

# How you like DEM apples

EGEE 520

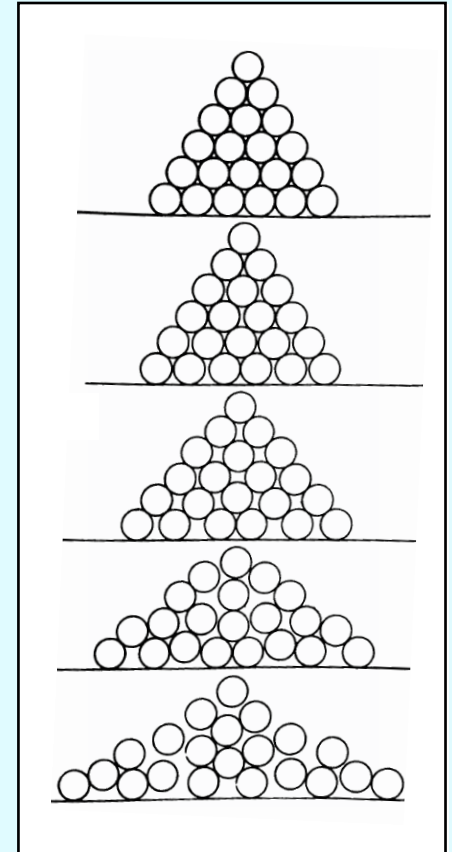
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# What is DEM

- Discrete Element Method
  - Particle Based
  - Explicit Time Steps
- Useful for:
  - Modeling Movement of Individual Particles
  - Rotation of Particles
  - Time Steps
  - Progressive Failure
- Not Useful for:
  - Complex Particle Geometries and Arrangements
  - Roughness, Texture
  - Grain Crushing, Particle Breakage
  - Non-Idealized Contacts

