4_1 Subsurface Fluid Flow

Recap:

Thermo - Defines behavior of fluids and minerals in the crust
Importance of First and Second Laws
Phase change is an important mechanism for heat transfer fluids

Movies:

Fractional Fluid Flow: https://www.youtube.com/watch?v=cNDUKylb4Ds

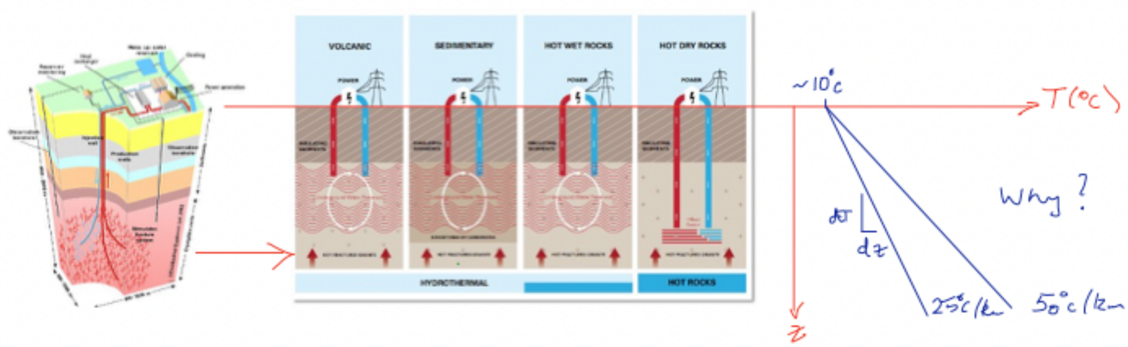
Resources: WG4

Reative Permeabilities: https://www.youtube.com/watch?v=A9c0vRU_Jko&feature=youtu.be

Darcy's Law: https://www.youtube.com/watch?v=mxPuiryMjJs&feature=youtu.be

Motivation:

1. **Motivation [10%]** Provide context for the topic. *Use of relevant public domain videos* are a useful method for this. Why is this particular topic or sub-topic important in the broad view of geothermal energy engineering?



Fluids present naturally or introduced.... and function as a heat transfer medium
Convective (rather than conductive) heat transfer often necessary
Fluids may be naturally occurring water/brine or artificially introduced CO_2 (v. exotic)

Scientific Questions:

2. **Scientific Questions to be Answered/Outline [10%]** What questions arise from the motivation. What are the sub-topical areas that address these scientific questions.

What controls fluid movement (modulated by permeability and storage)?

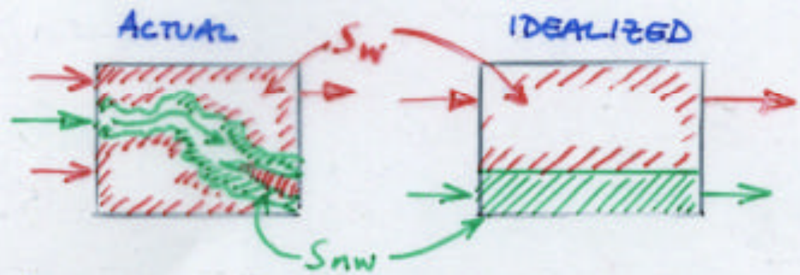
Impacts of multiple phases (vapor/liquid) and changes in permeability?

What rates of thermal recovery may result - and what are the controls?

3. SIMULTANEOUS FLOW OF TWO IMMISCIBLE FLUIDS

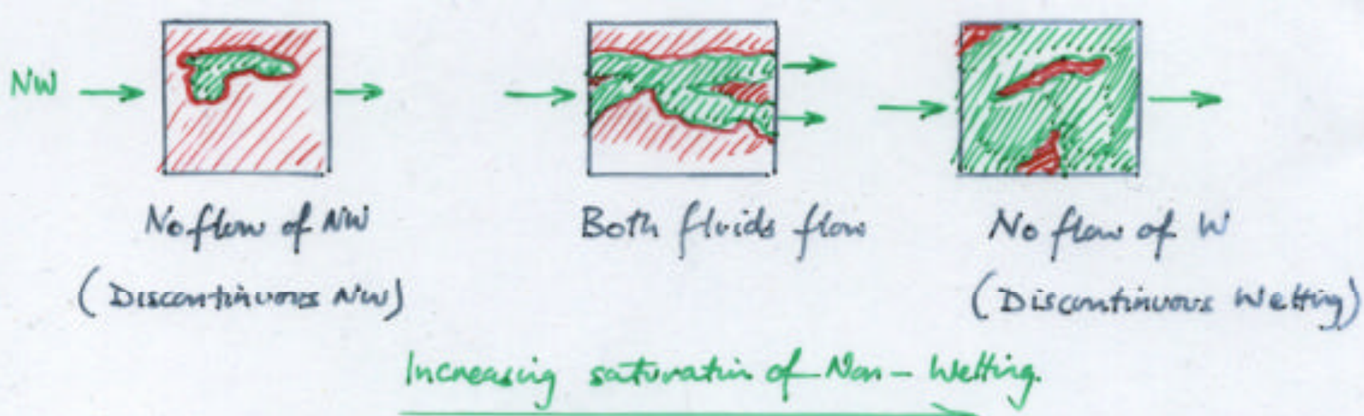
Simultaneous Flow of Two Fluids (water & vapor):

