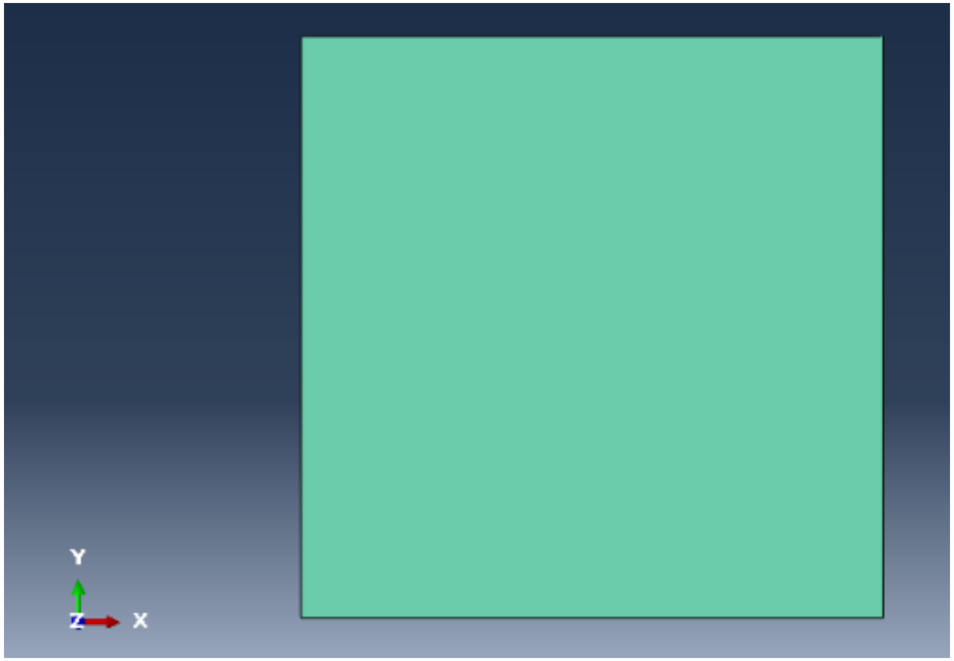


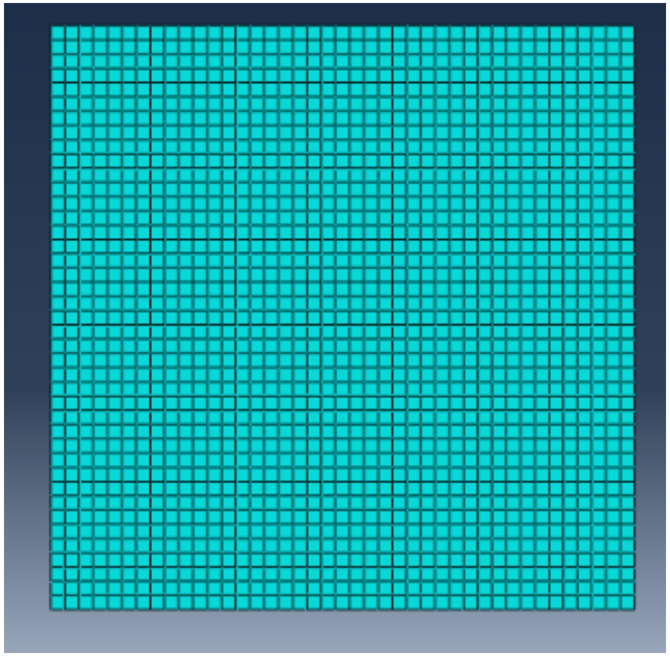
Numerical Simulation
ABAQUS



Numerical Example

- ABAQUS XFEM: 2D Edge Crack





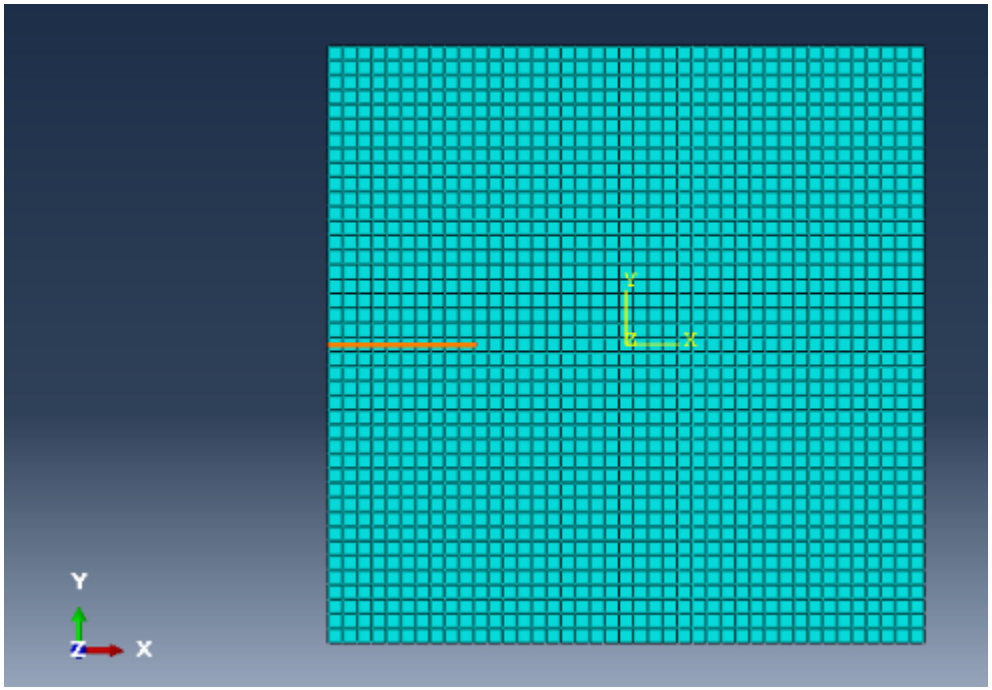
2D planner, shell, Square Plate: 4x4

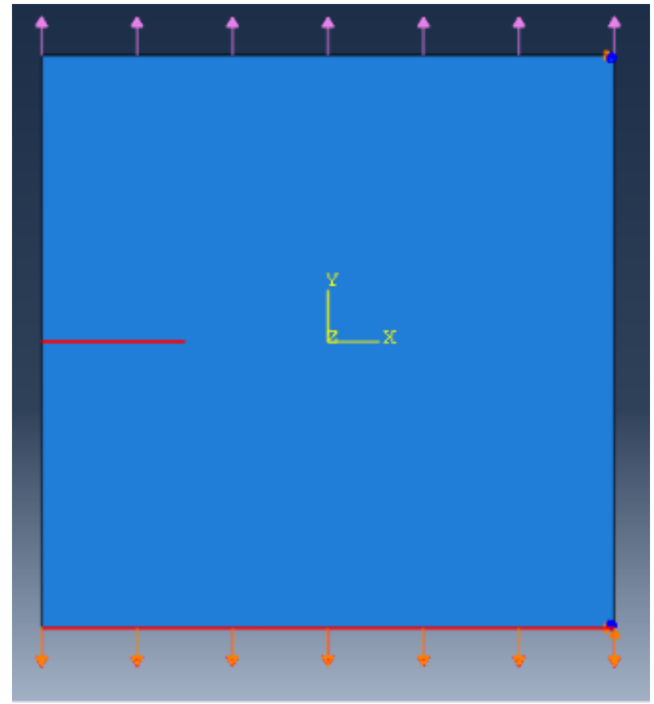
Material: Aluminum, Young's modulus:70 GPa, Poisson's ratio: 0.33