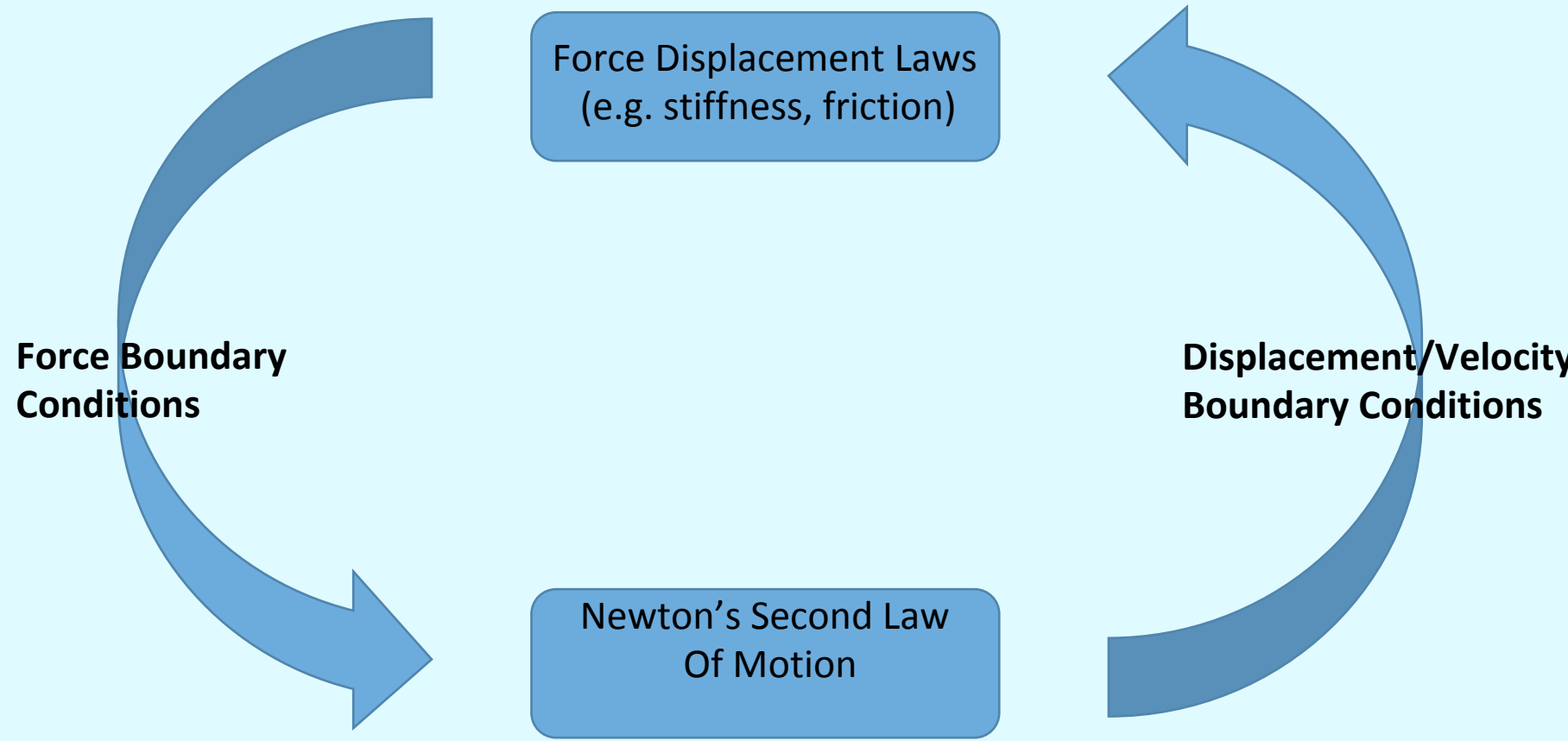
# Historical perspective

- First developed by Cundall in his 1971 thesis- “A computer model for simulating progressive large-scale movements in blocky rock systems”
  - Modeled the movement of particles as a collapse of cylinders and toppling of blocks
- Modified into a RBM (restricted Boltzmann machines) code (Cundall 1974), FORTRAN code (Cundall 1978), and a 3D model including fluid and pore pressure (Cundall and Hart 1985)
- Current Uses:
  - Agriculture
  - Food Transport
  - Pharmaceutical
  - Civil Engineering
  - Powder Metallurgy
  - Chemical Engineering



