[8:1] Thermodynamics and Energy

Recap

Transport Theorem: $\dfrac{DB_{sys}}{Dt} = \dfrac{\partial}{\partial t}\int_{cv} \rho b d\cancel{V} + \int_{cs} \rho b \mathbf{V}\cdot\hat{n} dA$ for $b = \dfrac{B}{m}$

Outline

First Law of Thermodynamics: $b = e$ and

$$\frac{\partial}{\partial t}\int_{cv} e\rho d\cancel{V} + \int_{cs} e\rho \mathbf{V}\cdot\hat{n} dA = \dot{Q}_{netin} + \dot{W}_{netin}$$

$$\frac{p_{out}}{\gamma} + \frac{\alpha_o V_{out}^2}{2g} + z_{out} + h_L = \frac{p_{in}}{\gamma} + \frac{\alpha_i V_{in}^2}{2g} + z_{in} + h_P$$

$$\dot{m}[(\breve{h}_{out} - \breve{h}_{in}) + \tfrac{1}{2}(v_{out}^2 - v_{in}^2) + g(z_{out} - z_{in})] = \dot{Q}_{netin} + \dot{W}_{netin}$$

$$h_P = \frac{w_{shaftin}}{g}; \quad w_{shaftin} = \frac{\dot{W}_{shaftin}}{\dot{m}}$$

SIMPLIFICATIONS TO THE GENERAL SYSTEM

$$\dot{Q}_{\substack{net\\in}} + \dot{W}_{\substack{net\\in}} = \frac{\partial}{\partial t}\int_{CV} e\rho \, d\forall + \oint_{CS} \left(e + \frac{P}{\rho}\right) \underbrace{\rho(\underline{V} \cdot \hat{n}) \, dA}_{\dot{m}}$$

Steady system : $\frac{\partial}{\partial t} \to 0$

Adiabatic : $\dot{Q}_{\substack{net\\in}} = 0$

'Stagnant' system : $\underline{V} = 0 \quad \therefore \quad \underline{V} \cdot \hat{n} = 0$

No transfer of power : $\dot{W}_{\substack{net\\in}} = 0.$

For "1-D" system: (Steady)

$$\dot{m}\left[(e_{out} - e_{in}) + \frac{1}{\rho}(p_{out} - p_{in})\right] = \dot{Q}_{\substack{net\\in}} + \dot{W}_{\substack{net\\in}}$$

OR Denoting enthalpy as $\check{h} = \check{u} + \frac{P}{\rho}$ $\qquad e = \check{u} + \frac{V^2}{2} + gz$

$$\boxed{\dot{m}\left[\check{h}_{out} - \check{h}_{in} + \frac{V_{out}^2 - V_{in}^2}{2} + g(z_{out} - z_{in})\right] = \dot{Q}_{\substack{net\\in}} + \dot{W}_{\substack{net\\in}}}$$

$\check{u}$ = Internal energy per unit mass.

e = Total energy

EXAMPLE 5.20

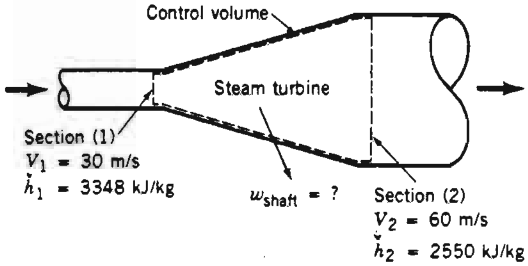
Steam enters a turbine with a velocity of 30 m/s and enthalpy, $\check{h}_1$, of 3348 kJ/kg (see Fig. E5.20). The steam leaves the turbine as a mixture of vapor and liquid having a velocity of 60 m/s and an enthalpy of 2550 kJ/kg. If the flow through the turbine is adiabatic and changes in elevation are negligible, determine the work output involved per unit mass of steam through-flow.

Note:

Work Joule, $J \equiv Nm$

Units: $\frac{P}{\rho} = \frac{N m^{-2}}{Kg/m^3} = \frac{J}{Kg}$

$\check{h} \equiv \check{u} + \frac{P}{\rho} \equiv \frac{ML^{-1}T^{-2}}{ML^{-3}}$



■ FIGURE E5.20

SOLUTION

We use a control volume that includes the steam in the turbine from the entrance to the exit as shown in Fig. E5.20. Applying Eq. 5.69 to the steam in this control volume we get

$$\dot{m}\left[\check{h}_2 - \check{h}_1 + \frac{V_2^2 - V_1^2}{2} + \underbrace{g(z_2 - z_1)}_{0 \text{ (elevation change is negligible)}}\right] = \underbrace{\dot{Q}_{\substack{\text{net}\\ \text{in}}}}_{0 \text{ (adiabatic flow)}} + \dot{W}_{\substack{\text{shaft}\\ \text{net in}}} \tag{1}$$