- Capillarity and capillary pressures govern the "equilibrium" penetration of fluids – static behavior
- Once penetrated, the individual phases may transfer and be transported
  - Pure phase (free product)
  - Dissolved form (later).



a) Note that flow within phase is not subject to capillarity (capillarity acts at fringes, only)

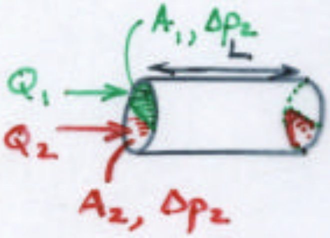
b) Each fluid establishes its own "tortuous" path → stable channels



3.1 MOTION EQUATIONS

Darcy's Law

Apply Darcy's Law



Establish steady flow at volumetric flow rates, Q_1 and Q_2.

$$q_1 = \left(\frac{k_1}{\mu_1}\right)\frac{\Delta p_1}{L}$$

$$q_2 = \left(\frac{k_2}{\mu_2}\right)\frac{\Delta p_2}{L}$$

$$q_\alpha = \left(\frac{k_\alpha}{\mu_\alpha}\right)\frac{\Delta p_\alpha}{L} \quad ; \quad q_\alpha = \frac{Q_\alpha}{A_\alpha}$$

Δp_α = effective pressure drop of α^{th} fluid

k_α = "effective" permeability of medium to fluid α.

k_α depends on: a) Porous medium (pore size & distribution and fractures).
b) Saturation, S_w and S_{nw}.

"Relative" permeabilities

$$k_{r_1} = \frac{k_1}{k}(S_1) \quad ; \quad k_{r_2} = \frac{k_2}{k}(S_2)$$

- Relative to single phase permeability of a fully saturating fluid
- Determine from "equilibrium" laboratory flow tests under different saturations.

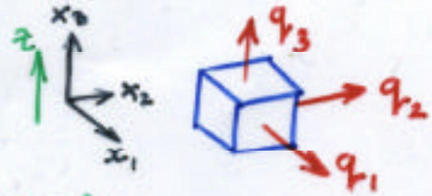
$$k_1 = k_{r_1}\, k$$

Effective permeability

Relative permeability

True permeability (L^2)

FLUID MOTION EQUATION (DARCY'S LAW)



i = flow direction, x_1, x_2, x_3. ("z" is // to x_3)

$$q_{i_1} = -\frac{k_1}{\mu_1}\left(\frac{\partial p_1}{\partial x_j} + \rho_1 g \frac{\partial z}{\partial x_j}\right) = -k\,\frac{k_{r1}}{\mu_1}\left(\frac{\partial p_1}{\partial x_j} + \rho_1 g \frac{\partial z}{\partial x_j}\right)$$

$\frac{K_2}{\rho_2 g}$

$$q_{i_2} = -\frac{\cancel{k_2}}{\mu_2}\left(\frac{\partial p_2}{\partial x_j} + \rho_2 g \frac{\partial z}{\partial x_j}\right) = -k\,\frac{k_{r2}}{\mu_2}\left(\frac{\partial p_2}{\partial x_j} + \rho_2 g \frac{\partial z}{\partial x_j}\right)$$

May also use piezometric head, h, for constant ρ_1 and ρ_2, but must be defined separately for each fluid, h_α

$$h_\alpha = \frac{p_\alpha}{\rho_\alpha g} + z \quad ; \qquad \alpha = 1, 2$$

$$\boxed{\begin{aligned} q_{i_1} &= -k\,\frac{k_{r1}}{\mu_1}\,\rho_1 g\,\frac{\partial h_1}{\partial x_j} \\ q_{i_2} &= -k\,\frac{k_{r2}}{\mu_2}\,\rho_2 g\,\frac{\partial h_2}{\partial x_j} \end{aligned}}$$

Similarity between:

permeability (L^2)

$$q = -k\,\frac{\rho g}{\mu}\,\frac{dh}{dx_j}$$

$$= -K\,\frac{dh}{dx_j}$$

Hydraulic Conductivity (L/T)