Mesh: Quad, Structured

Numerical Example

➢ ABAQUS XFEM: 2D Edge Crack

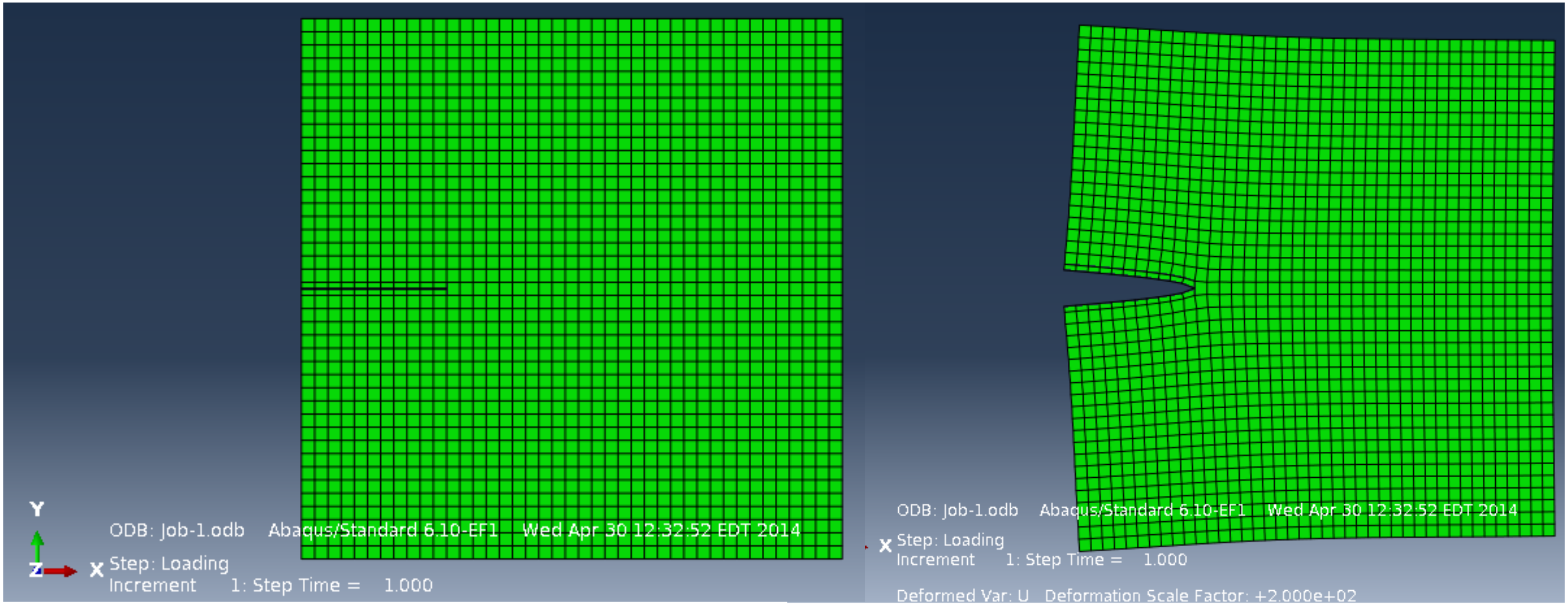




Crack: 2D Planar, wire, 1m Edge crack
Load: Top Pressure and bottom pressure : tensile stress: 10 MPa
BC: Fixed at bottom right corner, Roller at top right corner