The work output per unit mass of steam through-flow, $w_{\substack{\text{shaft}\\ \text{net in}}}$, can be obtained by dividing Eq. 1 by the mass flow rate, $\dot{m}$, to obtain

$$w_{\substack{\text{shaft}\\ \text{net in}}} = \frac{\dot{W}_{\substack{\text{shaft}\\ \text{net in}}}}{\dot{m}} = \check{h}_2 - \check{h}_1 + \frac{V_2^2 - V_1^2}{2} \tag{2}$$

Since $w_{\text{shaft net out}} = -w_{\text{shaft net in}}$, we obtain

$$w_{\substack{\text{shaft}\\ \text{net out}}} = \check{h}_1 - \check{h}_2 + \frac{V_1^2 - V_2^2}{2}$$

or

$$w_{\substack{\text{shaft}\\ \text{net out}}} = 3348 \text{ kJ/kg} - 2550 \text{ kJ/kg} + \frac{[(30 \text{ m/s})^2 - (60 \text{ m/s})^2][1 \text{ J/(N·m)}]}{2[1 \text{ (kg·m)/(N·s}^2)](1000 \text{ J/kJ})}$$

Thus

$$w_{\substack{\text{shaft}\\ \text{net out}}} = 3348 \text{ kJ/kg} - 2550 \text{ kJ/kg} - 1.35 \text{ kJ/kg} = 797 \text{ kJ/kg} \qquad \text{(Ans)}$$

Assumes a perfect system with no losses!!

Note that in this particular example, the change in kinetic energy is small in comparison to the difference in enthalpy involved. This is often true in applications involving steam turbines. To determine the power output, $\dot{W}_{\text{shaft}}$, we must know the mass flowrate, $\dot{m}$.

5.117

5.117 A pump transfers water from one large reservoir to another as shown in Fig. P5.117*a*. The difference in elevation between the two reservoirs is 100 ft. The friction head loss in the piping is given by $K_L \overline{V}^2/2g$, where $\overline{V}$ is the average fluid velocity in the pipe and K_L is the loss coefficient, which is considered constant. The relation between the total head rise H across the pump and the flowrate Q through the pump is given in Fig. 5.117*b*. If $K_L = 20$, and the pipe diameter is 4 in., what is the flowrate through the pump?

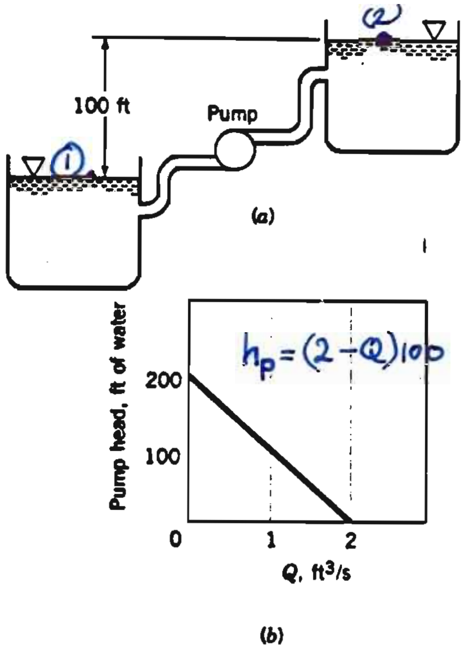


FIGURE P5.117

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L$$

For the flow from section (1) to section (2) Eq. 5.84 leads to

$$h_p = z_2 - z_1 + h_L \qquad (1)$$

From Fig. P5.117 b we conclude that

$$h_p = 200 - 100\,Q \qquad (2)$$

From the problem statement

$$h_L = K_L \frac{\overline{V}^2}{2g}$$

or since

$$\overline{V} = \frac{Q}{A} = \frac{Q}{\frac{\pi D^2}{4}}$$

we have

$$h_L = \frac{K_L Q^2}{2g\left(\frac{\pi D^2}{4}\right)^2} = \frac{(20)\left(Q\ \frac{ft^3}{s}\right)^2}{(2)\left(32.2\ \frac{ft}{s^2}\right)\left[\frac{\pi\,(4\ in.)^2}{\left(12\ \frac{in.}{ft}\right)^2(4)}\right]^2} = 40.78\,Q^2\ ft \qquad (3)$$

Combining Eqs. 1, 2 and 3 we obtain

$$40.78\,Q^2 + 100\,Q - 100 = 0 \qquad (4)$$

The root of Eq. 4 that makes physical sense is

$$Q = \underline{\underline{0.763\ \frac{ft^3}{s}}}$$

Darcy's Law

The flow of fluids in porous and fractured media are governed by Darcy's Law that states flow rate, v, is directly proportional to the driving gradient of total head, h. This is described as,

$$v = k\frac{\partial h}{\partial x}$$

where x is the longitudinal direction of flow and k is the hydraulic conductivity of the porous medium. The parameter, v, is often referred to as the Darcy velocity. The mean discharge, Q, across a plane of area A, oriented perpendicular to the direction of flow ($x-axis$) is defined as,

$$Q = Ak\frac{\partial h}{\partial x}$$

or when the partial derivatives are written as finite derivatives,

$$Q = Ak\frac{\Delta h}{\Delta x}.$$

In this the total head, h, is the sum of elevation and pressure head, and velocity head is assumed negligible. The total head at any point is given by the elevation of that point above an arbitrary datum, plus the pressure head that is experienced at that point.