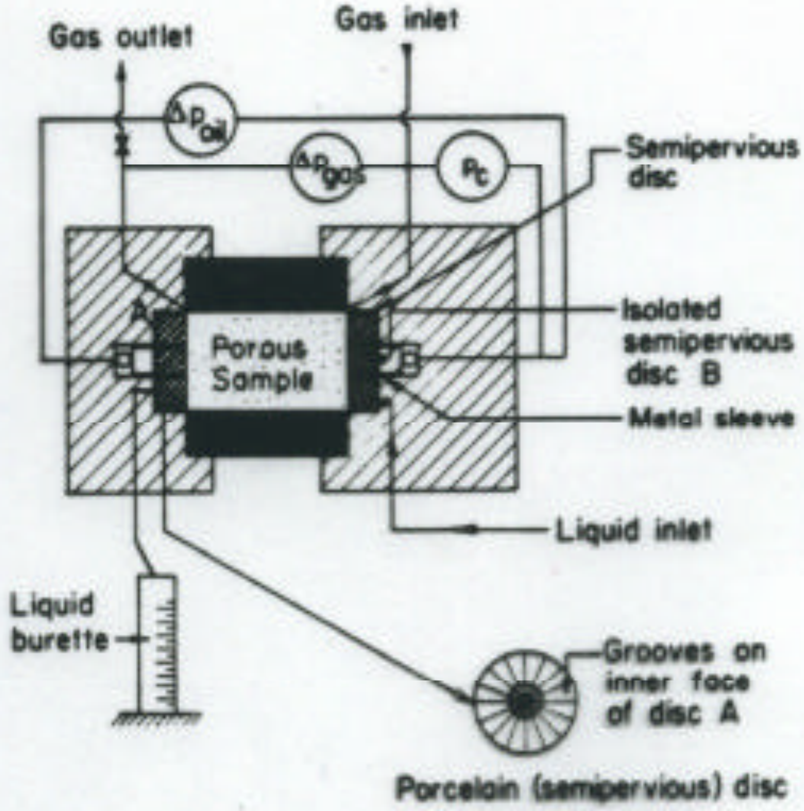


FIG. 9.2.15. Hassler's apparatus for relative permeability determination (after Osoba et al., 1951).

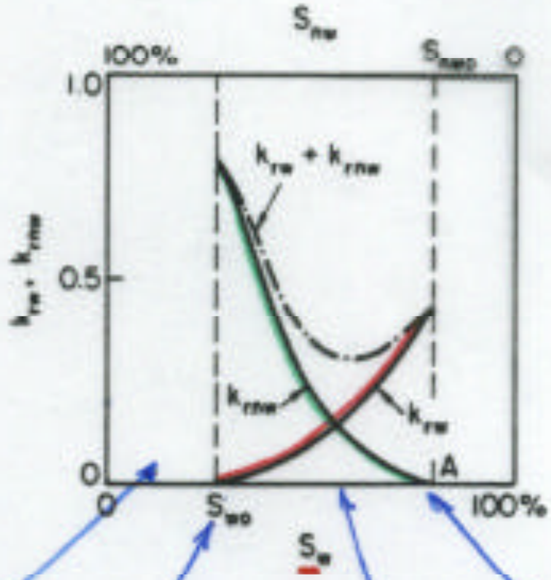


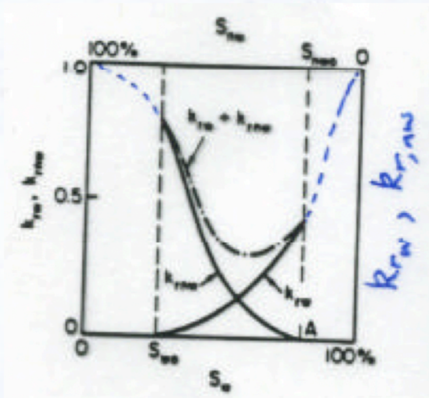
FIG. 9.3.1. Typical relative permeability curves (e.g., wetting fluid = water, nonwetting fluid = oil).

Relative Permeability

3.2 RELATIVE PERMEABILITY

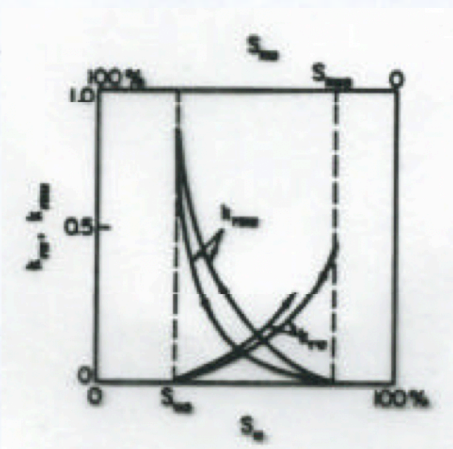
- $k_{r_{nw}} + k_{r_w} \neq 1 \Rightarrow k_{r_{nw}} + k_{r_w} < 1$.

  Most effective transmission is at 100% saturation (if accessible). Interference.



- Usually $k_{r_{nw}}$ closer to 1 than k_{r_w}
- Steep decline of k_{r_w} with increasing S_{nw} indicates larger pores occupied first by nonwetting phase.

Nonwetting phase occupies larger pores preferentially due to capillary pressure arguments.



- k to wetting fluid is always larger for open pored unconsolidated material.

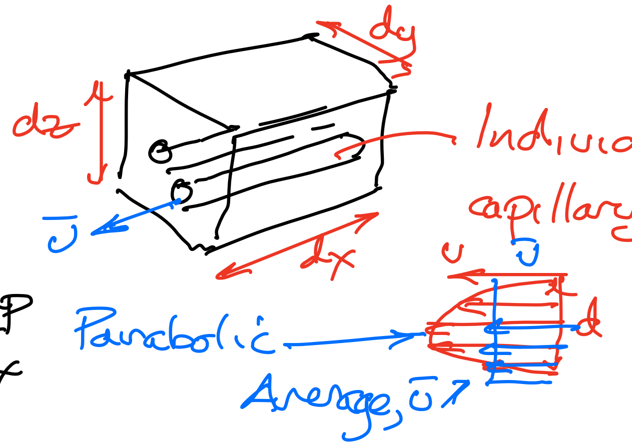
- k to non-wetting fluid is always smaller for open-pored unconsolidated material.