Numerical Example

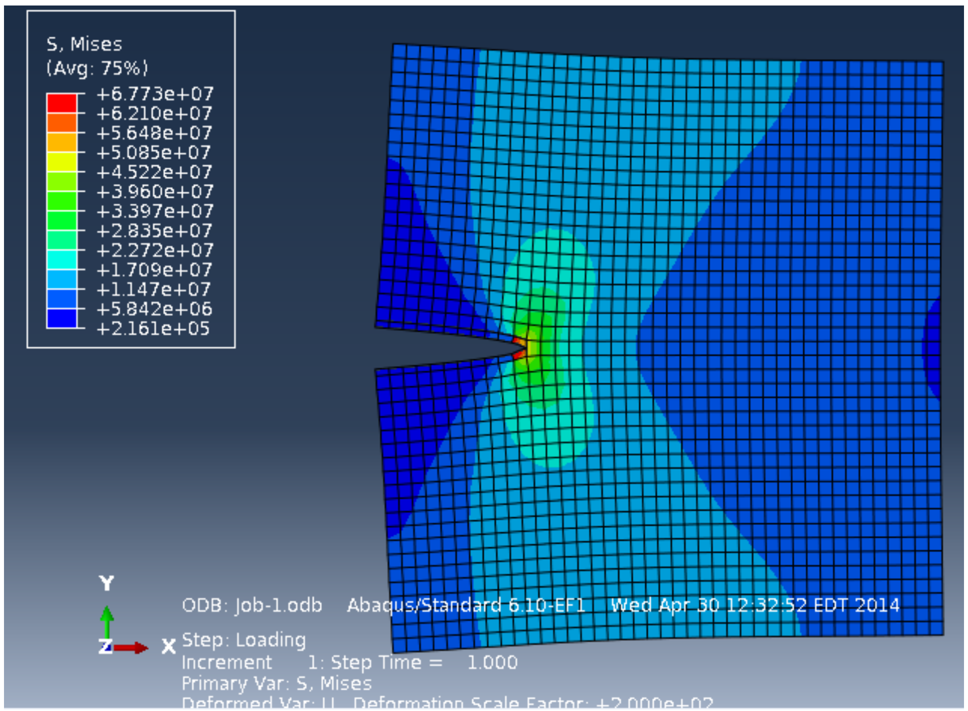
➤ ABAQUS XFEM: 2D Edge Crack - Results

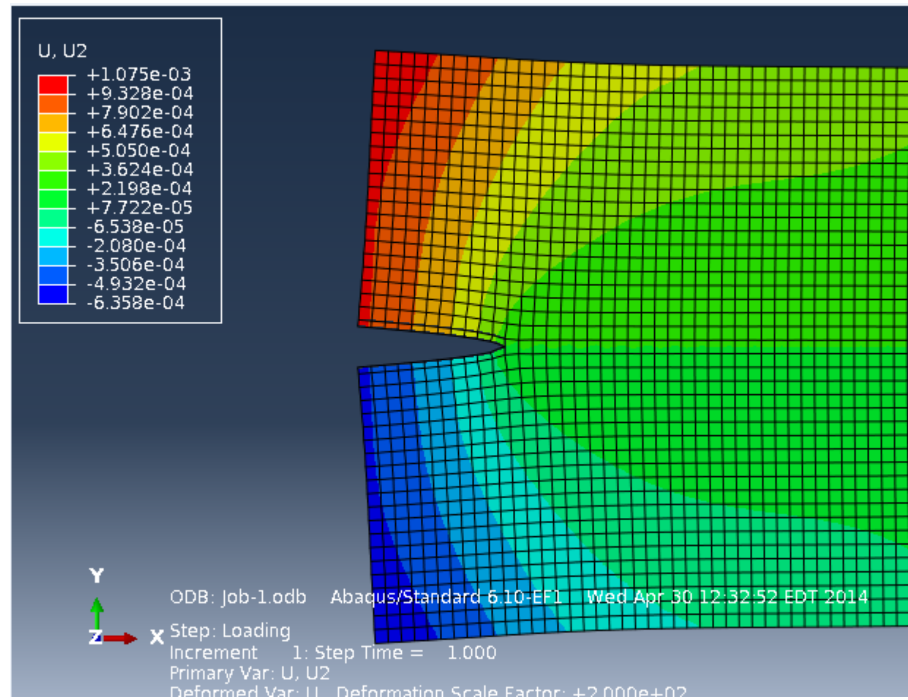


Undeformed and deformed shape

Numerical Example

➢ ABAQUS XFEM: 2D Edge Crack - Results

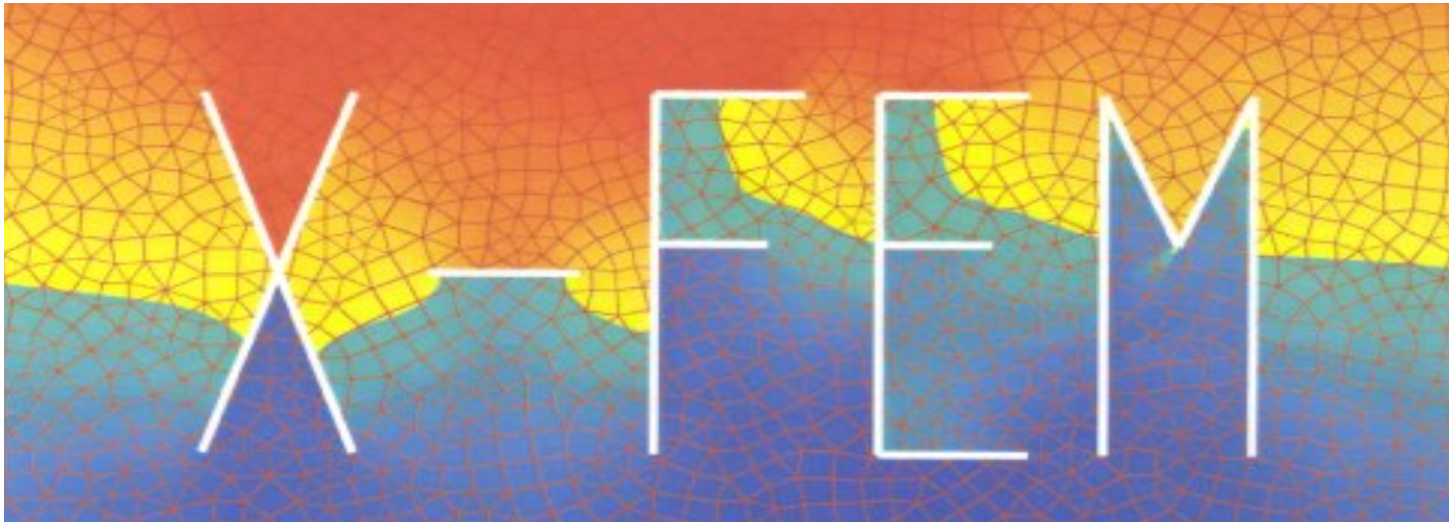




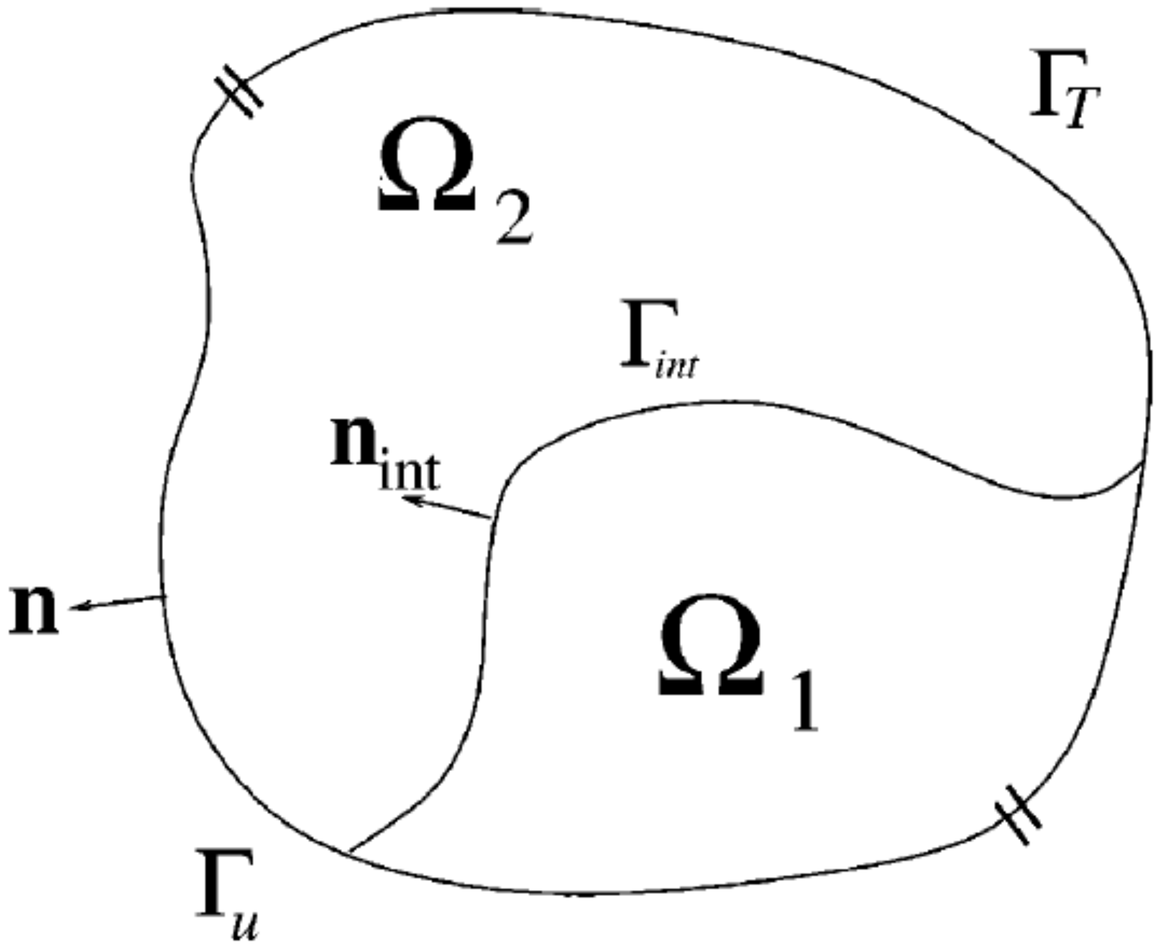
Stress and displacement contours on deformed shape

Extended finite element method for two-phase fluids

Adapted from Chessa, J., Belytschko, T.



Problem domain



Governing equations

- Isothermal, incompressible two-phase flow:

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t}+\nabla\cdot(\mathbf{u}\otimes\mathbf{u})-\mathbf{f}\right)-\nabla\cdot\boldsymbol{\sigma}=\mathbf{0}$$

$$\nabla\cdot\mathbf{u}=0$$

  - u: velocity field
  - ρ: density
  - f: applied body force
  - σ: Cauchy stress

Governing equations

- Isothermal, incompressible two-phase flow:

$$\boldsymbol{\sigma} = \boldsymbol{\tau} - p\mathbf{I}.$$

$$\boldsymbol{\tau} = 2\mu\mathbf{D}$$

$$\mathbf{D} = \frac{1}{2}(\nabla\mathbf{u} + \mathbf{u}\nabla)$$

  - τ: deviatoric stress
  - p: hydrostatic pressure
  - μ: viscosity
  - D: deformation tensor

Governing equations

- Isothermal, incompressible two-phase flow:

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot (\mathbf{u} \otimes \mathbf{u})\right) + \nabla p = \nabla \cdot (\mu \nabla \mathbf{u}) + \mathbf{f}$$

$$\nabla \cdot \mathbf{u} = 0$$

Fluid interphase tracking

- Level set method:

$$\phi(\mathbf{x},t)\begin{cases} >0 & \forall \mathbf{x}\in\Omega_1 \\ =0 & \forall \mathbf{x}\in\Gamma_{\text{int}} \\ <0 & \forall \mathbf{x}\in\Omega_2 . \end{cases}$$