Flow Nets

Despite the increasing use of computer methods, graphical methods remain an important, rapid and robust method of computing pressure distributions and flow rates in nominally homogeneous bodies. Flow net methods apply to flow in two-dimensional sections under steady state conditions. The material may be porous or porous-fractured providing it may be represented by an equivalent isotropic ($k_x = k_y$) or anisotropic hydraulic conductivity($k_x \neq k_y$). The method requires that a net of orthogonal trajectories is drawn to cover the saturated flow domain representing, respectively, streamlines and equipotentials.

Streamlines trace the path of a individual particle of fluid (in an average sense) as it transits the system.

Equipotentials locate a locus of constant total head, h. It can further be demonstrated that the streamlines represent boundaries that no fluid may cross, and therefore the groundwater surface is a streamline. .PP Any orthogonal grid of streamlines and equipotentials that simultaneously satisfy the boundary conditions of the flow system and the requirements for orthogonality also satisfy the conditions for groundwater flow. A simple example is illustrated in the Figure 1 where a net of curvilinear quadrilaterals is drawn that satisfies the constant head boundary conditions of heads h_0 and h_9. The phreatic surface (groundwater surface) represents the topmost streamline dropping uniformly between equipotentials such that $\Delta_{23} = \Delta_{45}$, etc. The lowermost streamline corresponds to a prescribed no-flow boundary beneath the system. It may be noted that the equipotentials remain orthogonal to the upper (phreatic) and lower bounding streamlines, and also to all intermediate streamlines.

From Darcy's law, the unidirectional flow confined between streamlines (characterized by the inset of Figure 1) may be represented as

$$Q_{12} = dw\ k\frac{(h_1 - h_2)}{dl}$$

where Q is the volumetric flow rate of a single streamtube. Noting the equidimensionality of the system as $dw = dl$ then it follows that

$$Q_{12} = k(h_1 - h_2)$$

and further realizing that fluid cannot leave the streamtube, then $Q_{12}=Q_{34}$ and the head drops between streamtubes must be uniform, as $h_2-h_1=h_4-h_3$, etc. Consequently, the total head along any equipotential may be evaluated. For equipotential h_4 the corresponding head is given as

$$h_4=(h_{in}-h_{out})\frac{5}{N_d}+h_{out}$$

where N_D is the number of potential drops in the system. This is 9 for this particular example. Total flow rate may also be determined by summing the contribution of each of the streamtubes. Due to the orthogonality of the flow net the flux contribution of each streamtube is identical. Consequently, for a total of N_S streamtubes, the total flow, Q_{total}, is given as

$$Q_{total}=\frac{N_S}{N_D}k(h_{in}-h_{out}).$$

This extremely simple technique is powerful and versatile, and gives surprisingly good estimates of pressure distributions and flow rates.

Flow Nets in Anisotropic Media

Where the system is described by different hydraulic conductivities in the $x-$ and $y-$ directions, the method may be extended. The following procedure must be adopted.

1 Redraw the flow section with the original (x,y) coordinates scaled to $(\bar{x},y)$ where $\bar{x}=x\sqrt{k_y/k_x}$.
2. Draw a flow net in the distorted geometry described in 1., above.
3. The head distribution is determined by applying the reverse distortion of 1. and 2. to return the geometry to its real form.
4. The flow rate may be evaluated from

$$Q_{total}=\frac{N_S}{N_D}\sqrt{k_x k_y}(h_{in}-h_{out})$$

In groundwater environments, where conductivity magnitudes and distributions are commonly poorly defined, or potentially indeterminate, approximate analysis by flow net sketching is of eminent use.