Hysteresis:

1. Wetting fluid surrounds grains and non-wetting fluid ∴ may move nw fluid even if no pressure gradient in nw fluid.

2. Since change in saturation requires change in wetted grain surface – wettability is hysteretic. ∴ permeabilities are hysteretic.

Capillary Relations for Permeability



Average flow velocity, $\bar{v} = \frac{d^2}{32\mu}\frac{dp}{dx}$

Flow volume rate per tube, Q_i : $Q_i = \frac{\pi d^2}{4}\frac{d^2}{32\mu}\frac{dp}{dx}$

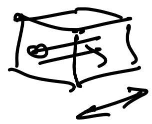
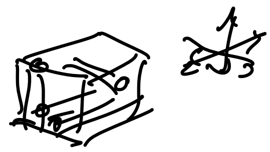
Area

No. of tubes: $n = \frac{V_v}{V_T} = \frac{A_v}{A_T}\cdot\frac{dx}{dx} = \frac{(\pi d^2/4)\times n_{tubes}\times dx}{dy\,dz\,.\,dx}$

$Q_{TOTAL} = Q_i \times n_{tubes}$

$$Q_T = \cancel{\frac{\pi d^2}{4}}\,\frac{d^2}{32\mu}\cdot\frac{dp}{dx}\times\frac{\cancel{dx}\,dy\,dz\,.\,n}{\cancel{(\pi d^2/4)}\,\cancel{dx}}$$

$$Q_T = A\frac{k}{\mu}\frac{dp}{dx} = dy\,dz\cdot\frac{d^2 n}{32\mu}\cdot\frac{dp}{dx}$$

Thus: $k = \frac{d^2 n}{32}$ for  or $\frac{d^2 n}{96}$ for 

Equivalency of Permeability (k)and Hydraulic Conductivity (K)

Permeability (k : m^2) intrinsic property of rock

Hydraulic conductivity (K : m/s) – rock + fluid

$$\frac{k}{\mu_f} = \frac{K}{\rho_f g} \quad : \quad \frac{m^2}{Pa.s} \doteq \frac{(m/s)}{kg/m^3 . m/s^2}$$

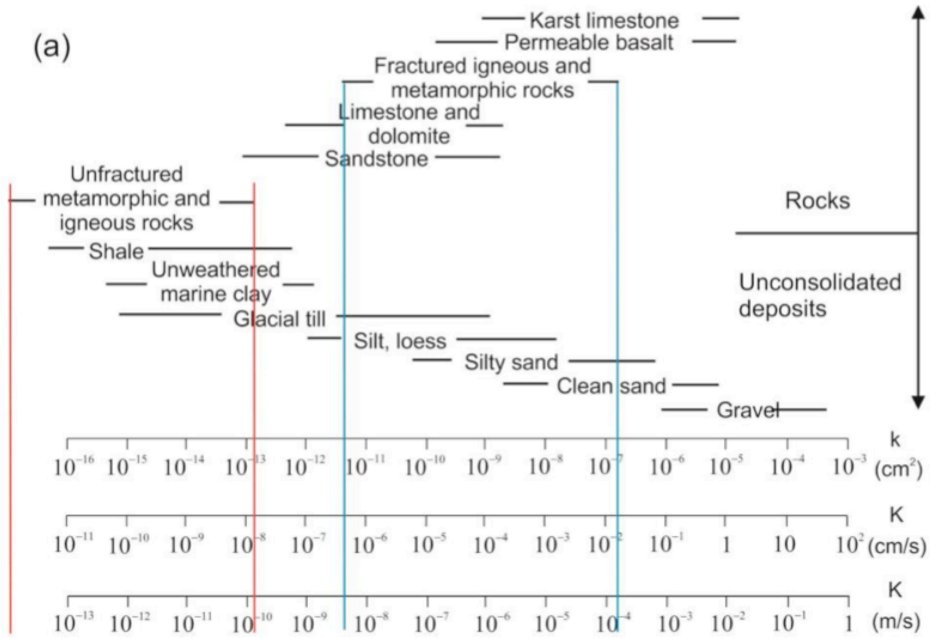


Figure 2. (a) Values of permeability (*k*) and hydraulic conductivity (*K*) for different geological materials. Notice that the *K* values for the fractured igneous and metamorphic rocks (between the blue bars) vary 5 orders of magnitude. Unfractured crystalline rocks have extremely low *K* values and are comparable to shale and clay (modified from Freeze and Cherry 1979).