- Density and viscosity global definitions:

$$\rho(\phi)=\begin{cases} \rho_1 & \phi\geqslant 0 \\ \rho_2 & \phi<0 \end{cases} \quad \forall \mathbf{x}\in\Omega$$

$$\rho(\phi)=\rho_2+\mathcal{H}(\phi)(\rho_1-\rho_2) \quad \forall \mathbf{x}\in\Omega$$

$$\mu(\phi)=\begin{cases} \mu_1 & \phi\geqslant 0 \\ \mu_2 & \phi<0 \end{cases} \quad \forall \mathbf{x}\in\Omega$$

$$\mu(\phi)=\mu_2+\mathcal{H}(\phi)(\mu_1-\mu_2) \quad \forall \mathbf{x}\in\Omega$$

Fluid interphase tracking

- Immiscibility condition (no flow across the interphase):

$$\frac{\partial}{\partial t}\phi(\mathbf{X},t)=0 \quad \forall \mathbf{X}\in\Gamma_{\text{int}}$$

  - X: position of a point that remains in the interphase.

Fluid interphase tracking

- Expanding the immiscibility condition to include the spatial coordinates, we present the standard version of the level set equation:

$$\frac{\partial \phi}{\partial t} + \mathbf{u} \cdot \nabla \phi = 0.$$

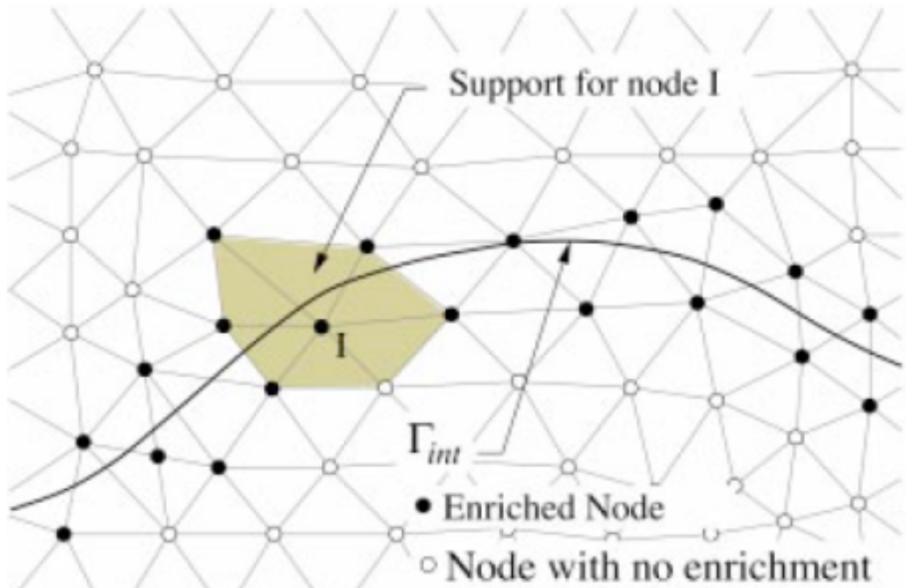
- This equation is used to update the level set, and consequently update the interphase location.

Enriched FEM with discontinuous gradient

- The domain Ω is subdivided into elements Ω_e associated with a set of nodes x_I, I=1:n.
- The shape function associated with each node I is denoted by $N_I(\mathbf{x})$ and the set of all nodes by N.

Enriched FEM with discontinuous gradient

- The support of $N_I(\mathbf{x})$, which is the area over which it is non-zero, is limited to the elements connected to node I (i.e. the support is compact).



Enriched FEM with discontinuous gradient

- The enriched approximation of the velocity field is given by:

$$\mathbf{u}^h(\mathbf{x},t)=\sum_{I\in\mathcal{N}} N_I(\mathbf{x})\mathbf{U}_I(t)+\sum_{J\in\mathcal{N}_{\text{enrich}}} N_J^{\text{enrich}}(\mathbf{x},t)\mathbf{A}_J(t)$$

where

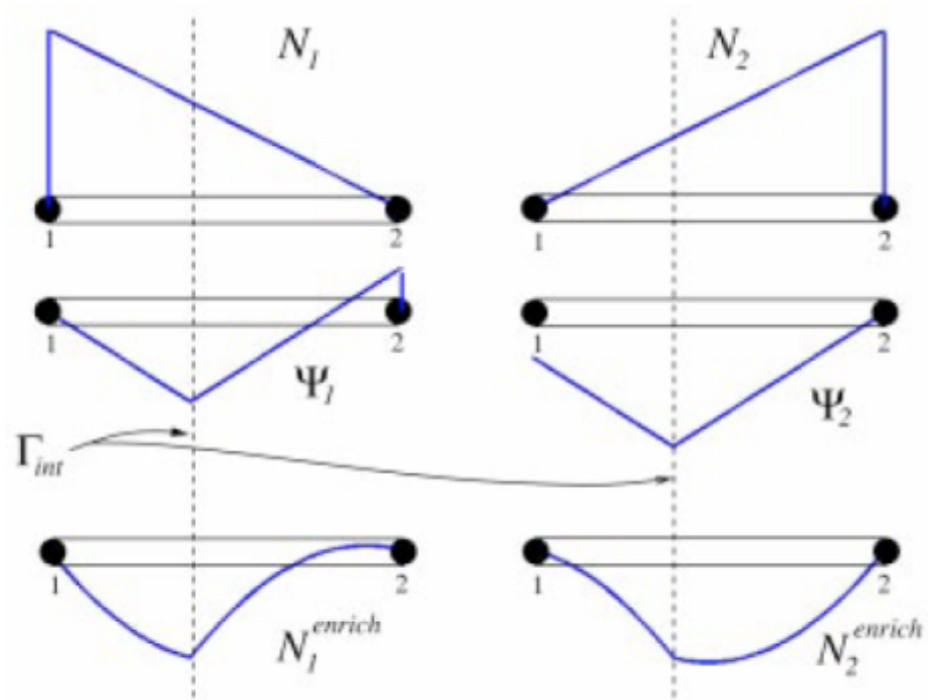
$$N_J^{\text{enrich}}(\mathbf{x},t)=N_J(\mathbf{x})\Psi_J(\mathbf{x},t)$$

Enrichment function ← $$\Psi_J(\mathbf{x},t)=|\phi^h(\mathbf{x},t)|-|\phi^h(\mathbf{x}_J,t)|.$$ → Not essential, but yields: $u^h(x_I,t)=U_I(t)$

- U_I are the nodal parameters for standard FEM approximation
- A_J are additional nodal parameters at enriched node J.