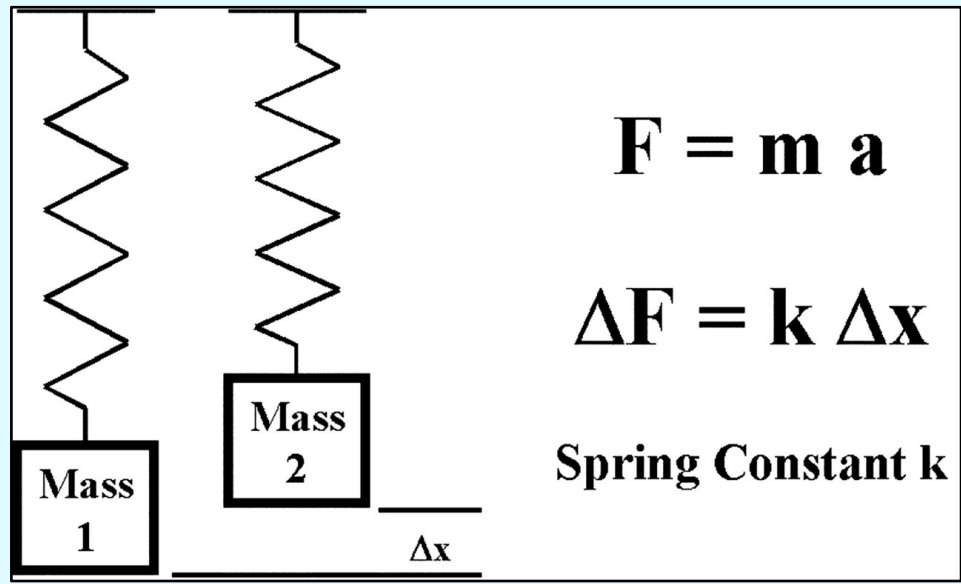
# General Principles

- Newton's Second Law of Motion
- Force Displacement Law

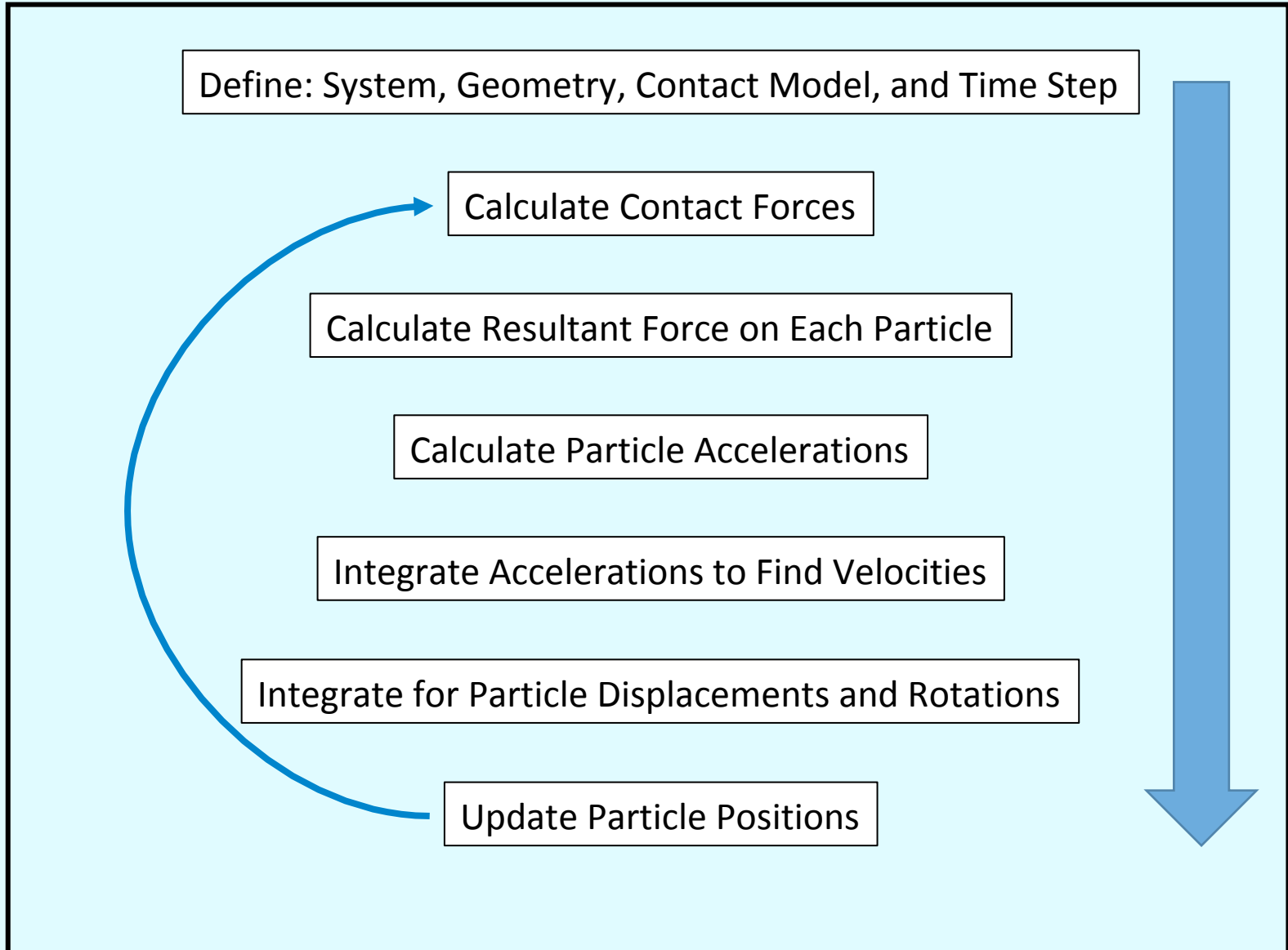


# Governing Equations

- DEM uses two types of governing laws:
- Force-Displacement Law
  - Hooke's law, friction etc...
- Newton's Second Law of Motion
  - $F=MA$
- Time Step
  - Discrete
  - Critical Time Step



# DEM Process Flow



# Hand Calculation

## Time Step

$$\Delta T = \sqrt{M/K}$$

## Forces on Grains

$$\Delta F_n = K_n(\Delta n)_{t1} = K_n(V)(\Delta t)$$

## Newton's Second Law

$$F = ma$$

$$\ddot{X} = F_{(x)}/m$$

## Integrate to find velocity

$$\dot{X}_{t2} = (F_x/m)\Delta t$$

## Integrate again to find relative displacements

$$(\Delta n_{(A)})_{t2} = (v - [F_{(x)}/m]\Delta t)\Delta t$$

$$(\Delta n_{(B)})_{t2} = ([F_{(x)}/m_{(x)}]\Delta t - [F_{(y)}/m_{(y)}]\Delta t)\Delta t$$