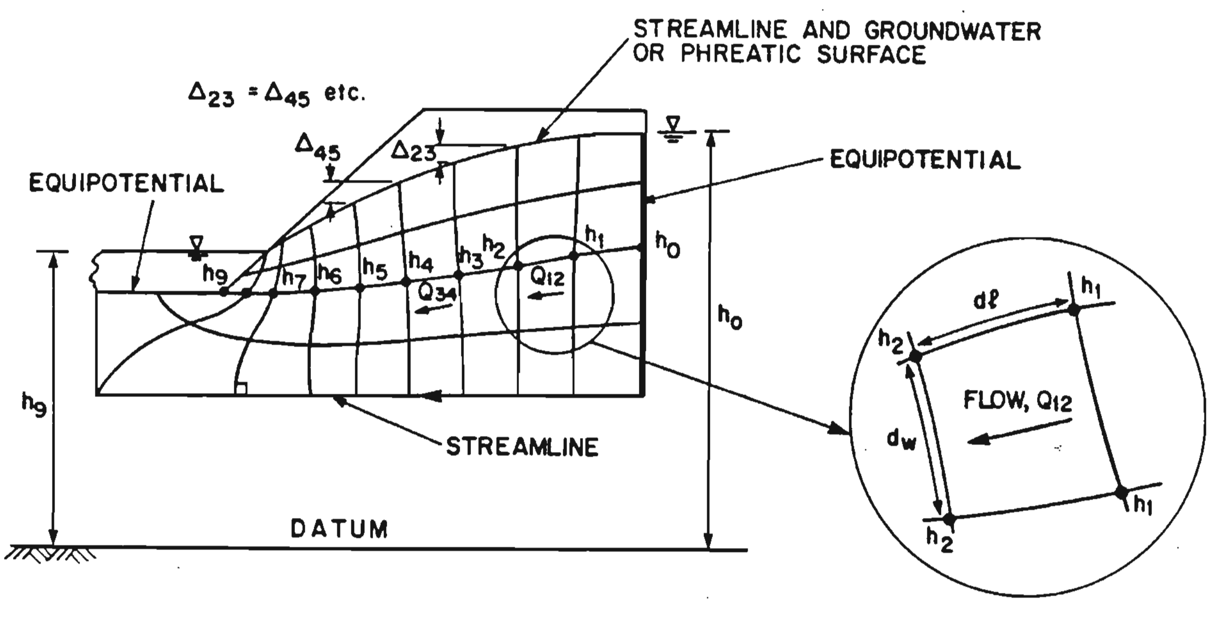


FIGURE 1

[9:3] Dimensional Analysis

Recap

Buckingham Pi & $\mathbf{Re} = \dfrac{\rho V l}{\mu}; \quad \mathbf{Fr} = \dfrac{V}{\sqrt{gl}}; \quad \mathbf{Eu} = \dfrac{p}{\rho V^2}$

Outline

Relevance of dimensionless terms

Use of models

Similitude

- Geometric
- Kinematic
- Dynamic

BUCKINGHAM PI THEOREM – Formalism for selecting groupings.

"If an equation involving k variables is dimensionally homogeneous, it can be reduced to a relationship among $k-r$ dimensionless products (groups), where r is the minimum number of reference dimensions used to describe the variables."

Pipe flow: Variables: $[V; \rho; \mu; b; \Delta p_L]$ $k = 5$

Ref. dimensions: M L T $r = 3$

$k - r \rightarrow 2$

Re and friction f.

Solution Steps:

1. List all variables – i.e. Non dimensional & dimensional.

   eg. geometry – b or D
   fluid properties μ, σ ... ρ, γ
   driving forces $g, \Delta p_L$

   Be careful not to oversupply eg. $\gamma = \rho g$ $\rightarrow k$

2. Express each variable in terms of its dimensions

   eg. $\rho \doteq ML^{-3}$ etc $\rightarrow r$

3. Determine the number of Π terms $\Pi = k - r$

4. Select the number of repeating variables.
Remove from the list of variables some that may be combined to give a Π (dimensionless) term. Must include all ref. dimensions (MLT) in each group.
Do <u>not</u> choose the <u>dependent</u> variable as one of the repeating variables.
(we want to isolate behavior of dependent variable)

5. Form a Π term by multiplying the nonrepeating variables by the product of the repeating variables.

6. Repeat <u>step 5</u> for all the non-repeating variables.

7. Check the resulting Π terms to ensure non-dimensionality.

8. Express final relationship as

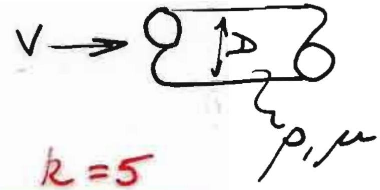
$$\Pi_1 = \phi\,[\Pi_2, \Pi_3 \ldots, \Pi_{k-r}]$$

Contains dependent variable in numerator

$$\Downarrow$$

Run experiment to determine the form of ϕ relating the non-dimensional terms.

PIPE FLOW



Dependent variable

Step 1: $\Delta p_e = f[D; \rho; \mu; V]$ $k = 5$

Step 2:

$\Delta p_e \doteq ML^{-1}T^{-2}\,(L^{-1})$ ← * Note Δp_e is $\frac{dp}{dx}$ not p.

$D \doteq L$

$\rho \doteq ML^{-3}$

$\mu \doteq ML^{-1}T^{-1}$

$V \doteq LT^{-1}$

$r = 3$ [ie ML & T]

Step 3: No. of π terms $\equiv k - r = 2$

Step 4: Select repeating varables fm $[D, \rho, \mu, V]$

$r = 3$ $\therefore$ need three repeating variables

– pick the dimensionally simplest:

$$\boxed{D;\ V;\ \rho}$$

L LT^{-1} ML^{-3}

Check that they are dimensionally independent – i.e.
none of D, V and ρ has same dimensions (units).

Step 5 → over.