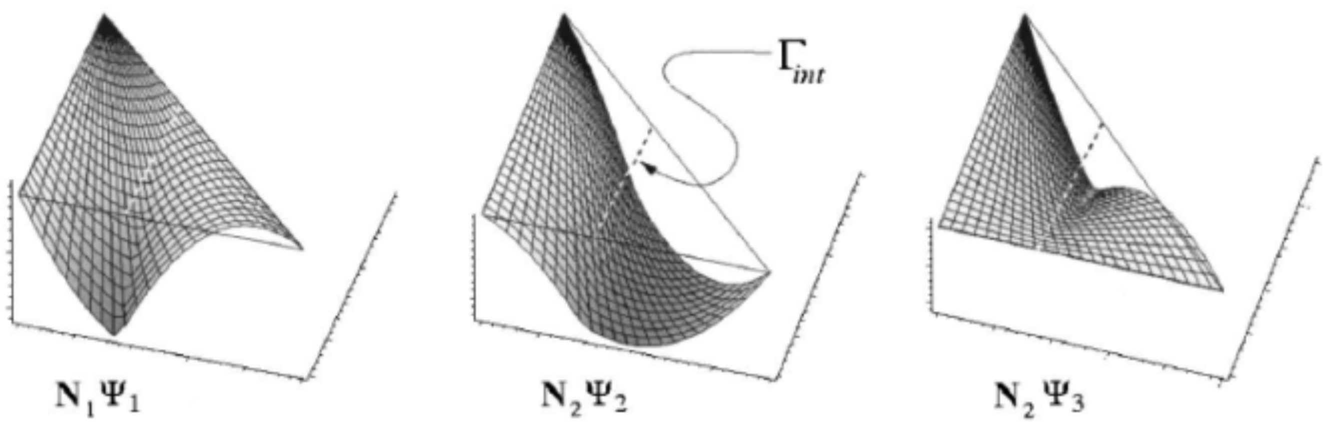
Enriched FEM with discontinuous gradient

- Example of an enriched finite element shape function in 1-D



Enriched FEM with discontinuous gradient

- Example of an enriched finite element shape function for a three-node linear triangular element

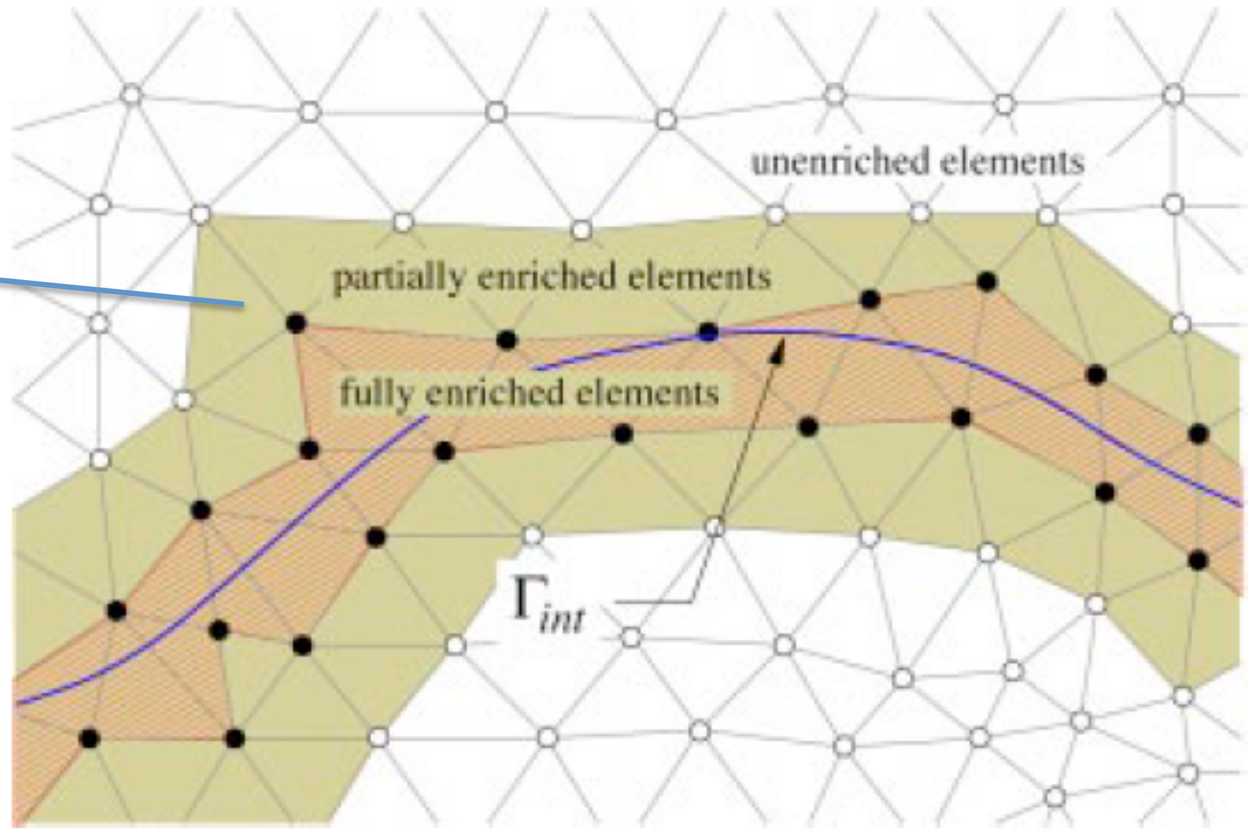


- Note: each enrichment function vanishes along two of the edges, and there is a kink along the third one.

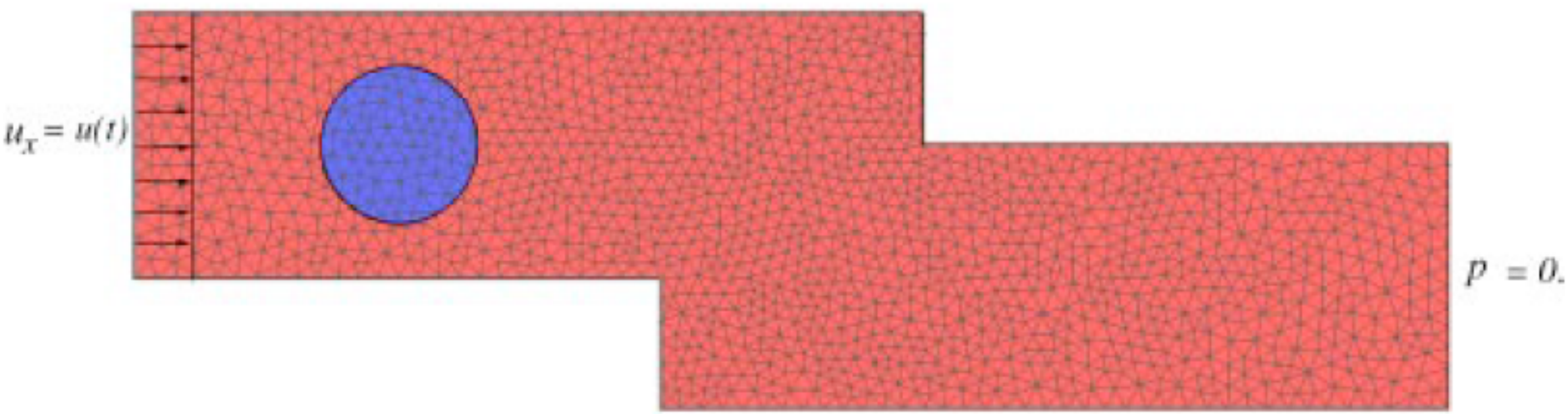
Enriched FEM with discontinuous gradient

- Two types of enrichments:

Blending between fully enriched and unenriched elements

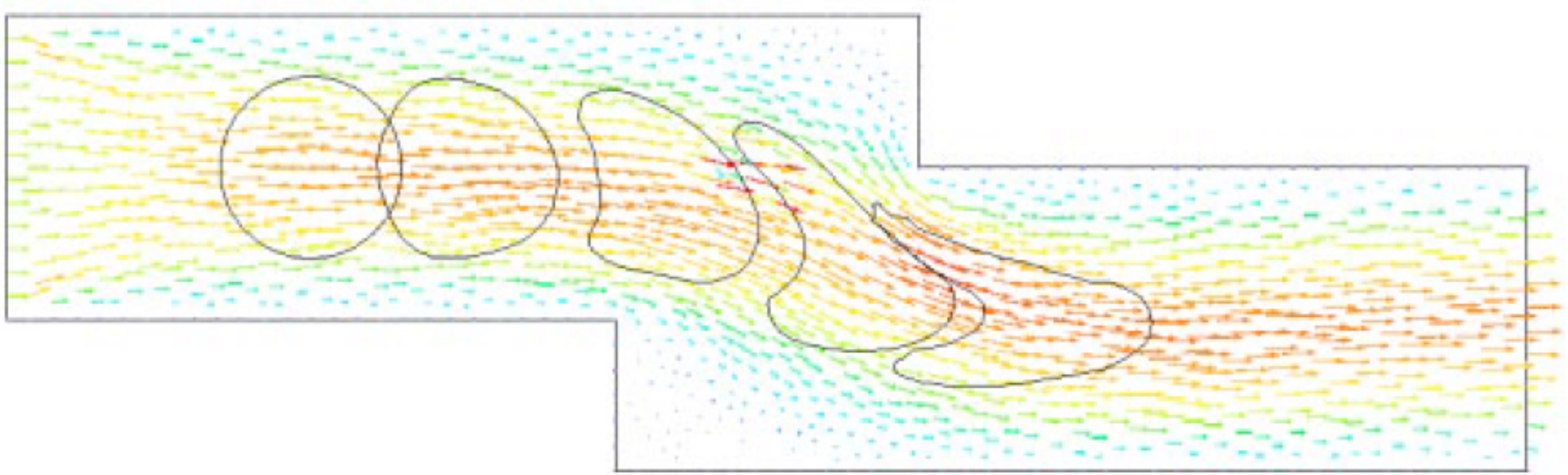


- Interstitial fluid in a "jogged" channel



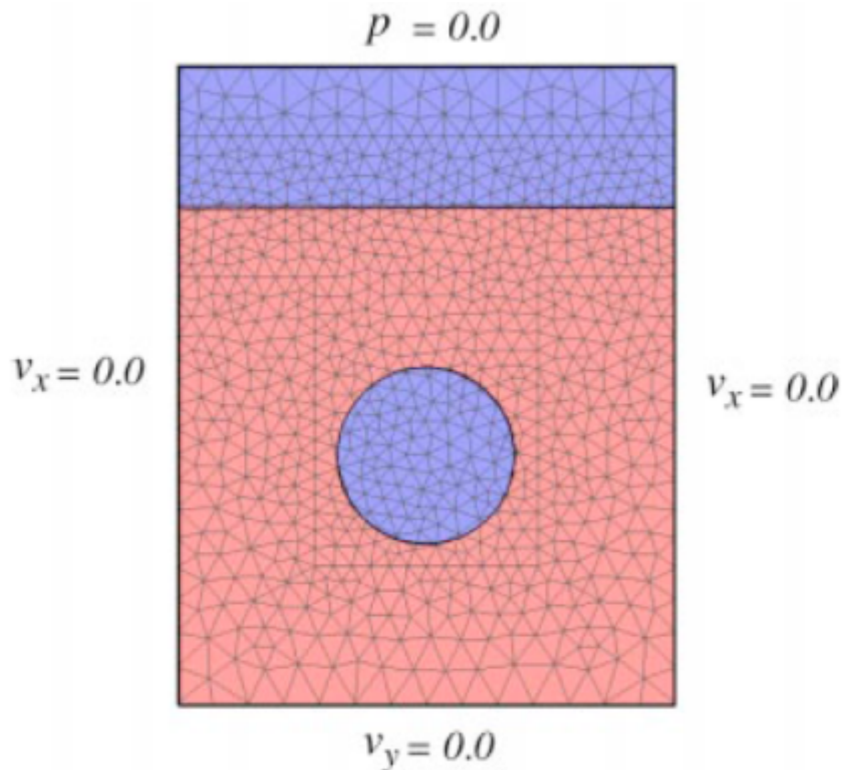
Numerical examples of two-phase fluid flow problems