$$(\Delta n_{(C)})_{t2} = ([F_{(y)}/m_{(y)}]\Delta t - [-v])\Delta t$$



# Realistic Grain Orientation

## Unit Vector-Normal Direction

$$e_i = (y_i - x_i) / D = (\cos\alpha, \sin\alpha)$$

## Shear Direction

$$t_i = (e_2, -e_1)$$

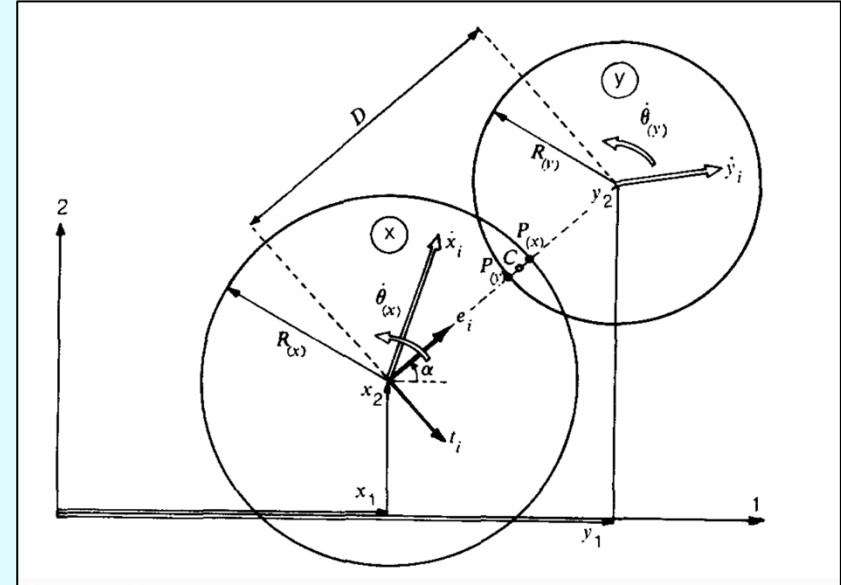
Velocity:

$$\dot{X}_i = (\dot{x}_i - \dot{y}_i) - (\dot{\theta}_{(x)} R_{(x)} + \dot{\theta}_{(y)} R_{(y)}) t_i$$

Velocity Components:

$$\dot{n} = \dot{X}_i e_i = (\dot{x}_i - \dot{y}_i) e_i - (\dot{\theta}_{(x)} R_{(x)} + \dot{\theta}_{(y)} R_{(y)}) t_i$$

$$\dot{s} = \dot{X}_i t_i = (\dot{x}_i - \dot{y}_i) t_i - (\dot{\theta}_{(x)} R_{(x)} + \dot{\theta}_{(y)} R_{(y)}) t_i t_i$$





# Putting it all Together

## Force-Displacement

$$\Delta F_n = K_n(\Delta n)_{t1}$$

$$= K_n(V)(\Delta t)$$

$$= K_n[(\dot{x}_i - \dot{y}_i)e_i] \Delta t$$

$$\Delta F_s = K_s[(\dot{x}_i - \dot{y}_i)t_i - (\dot{\theta}_{(x)}R_{(x)} + \dot{\theta}_{(y)}R_{(y)})] \Delta t$$

## Force Increment Sum

$$(F_n)_N = (F_n)_{N-1} + \Delta F_n; (F_s)_N = (F_s)_{N-1} + \Delta F_s$$

## Failure

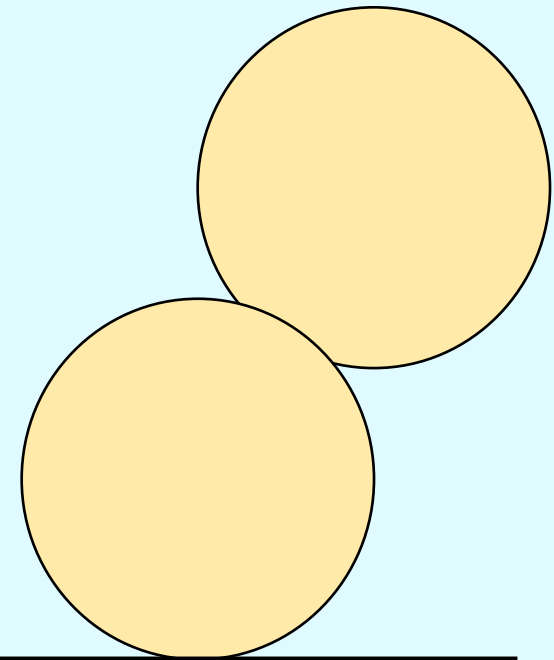
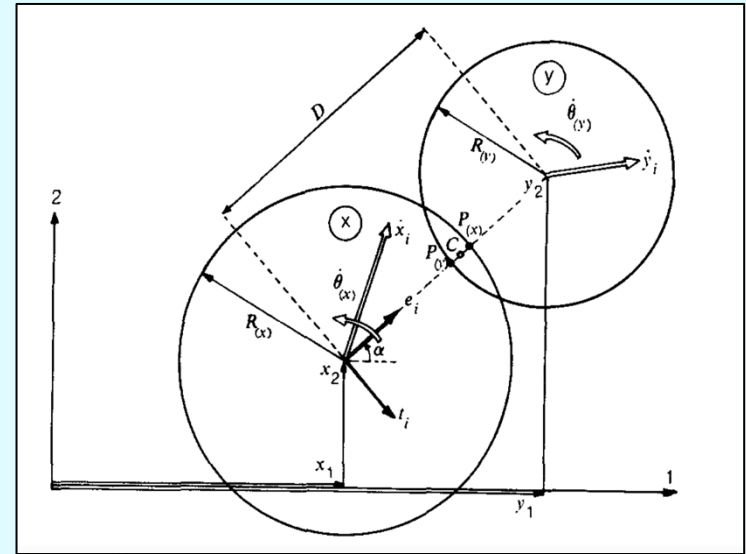
$$(F_s)_{\max} = F_n \tan \Phi_\mu + c$$