TABLE 4.1
Permeabilities for Some Representative Geological Materials

	Highly Fractured Rock	Well-Sorted Sand, Gravel	Very Fine Sand and Sandstone	Fresh Granite
κ (cm^2)	10^{-3}–10^{-6}	10^{-5}–10^{-7}	10^{-8}–10^{-11}	10^{-14}–10^{-15}
κ (millidarcy)	10^{8}–10^{5}	10^{6}–10^{4}	10^{3}–1	10^{-3}–10^{-4}

Kozeny–Carman Equation

The factors that determine permeability were formally quantified by Kozeny (1927) and later modified by Carman (1937, 1956). The final form of the equation they developed is

$$\kappa = \frac{\left[n^3/(1-n)^2\right]}{(5\times S_A)^2} \tag{4.2a}$$

where:
n is the porosity, as a fraction
S_A is the specific surface area of the pore spaces per unit volume of solid (cm^2/cm^3)

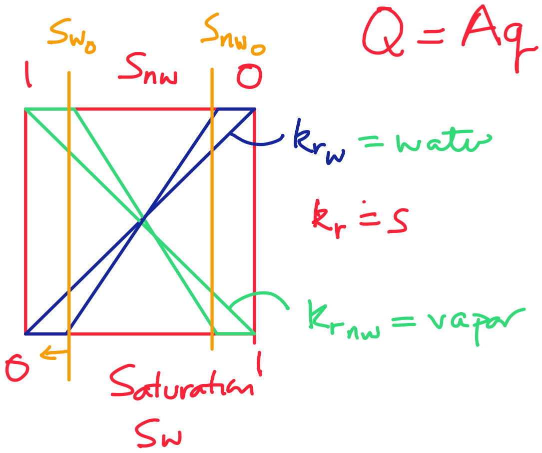
Equation 4.2a is known as the Kozeny–Carman equation. This equation allows the dependence of the permeability on the porosity of a porous sample to be determined. Implicit in this relationship are all of the factors discussed above regarding flow in the porous rocks. Of particular importance for permeability is the tortuosity of the flow path—the more tortuous the network of pores through which fluid must flow, the lower will be the permeability. Tortuosity can be accounted for by recasting Equation 4.2a as

$$\kappa = c_0 \times T \times \frac{\left[n^3/(1-n)^2\right]}{S_A^2} \tag{4.2b}$$

where:
T is the tortuosity which is equivalent to the ratio of a straight path of length L connecting two points to the actual path followed along some tubular route L_t, that is, L/L_t
c_0 is a constant characteristic of the system

Generally, $c_0 \times T = 0.2$, thus reducing Equation 4.2a to 4.2b.

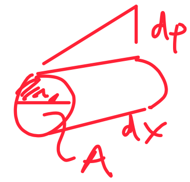
4-2 - RECAP



$k_r \doteq S$

$S_w + S_{nw} = 1$

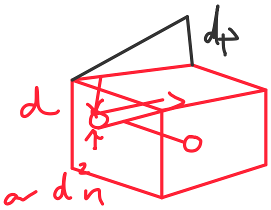
$$Q = Aq = A\, k_{r\alpha}\, \frac{k}{\mu}\, \frac{dp}{dx}$$