- Interstitial fluid in a “jogged” channel



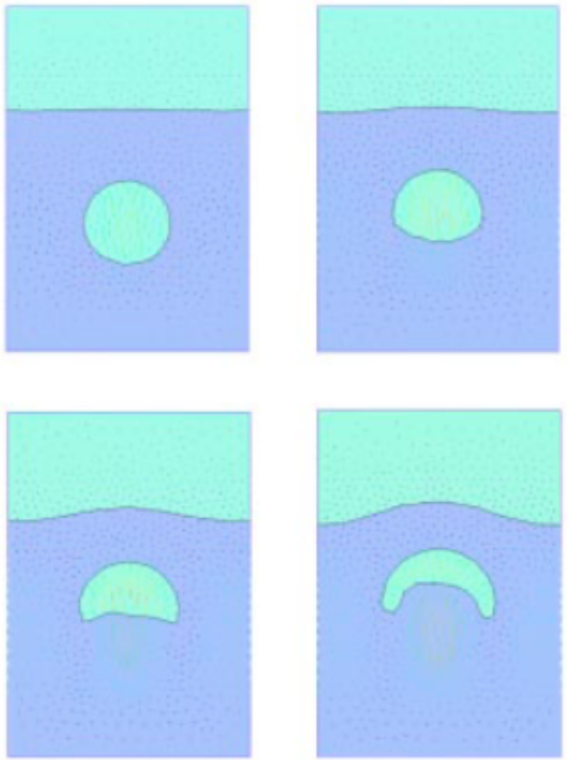
Phase interface for several time-steps

- Bubble rising to a free surface



Initial configuration

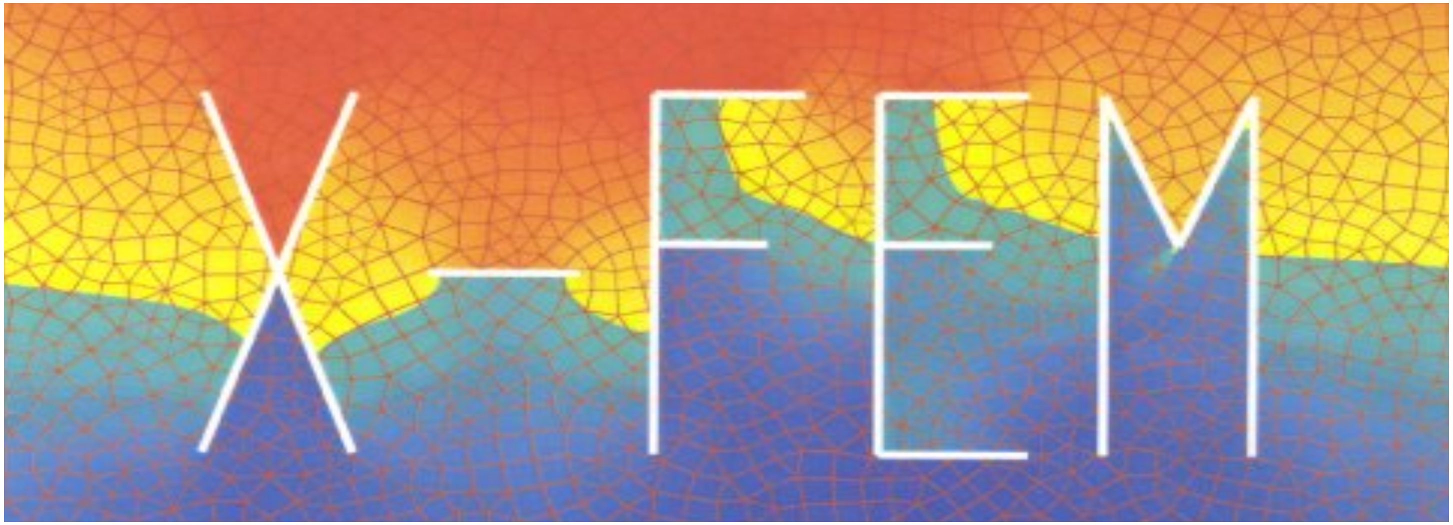
- Bubble rising to a free surface



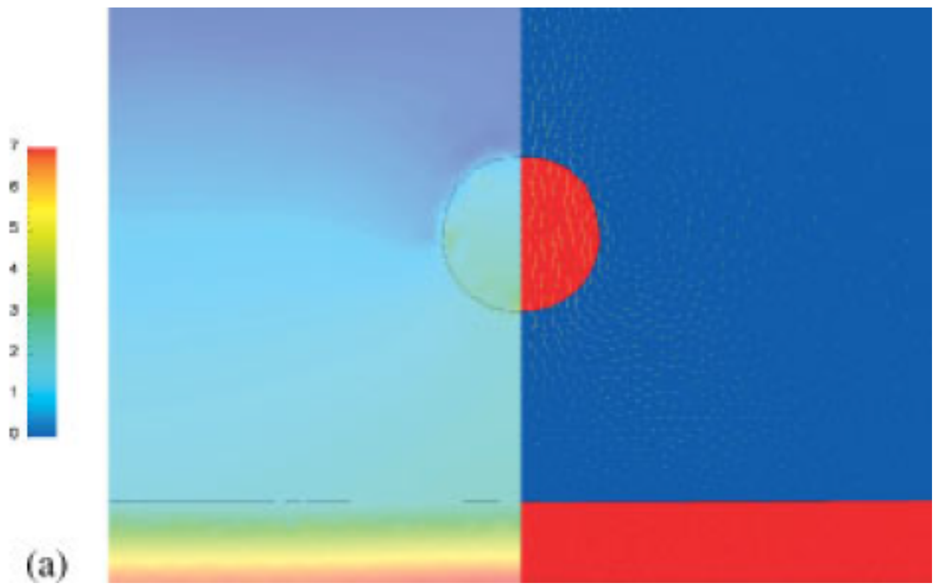
Phase interface for several time-steps

Additional numerical examples:
An enriched finite element method and level sets for axisymmetric two-phase flow with surface tension

Adapted from Chessa, J., Belytschko, T.



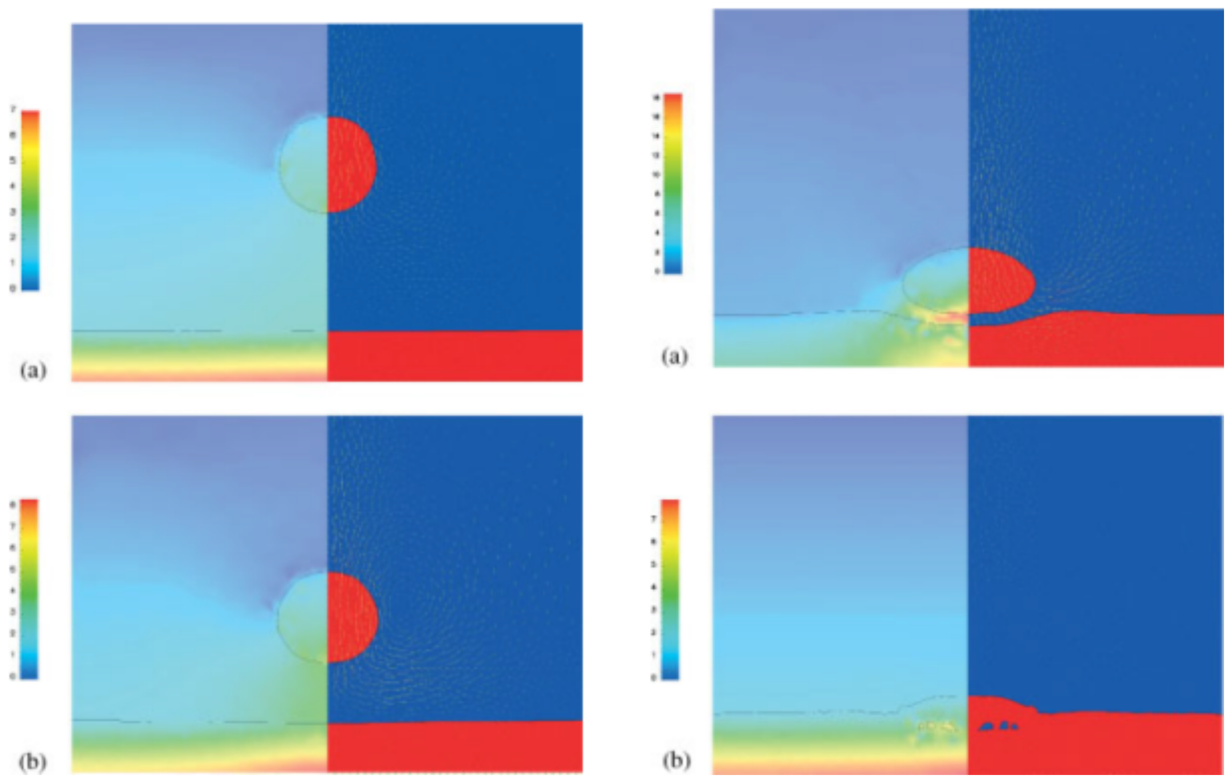
- Droplet falling onto a thin film



Initial configuration

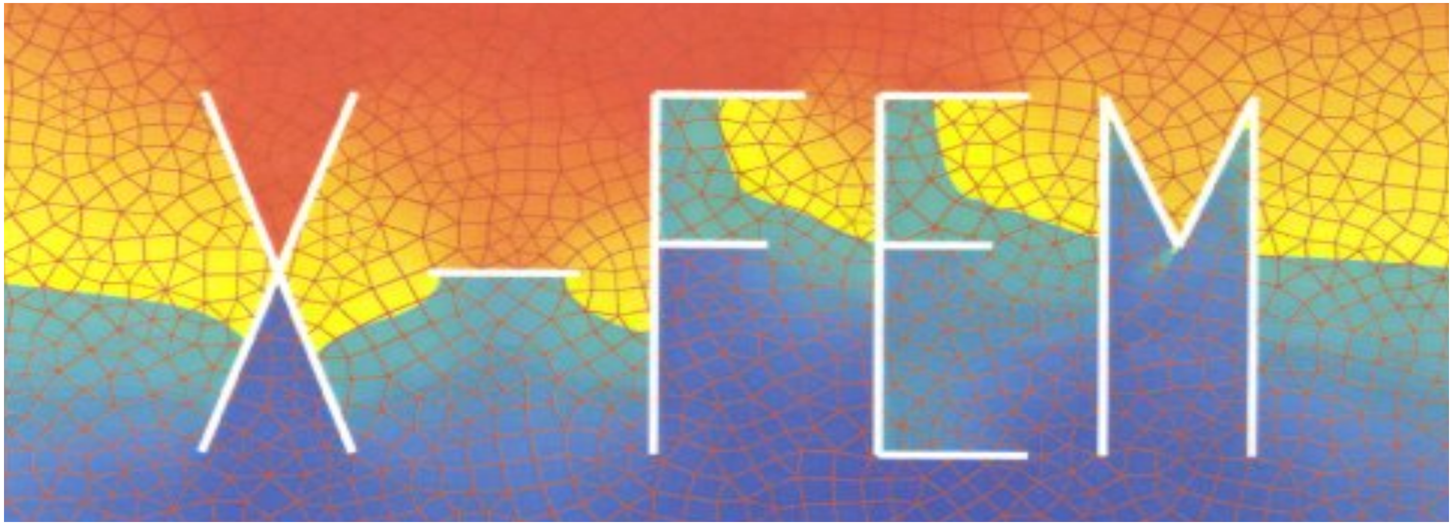
Numerical examples of two-phase fluid flow problems