## Damping-Contact

$$C_n = \beta K_n; C_s = \beta K_s$$

## Damping-Global

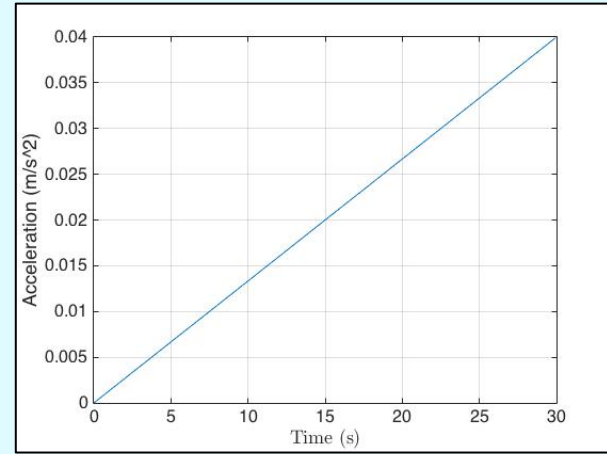
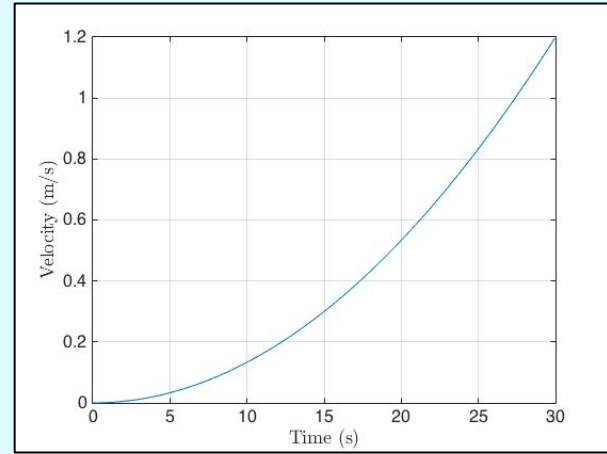
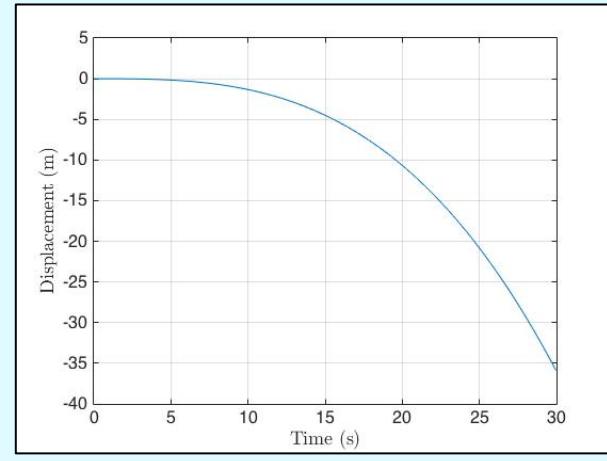
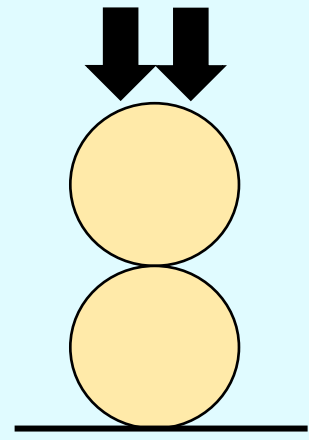
$$C = \alpha m_{(x)}; C^* = \alpha I_{(x)}$$