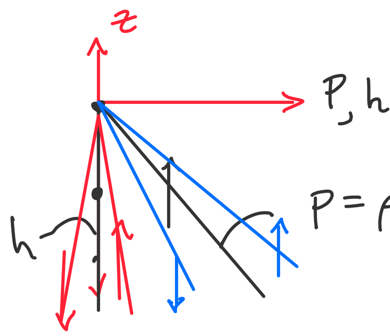


m^2 m/s

$$\frac{k}{\mu} = \frac{K}{\rho g}$$

$$k = \frac{d^2 n}{32} \sim \frac{d^2 n}{96}$$





$$P = \rho g \cdot (-z)$$

$$h = \cancel{z} + \cancel{\frac{P}{\rho g}} + \cancel{\frac{v^2}{2g}}$$

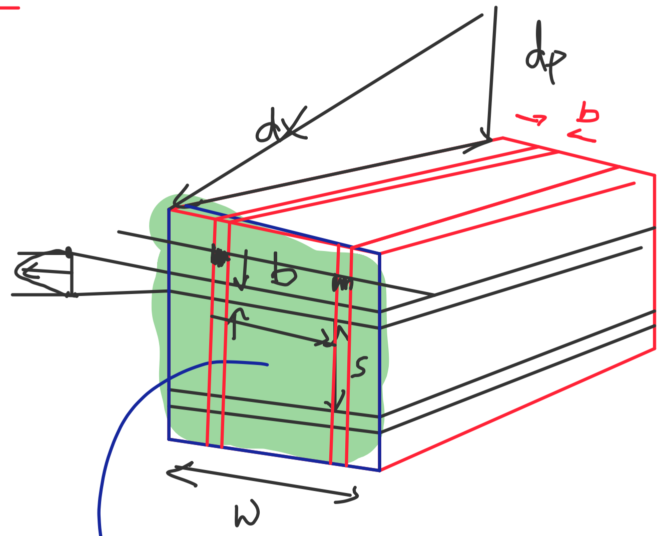
0

Permeability Fractured –

Conservation of Mass & Momentum –

Simple flow models

FRACTURED ROCKS



$$k = \frac{b^2}{12}$$

$$Q = b \cdot \overbrace{\frac{b^2}{12}}^{k} \frac{1}{\mu} \frac{dp}{dx} w$$

$$\text{no. of tubes} = \frac{1}{s}$$

$$k = \frac{b^3}{12s}$$

$$Q = A \frac{k}{\mu} \frac{dp}{dx} \quad (m^2)$$

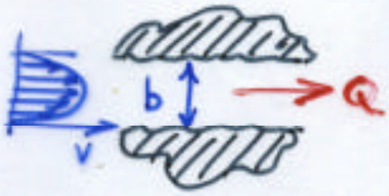
$$\frac{K}{\rho g} = \frac{k}{\mu}$$

$$\underset{(m/s)}{K} = \frac{\rho g}{\mu} \frac{b^3}{12s}$$

$$k_{2sets} = \frac{b^3}{12s} + \frac{b^3}{12s} = \frac{2}{12} \frac{b^3}{s}$$

PERMABILITY/CONDUCTIVITY OF FRACTURES

Flow in fractures



$$\bar{v} = -\overbrace{\frac{gb^2}{12\nu}}^{k}\frac{dh}{dx}$$

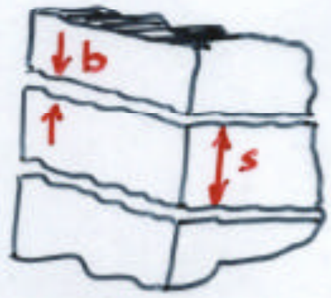
$\bar{v}$ = average velocity. ν = kinematic viscosity of fluid

$$\nu = \mu/\rho$$

Equivalent flow rate per unit width: for single fracture

$$Q = b\,\frac{gb^2}{12\nu}\,\frac{dh}{dx}$$

Multiple fractures arranged in parallel:



Total of N fractures per unit height:

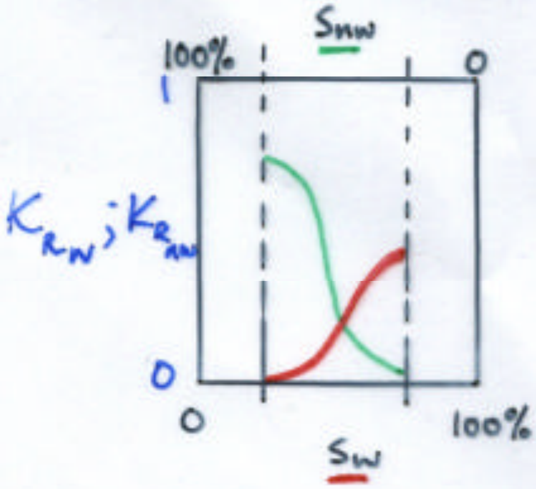
$N = 1/s$

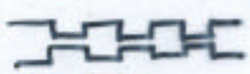
$$Q = \underbrace{\frac{gb^3}{12\nu}\,\frac{1}{s}}_{K_b}\,\frac{dh}{dx}$$

Equivalent conductivity for multiple sets: $K_b = \frac{gb^3}{6\nu s}$

Enables b to be evaluated if K known (measured).

Relative permeability of fractures



- Similar behavior to porous medium
- Distribution of apertures 

  Large apertures saturate first

Capillary pressure or head $h_c = \frac{2\sigma}{b\gamma_w}$

Fracture Permeability - Nomograph

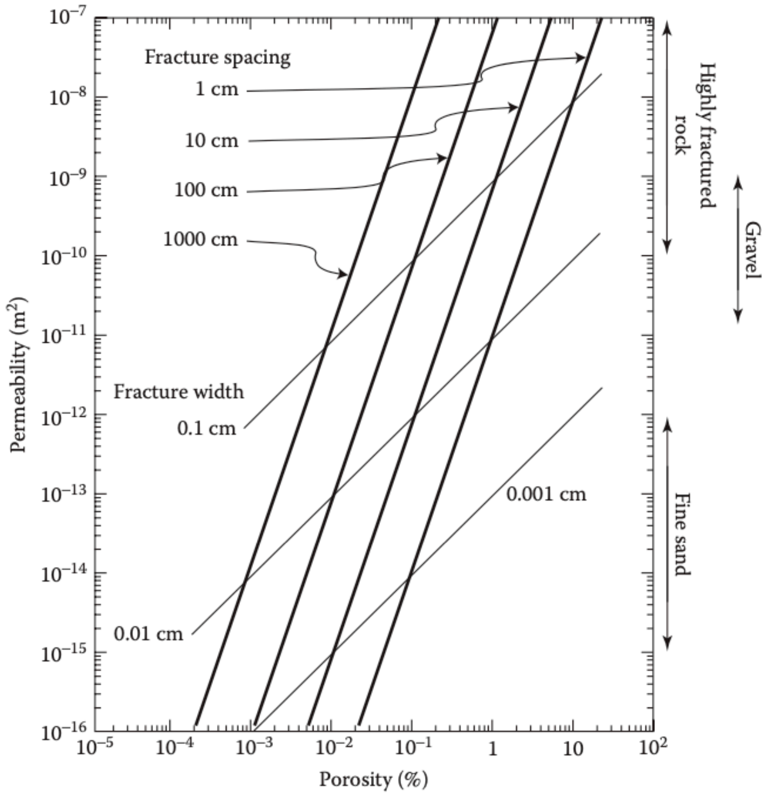


FIGURE 4.6 Theoretical relationship between fracture permeability (air) and fracture porosity. The bulk porosity and permeability for a given fracture width (or aperture) and spacing of those fractures is found by locating the intersection of the width and spacing of interest. It is clear that permeability is a function of both fracture width and spacing, both of which affect bulk porosity. (Modified from Reservoir Characterization Research Lab, University of Texas, Austin, TX, available at http://www.beg.utexas.edu/indassoc/rcrl/rckfabpublic/petrovugperm.htm; Lucia, F.J., *American Association of Petroleum Geologists*, 79, 1275–1300, 1995.)