Step 5: Form the Π terms.

dependent variable · repeating variables

Variable 1: $$\Pi_1 = \underbrace{\Delta p_\ell}_{\text{dependent variable}}\,\underbrace{D^a V^b \rho^c}_{\text{repeating variables}}$$

*note Δp_ℓ is $\frac{dp}{dx}$ not p

To be dimensionless: $$(ML^{-2}T^{-2})(L)^a(LT^{-1})^b(ML^{-3})^c \doteq M^0L^0T^0$$

Determine exponents:

$$\begin{aligned} 1 \qquad\qquad\qquad + c &= 0 \quad (\text{for } M)\\ -2 + a + b - 3c &= 0 \quad (\text{for } L)\\ -2 \qquad - b \qquad\quad &= 0 \quad (\text{for } T)\end{aligned}$$

Solve system for a, b, c. $\longrightarrow$ $c = -1$

$b = -2$

$a = 1$

Resubstitute into Π_1 as: $$\Pi_1 = \frac{\Delta p_\ell D}{V^2\rho}$$

Step 6:

Variable 2: Add remaining variable (μ) to Π_1 term

$$\Pi_2 = \mu\, D^a V^b \rho^c$$

To be dimensionless: $$(ML^{-1}T^{-1})(L)^a(LT^{-1})^b(ML^{-3})^c = M^0L^0T^0$$

$$\begin{aligned} 1 \qquad\qquad\qquad + c &= 0 \quad (\text{for } M)\\ -1 + a + b - 3c &= 0 \quad (\text{for } L)\\ -1 \qquad - b \qquad\quad &= 0 \quad (\text{for } T)\end{aligned}$$

$c = -1$; $b = -1$; $a = -1$ $\quad\therefore\ \Pi_2 = \dfrac{\mu}{DV\rho}$

Step 6 ....

If there were any more variables, say σ (surface tension)
then the next Π group is:

$$\Pi_3 = \sigma \, D^a V^b \rho^c$$

etc.....

Step 7: Check non-dimensionality of Π terms

Step 8: Final relationship:

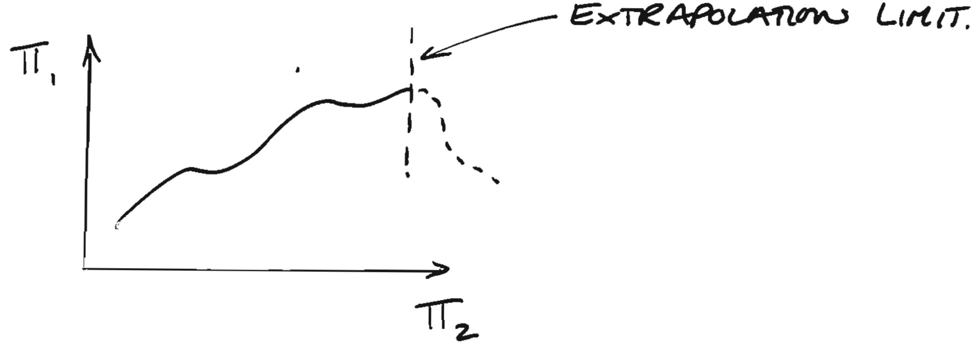
$$\frac{\Delta p_L D}{V^2 \rho} = \phi \left[\underbrace{\frac{\rho V D}{\mu}}_{Re} \right]$$

Note, since the form of ϕ is not defined, we can use Re or $\frac{1}{Re}$

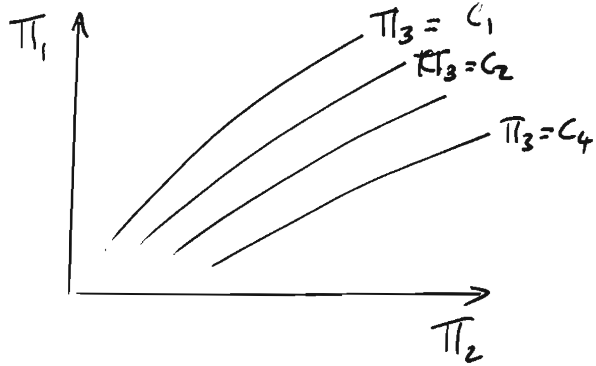
since nondimensional!!

RELEVANCE OF THE NUMBER OF Π TERMS

TWO TERMS:



THREE TERMS:



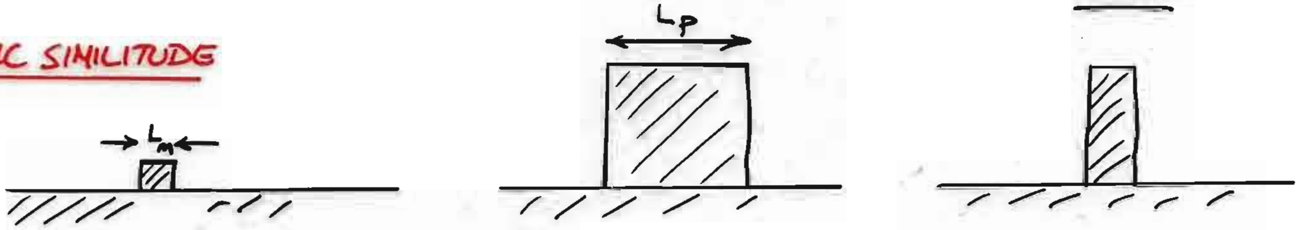
Four terms needs more imaginative representation.