- Droplet falling onto a thin film



Phase interface for several time-steps

Propagating Crack



Crack growth criteria

1. Minimum strain energy density criteria, [Sih 1974]

- Strain energy function

$$S = a_{11}K_I^2 + 2a_{12}K_IK_{II} + a_{22}K_{II}^2 + a_{33}K_{III}^2$$

$$a_{11} = \frac{\kappa+1}{16\mu\lambda\kappa^2 cos\theta}\left[2(1-2\nu)+\frac{\kappa-1}{\kappa}\right]$$

$$a_{12} = \frac{(\kappa^2-1)^{1/2}}{8\mu\lambda\kappa^2 cos\theta}\left[\frac{1}{\kappa}-(1-2\nu)\right]$$

$$a_{22} = \frac{1}{16\mu\lambda\kappa^2 cos\theta}\left[4(1-\nu)(\kappa-1)+\frac{1}{\kappa}(\kappa+1)(3-\kappa)\right]$$

$$a_{12} = \frac{1}{4\mu\lambda\kappa cos\theta}$$

- Assumptions

1) Crack initiates when the minimum S reaches to some critical value
2) Crack extends in a direction which strain energy density factor possess a minimum value.

$$\left(\frac{\partial S}{\partial\theta}\right)_{\theta=\theta_{cr}} = 0 \quad , \quad \left(\frac{\partial^2 S}{\partial\theta^2}\right)_{\theta=\theta_{cr}} > 0$$

- For linear elastic fracture mechanics

Crack growth criteria

2. Maximum energy release rate criteria, [Nuismer 1975]

- Determining the energy release rate at the crack tip

$$
\begin{aligned}
G &= J_1 cos\theta + J_2 sin\theta \\
\theta_{cr} &= atan\left(\frac{J_1}{J_2}\right) \\
J_k &= \int_\Gamma \left[\frac{1}{2}\sigma_{ij}\epsilon_{ij}\delta_{jk} - \sigma_{ij}\frac{\partial u_i}{\partial x_k}\right] n_j d\Gamma
\end{aligned}
$$

- Assumptions

1) Crack initiates when G reaches some critical value
2) Crack will grow in a radial direction from the crack tip along which the energy release rate is maximum

- For traction free cracks

Crack growth criteria

3. Maximum hoop stress or maximum principal stress criteria, [Erdogan and Sih 1963] [Liang et al. 2003]
   - Most commonly used criteria in LEFM
   - Based on the evaluation of mixed mode stress intensity factors K_I and K_{II}
   - Assumptions

   1) Crack initiates when the maximum hoop stress reaches to a critical value
   2) Crack will grow in a direction in which circumferential stress is maximum

$$\theta_{cr} = 2atan\frac{1}{4}\left(\frac{K_I}{K_{II}} \pm \sqrt{\frac{K_I}{K_{II}}^2 + 8}\right)$$

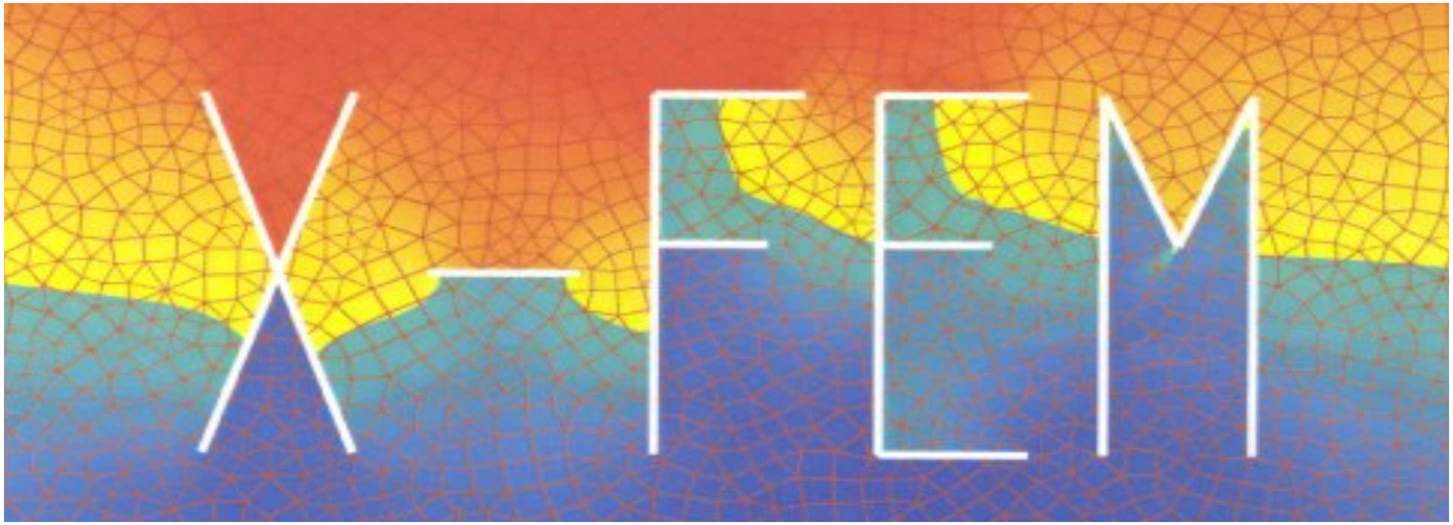
$$\theta_{cr} = 2\arctan\frac{-2K_{II}/K_I}{1+\sqrt{1+8(K_{II}/K_I)^2}}$$

   - For traction free crack surfaces

Crack growth criteria

4. Global energy based criterion, [Meschke and Dumstorff 2007]
   - Crack propagation is determined by minimizing the total energy of the body
5. Virtual crack extension method, [Hwang and Ingraffea 2007]
   - Modeling multiply crack systems

Example application



Example application

XFEM has implemented in software, such as

- ALTAIR RADIOSS
- ASTER
- ABAQUS

And also with other commercial software with a few plugins and actual core implementations

- ANSYS
- SAMCEF
- OOFELIE
- COMSOL

Example application

- Workflow:

1) Define initial crack geometry
    - 3D – face or shell part instance
    - 2D – edge or wire part instance
2) Define an enrichment region where crack initiation and growth can occur
3) Define damage Criteria in the material model
4) Specific output variables
    - PHILSM –fracture
    - STATUSXFEM –state

Example application

- Tips to help the XFEM analysis for reasonable converge

1) Reduce increment size and increase increment number
2) Increase Maxps (maximum principle stress) Damage Tolerance
3) Add damage stabilization
4) General solution controller
   - Time increments >> discontinues increment
5) In step module, add automatic stabilization

Animations

- XFEM fracture modeling with Abaqus
  - http://www.simulia.com/services/training/wbtAbaqus69/
- Crack Propagation on 2D by Abaqus
  - https://www.youtube.com/watch?v=cJSmehK-rpY
  - https://www.youtube.com/watch?v=6inDhNQXsQI
- Fluid Mixing with XFEM
  - https://www.youtube.com/watch?v=TQ_kSswzJJM
  - https://www.youtube.com/watch?v=9dIQlD5f8sY
- Multi-Crack with XFEM
  - https://www.youtube.com/watch?v=hwECn75CFO4
  - https://www.youtube.com/watch?v=GnKCN8nenQs

Thanks for your attention!

Any Questions