Crustal Permeability at Depth

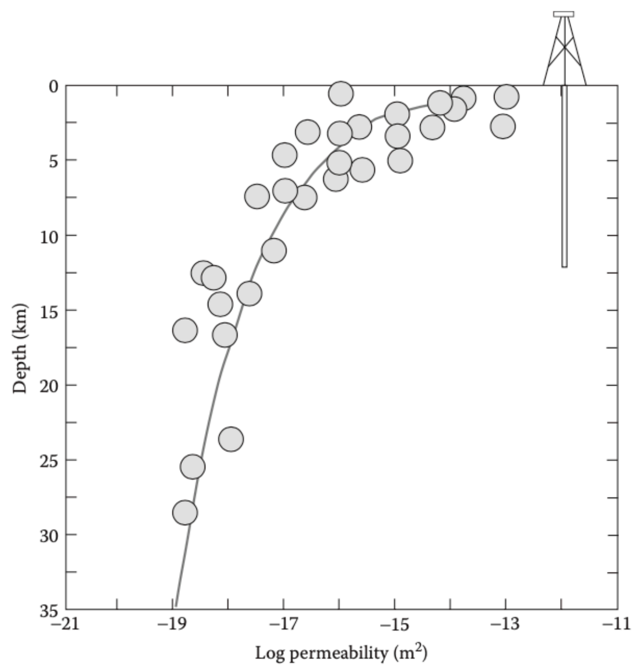
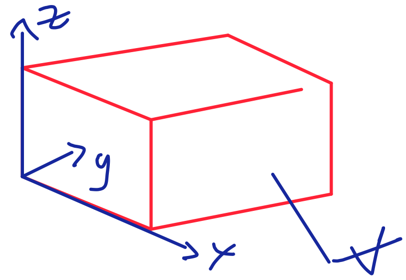


FIGURE 4.8 The variation of permeability as a function of depth. For reference, the depth of the deepest drilled oil well in the world is also portrayed. (Modified from Manning, C.E. and Ingebritsen, S.E., *Reviews of Geophysics*, 37, 127–150, 1999.)

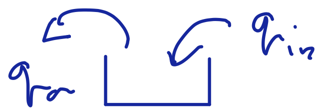
Conservation of Mass/Momentum



$$\frac{\partial}{\partial t}(\rho \cancel{V_v}) + \frac{\partial}{\partial x}\rho \cancel{q}_x = 0$$

$$V_v \to n V_T$$

$$q = \cancel{k_r}\,\frac{k}{\mu}\frac{dp}{dx}, \quad k_r = 1$$



$$n = \frac{V_v}{V_T}$$

$$\frac{\partial}{\partial t}(\rho n) - \frac{\partial}{\partial x}\frac{k}{\mu}\frac{dp}{dx}\rho = 0$$

$$\frac{\partial p}{\partial p}\rho\frac{\partial n}{\partial t} + n\frac{\partial \rho}{\partial t}\frac{\partial p}{\partial p} - \frac{\partial^2 p}{\partial x^2}\rho\frac{k}{\mu}$$

$$\cancel{\frac{\rho}{\rho}}\frac{\partial n}{\partial p}\frac{\partial p}{\partial t} + \frac{n}{\rho}\frac{\partial \rho}{\partial p}\frac{\partial p}{\partial t} - \frac{\partial^2 p}{\partial x^2}\frac{k}{\mu}\frac{\rho}{\rho}$$

$$\left(\frac{\partial n}{\partial p} + \frac{n}{\rho}\frac{\partial \rho}{\partial p}\right)\frac{\partial p}{\partial t} = \frac{k}{\mu}\frac{\partial^2 p}{\partial x^2}$$

Compressibility of reservoir $\left(\frac{1}{Pa}\right)$

$$n \cdot \frac{1}{\rho}\frac{\partial \rho}{\partial p}$$

$C_{water} \to E_{water} \sim 2\ GPa$

$C_{vapor} \to E_{gas} \sim P$

$E = 10\ GPa = 10 \times 10^9\ Pa$

$C = \frac{1}{E}$

$$\left(C_m + n\,C_{fluid}\right)\frac{\partial p}{\partial t} = \frac{k}{\mu}\frac{\partial^2 p}{\partial x^2} + \frac{k}{\mu}\frac{\partial^2 p}{\partial^2 y} + \cdot$$

Heads, h

$$\frac{K}{\rho g} = \frac{k}{\mu}$$

$$h = \frac{P}{\rho g} + z$$

$$S_w + S_{nw} = 1$$

$$P_c = P_{nw} - P_w$$

$$\left(C_m + c_f n\right)\frac{\partial p}{\partial t} = \frac{k}{\mu}\frac{\partial^2 p}{\partial x^2} \leftarrow$$ Petroleum Geotherm

$$S_s\frac{\partial h}{\partial t} = K\frac{\partial^2 h}{\partial x^2} \leftarrow$$ Groundwater