# Numerical Model

Input:

- Stiffness:
  - $K=0.002\text{N/m}$
- Mass
  - $1.5\text{e-}5\text{kg}$
- Time Step
  - $0.08\text{s}$
- Velocity-initial
  - $1\text{e-}5\text{m/s}$
- Radius
  - $0.001\text{m}$



Output:

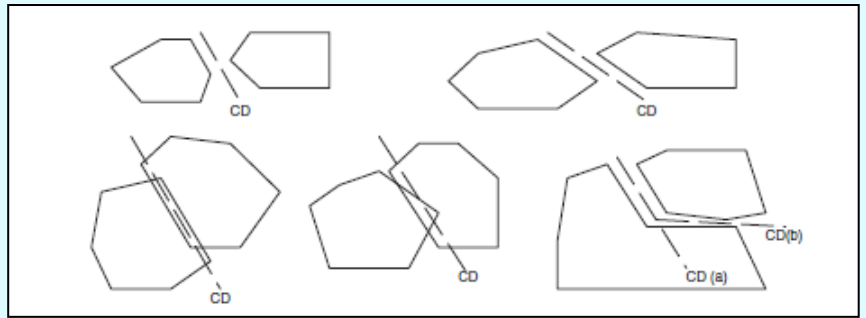
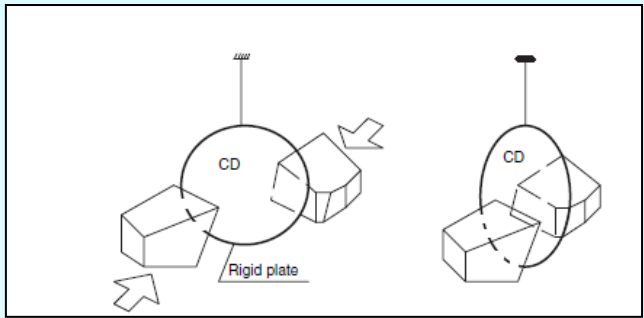
Time (s)	Displacement (m)	Velocity (m/s)	Acceleration (m/s <sup>2</sup> )
0.1732	-5.20E-06	4.00E-05	2.31E-04
15.0688	-4.5621	0.3028	0.0201
29.7047	-34.947	1.1765	0.0396

# Other Approaches

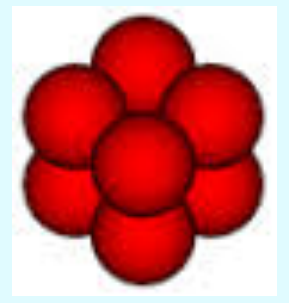
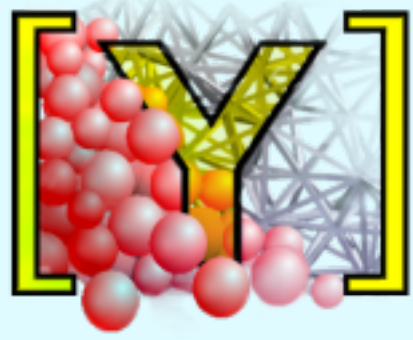
- Varying Contact Types
- Utilize Thousands of Grains
- Propagation and Mechanics of Fractures

**Table 8.2** Types of contacts for 2D polygons and 3D polyhedral blocks

Block shape	Contact type
General 2D polygons (convex or concave, singly or multiply connected)	Vertex-to-vertex
	Vertex-to-edge
	Edge-to-edge
Convex 3D polyhedra	Vertex-to-vertex
	Vertex-to-edge
	Vertex-to-face
	Edge-to-edge
	Edge-to-face
	Face-to-face