USE OF MODELS

Require to match similitude between model and prototype:

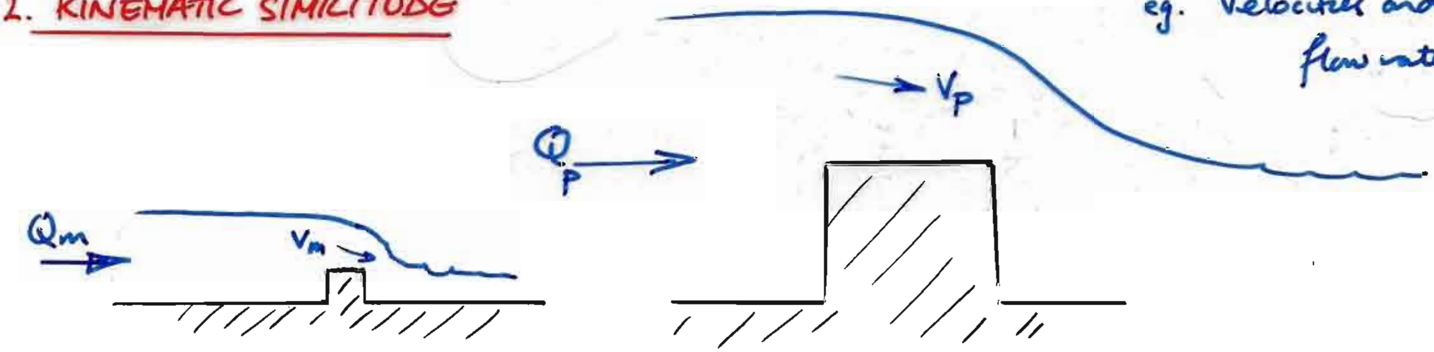
Model | Prototype | Distorted Model

1. GEOMETRIC SIMILITUDE



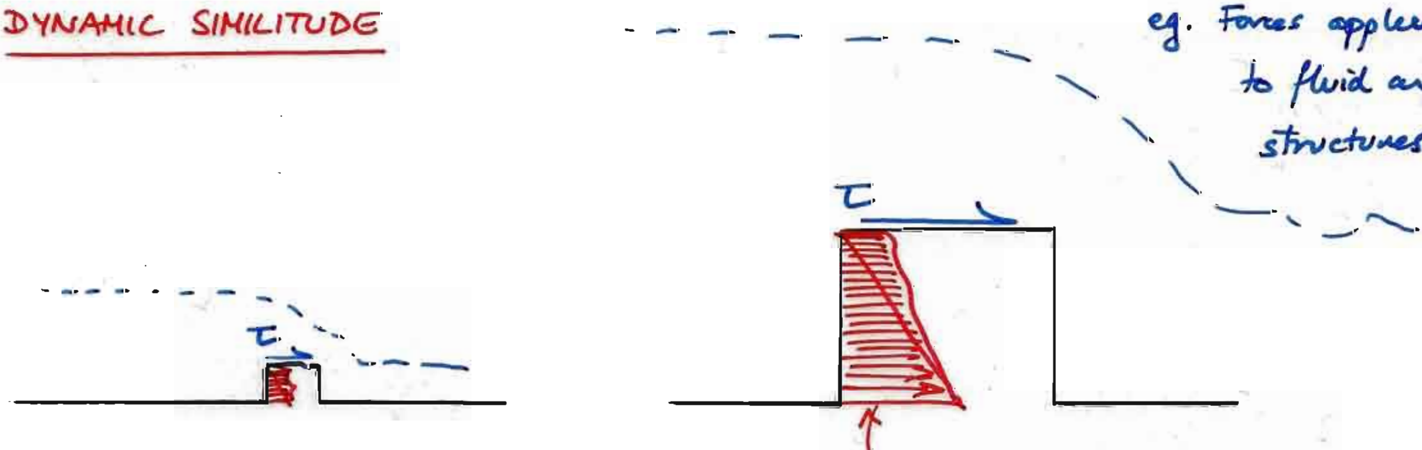
2. KINEMATIC SIMILITUDE

eg. Velocities and flow rates



3. DYNAMIC SIMILITUDE

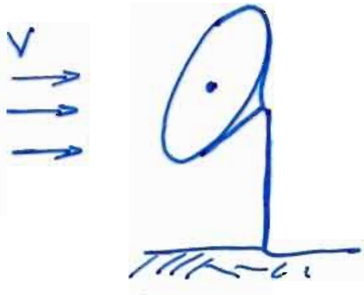
eg. Forces applied to fluid and structures.



pressure distribution on flow obstruction

7.47

7.47 The drag on a 2-m-diameter satellite dish due to an 80 km/hr wind is to be determined through a wind tunnel test using a geometrically similar 0.4-m-diameter model dish. Assume standard air for both model and prototype. **(a)** At what air speed should the model test be run? **(b)** With all similarity conditions satisfied, the measured drag on the model was determined to be 170 N. What is the predicted drag on the prototype dish?