# Open Source DEM Programs



Newton



# Input Parameters

## Wet Particles

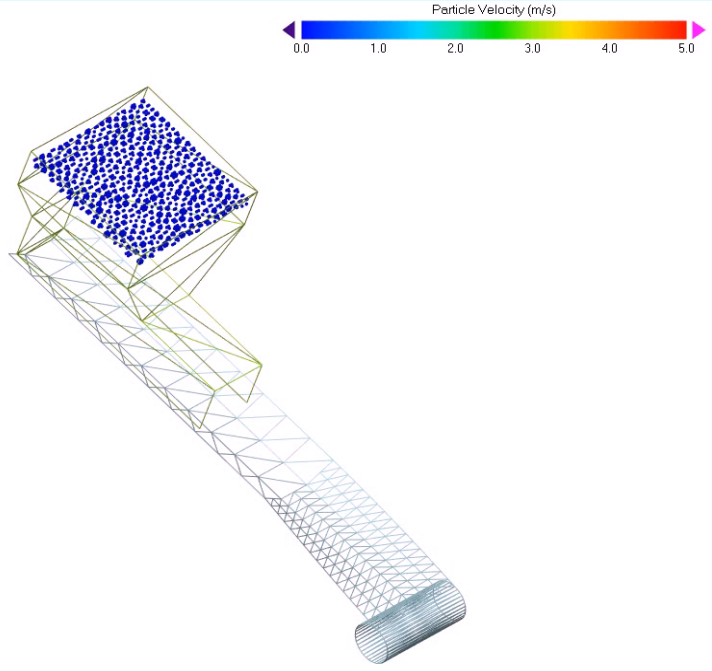
<b>General Properties</b>	<b>A</b>
Particle-Particle Friction Coefficient (0-1)	0.85
Particle-Boundary Friction Coefficient (0-1)	0.75
Coefficient of Restitution (0.075-1)	0.100
Rotational Damping (0-10)	1.00
<b>Ratchet Effect</b>	
Use Ratchet Effect	YES
Particle-Particle Cohesion Factor (0-0.25)	0.150
Particle-Boundary Cohesion Factor (0-0.25)	0.150
<b>Liquid Bridge</b>	
Use Liquid Bridge	YES
Surface Tensions (0.05 - 0.50 J/m <sup>2</sup> )	0.50
Water Content (%)	15.0
Boundary Surface Tension Multiplier (0-10)	2.00
Equivalent Sphere Size Ratio	15.00

## Dry Particles

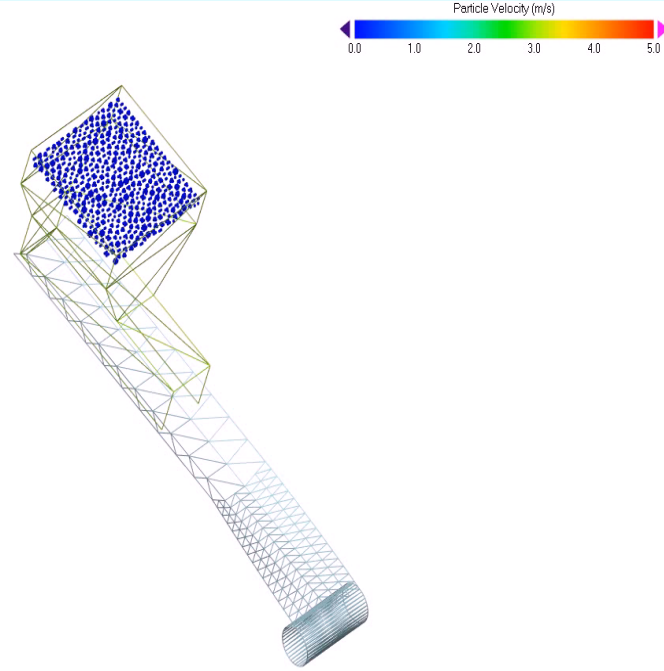
<b>General Properties</b>	<b>A</b>
Particle-Particle Friction Coefficient (0-1)	0.30
Particle-Boundary Friction Coefficient (0-1)	0.20
Coefficient of Restitution (0.075-1)	0.150
Rotational Damping (0-10)	2.00
<b>Ratchet Effect</b>	
Use Ratchet Effect	NO
Particle-Particle Cohesion Factor (0-0.25)	
Particle-Boundary Cohesion Factor (0-0.25)	
<b>Liquid Bridge</b>	
Use Liquid Bridge	NO
Surface Tensions (0.05 - 0.50 J/m <sup>2</sup> )	
Water Content (%)	
Boundary Surface Tension Multiplier (0-10)	
Equivalent Sphere Size Ratio	