(a) From Eq. 7.19, Reynolds number similarity is required. Thus,

$$\frac{V_m D_m}{\nu_m} = \frac{VD}{\nu}$$

where D is the dish diameter. It follows that

$$V_m = \frac{\nu_m}{\nu} \frac{D}{D_m} V$$

and with $\nu_m / \nu = 1$

$$V_m = \left(\frac{2\ m}{0.4\ m}\right)\left(80\ \frac{km}{hr}\right) = \underline{\underline{400\ \frac{km}{hr}}}$$

(b) From Eq. 7.19,

$$\frac{\mathcal{D}_m}{\frac{1}{2}\rho_m V_m^2 D_m^2} = \frac{\mathcal{D}}{\frac{1}{2}\rho V^2 D^2}$$

so that (with $\rho_m = \rho$)

$$\mathcal{D} = \frac{V^2}{V_m^2} \frac{D^2}{D_m^2} \mathcal{D}_m$$

$$= \frac{\left(80\ \frac{km}{hr}\right)^2}{\left(400\ \frac{km}{hr}\right)^2} \frac{(2m)^2}{(0.4m)^2} (170\ N) = \underline{\underline{170\ N}}$$

(Note that $\mathcal{D} = \mathcal{D}_m$ in this problem, since from the condition of Reynolds number similarity, $V^2/V_m^2 = D_m^2/D^2$. This is not true in general.)

Pipe Flow [10-11]

$$\tau_w = \frac{\rho V^2}{8} f; \quad h_L^{major} = f\left(\frac{l}{D}\right)\frac{V^2}{2g}; \quad h_P = \frac{Power}{\gamma Q}$$

$$\frac{p_1}{\gamma} + \frac{v_1^2}{2g} + z_1 + h_P = \frac{p_2}{\gamma} + \frac{v_2^2}{2g} + z_2 + \sum h_L^{major} + \sum h_L^{minor}$$

$$h_L^{minor} = K_L \frac{V^2}{2g}; \quad l_{eq}^{minor} = \frac{K_L D}{f}; \quad K_L = \frac{\Delta p}{\frac{1}{2}\rho V^2}$$

Non-circular: Laminar: $[f = \dfrac{C}{\mathbf{Re}_h}; D_h = \dfrac{4A}{P}]$ Turbulent: [Use Moody; $f = \varphi(\dfrac{\varepsilon}{D_h})$]

Series: $h_L = h_{L_1} + h_{L_2} + ... + h_{L_n}$; Parallel: $h_{L_1} = h_{L_2} = ... = h_{L_n}$

Flow meters: $Q = CA\sqrt{\dfrac{2(p_1 - p_2)}{\rho(1-\beta^4)}}$; $\beta = \dfrac{D_2}{D_1}$

ENERGY CONSIDERATIONS

General energy equation:

$$\frac{P_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_L$$

$\alpha \geq 1$ and h_L = viscous losses.

For constant section pipe flow $V_1 = V_2$ and

$$\left(\frac{P_1}{\gamma} + z_1\right) = \left(\frac{P_2}{\gamma} + z_2\right) + h_L$$

$$h_L = \frac{2\tau_w l}{\gamma r}$$

$$h_L = \frac{4 l \tau_w}{\gamma D}$$

Applies equally well to laminar & turbulent.

Substituting $f = \dfrac{8\tau_w}{\rho V^2}$ then $\tau_w = \dfrac{\rho V^2}{8} f$

$$h_L = f\left(\frac{l}{D}\right)\frac{V^2}{2g}$$

Equation ok for laminar and turbulent if (f) is determined correctly. (Moody).

EXAMPLE CALCULATIONS

BASIC CALCULATION TYPES

TYPE	GEOMETRY $D, \ell, \varepsilon/D$	FLOWRATE q or V	PRESSURE DROP Δp or h_L
I	—	—	Determine
II	—	Determine	—
III	Determine	—	—

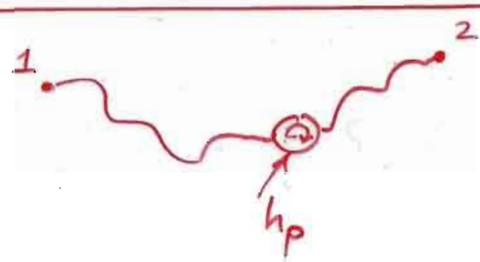
Iterative soln. (II, III) since $f = \phi[Re]$? $Re = \frac{\rho V D}{\mu}$?