# Animation Results

Time = 0.00 sec



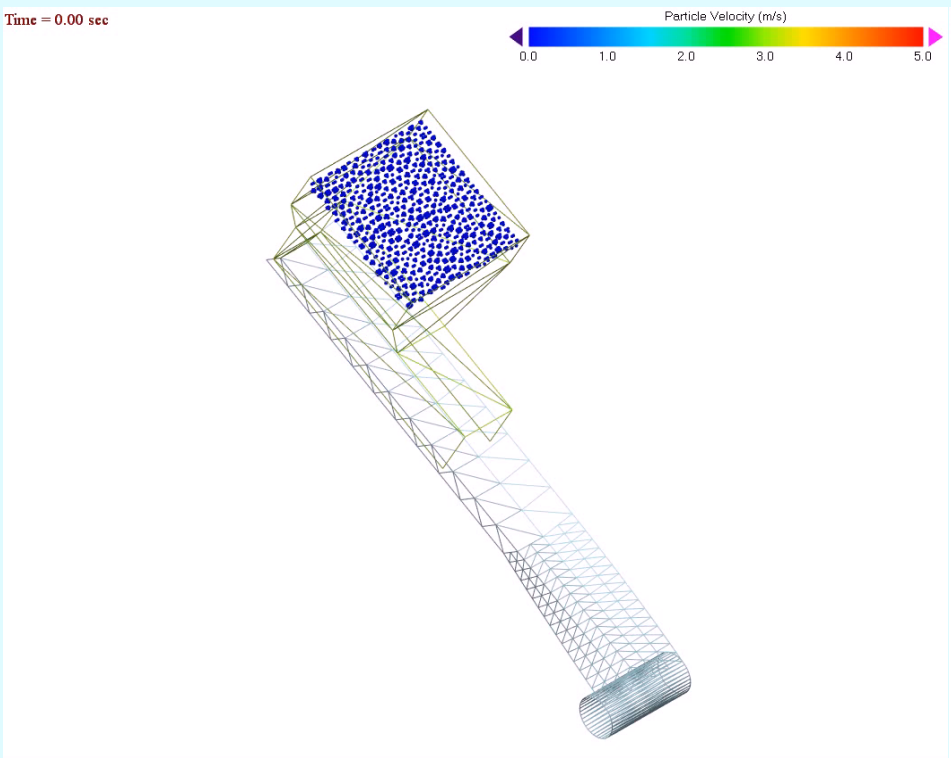
Time = 0.00 sec



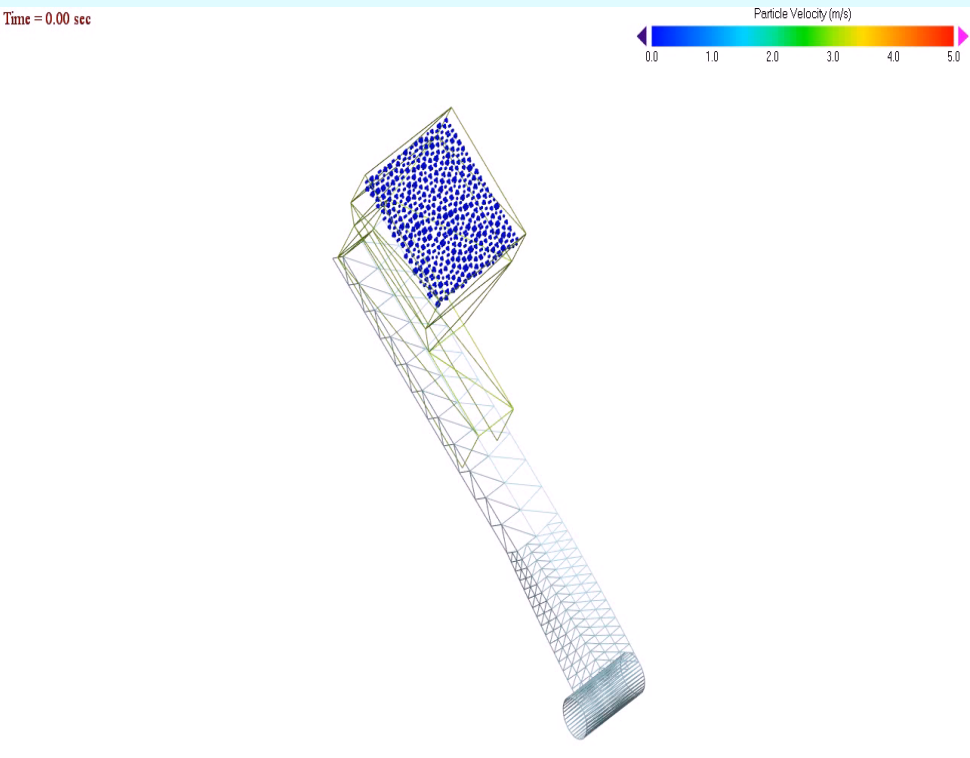
Dry particles on 20° decline

Wet particles on 20° decline

# Animation Results



Dry particles on 30° decline



Wet particles on 30° decline

# Example Animations

[L-shaped box and Funnel](#)

[Bruising on Pears](#)

[Fault Motion](#)



Thank You!