All other parameters given/known

BASIC EQUATIONS

(1)

$$\boxed{\frac{P_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p = \frac{P_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_L}$$



$\alpha_1 = 1$ for turbulent flow

h_p = head provided by pumps

h_L = head loss

(2)

$$h_L = \sum f \frac{\ell}{D} \frac{V^2}{2g}$$ for "major" loss pipe sections

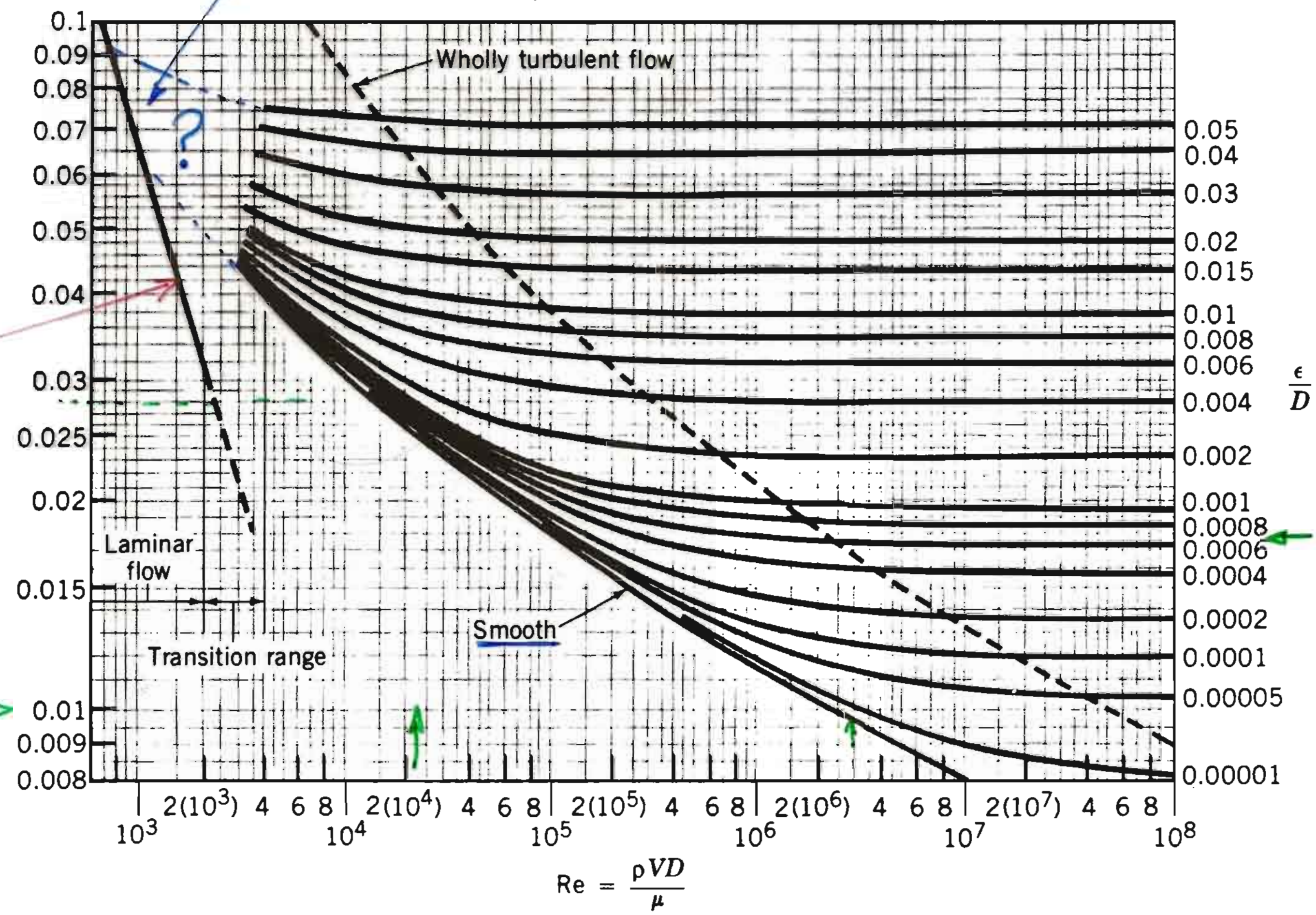
$$h_L = \sum K_L \frac{V^2}{2g}$$ for bends, elbows, valves etc.... "minor" losses

$$\boxed{f = \phi\left(Re ; \frac{\varepsilon}{D}\right)}$$

Transitional range

Laminar flow : $f = \frac{64}{Re}$; Complete turbulent flow : $f = \phi\left(\frac{\varepsilon}{D}\right)$

$f = \frac{64}{Re}$ f



■ **FIGURE 8.23** **Friction factor as a function of Reynolds number and relative roughness for round pipes—the Moody chart (Data from Ref. 7 with permission).**

Colebrook Formula (Non-laminar range, only)

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f}}\right)$$

(Laminar $f = \frac{64}{Re}$)

$$\boxed{h_L = f\,\frac{l}{D}\,\frac{V^2}{2g}}$$