3.3 Mass Conservation in Multiphase Flow

Continuity equation: $\frac{\partial}{\partial t}(n S_\alpha \rho_\alpha) + \frac{\partial}{\partial x_i}(\rho_\alpha q_{i\alpha}) = 0$ $\qquad \alpha = 1, 2$

or $\alpha = 1, 2, 3.$

For an incompressible fluid and medium $\frac{\partial}{\partial t}(n \text{ and } \rho) = 0$

Substitute q_α from relative permeability relation:

Results in 4 equations:

$$n\frac{\partial S_1}{\partial t} - \frac{\partial}{\partial x_i}\left[k\frac{k_{r1}}{\mu_1}\left(\frac{\partial p_1}{\partial x_j} + \rho_1 g\frac{\partial z}{\partial x_j}\right)\right] = 0$$

$$n\frac{\partial S_2}{\partial t} - \frac{\partial}{\partial x_i}\left[k\frac{k_{r2}}{\mu_2}\left(\frac{\partial p_2}{\partial x_j} + \rho_2 g\frac{\partial z}{\partial x_j}\right)\right] = 0$$

$$S_1 + S_2 = 1$$

$$p_2 - p_1 = p_c(S_1)$$

Solve for 4 unknowns: S_1, S_2, p_1, p_2

with $h_1 = z + \frac{p_1}{\rho_1 g}$; $h_2 = z + \frac{p_2}{\rho_2 g}$

Solve using numerical techniques.

SIMPLIFIED RELATIONS FOR FLOW

HYDROLOGY

$$K\frac{\partial^2 h}{\partial x^2} = S_s \frac{dh}{dt}$$

K = hydraulic conductivity
coefficent of permeability
(m/s).

z ; $P \rightarrow h = \frac{P}{\gamma_f} + z + \cancel{\frac{v^2}{\rho g}} \approx 0$; $z = 0$

S_s = specific storage

$$S_s = \rho g(\alpha + n\beta)$$

Reservoir rock compressibility $\sim (1/10\,GPa)$

Fluid compressibility

PETROLEUM ENG/GEOTHERMAL RES. ENG.

$$\frac{K}{\rho g}\frac{\partial^2 h}{\partial x^2} = \frac{S_s}{\rho g}\frac{dh}{dt}$$

with $\frac{K}{\rho g} = \frac{k}{\mu}$; $p = h\cdot\rho g = h\gamma$

Substituting:

$$\frac{k}{\mu}\frac{\partial^2 p}{\partial x^2} = \bar{\beta}\frac{\partial p}{\partial t}$$

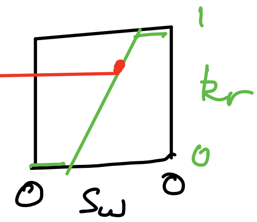
$$\bar{\beta} = (\alpha + n\beta)$$

QED

STEADY STATE FLOW ($\partial p/\partial t = 0$)

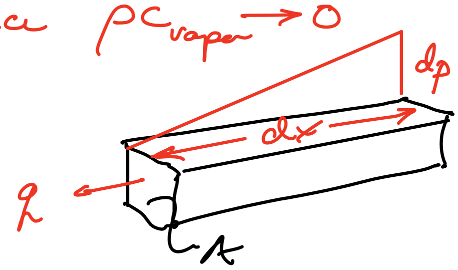
$k_r \sim 1$

$$q = -k_r(S_{\text{water}})\frac{k}{\mu}\frac{dp}{dx}$$



Less interested in q_{vapor} since $\rho c_{\text{vapor}} \rightarrow 0$

$$Q = qA = A\frac{k}{\mu}\frac{dp}{dx}$$



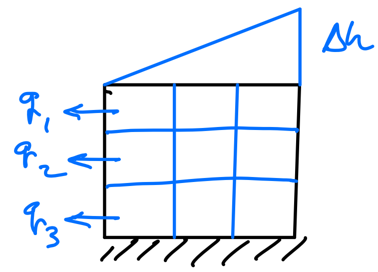
$$\text{Power} = Q\rho c\Delta T = \frac{m^{\cancel{3}}}{s}\cdot\frac{\cancel{kg}}{\cancel{m^3}}\frac{J}{\cancel{kg}\cdot\cancel{K}}\cdot\cancel{K} = \frac{J}{s} = W$$

GEOMETRY OF FLOW

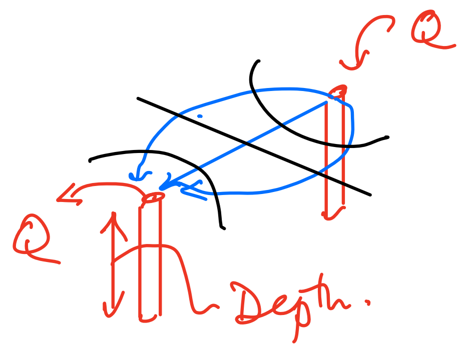
Flow Nets

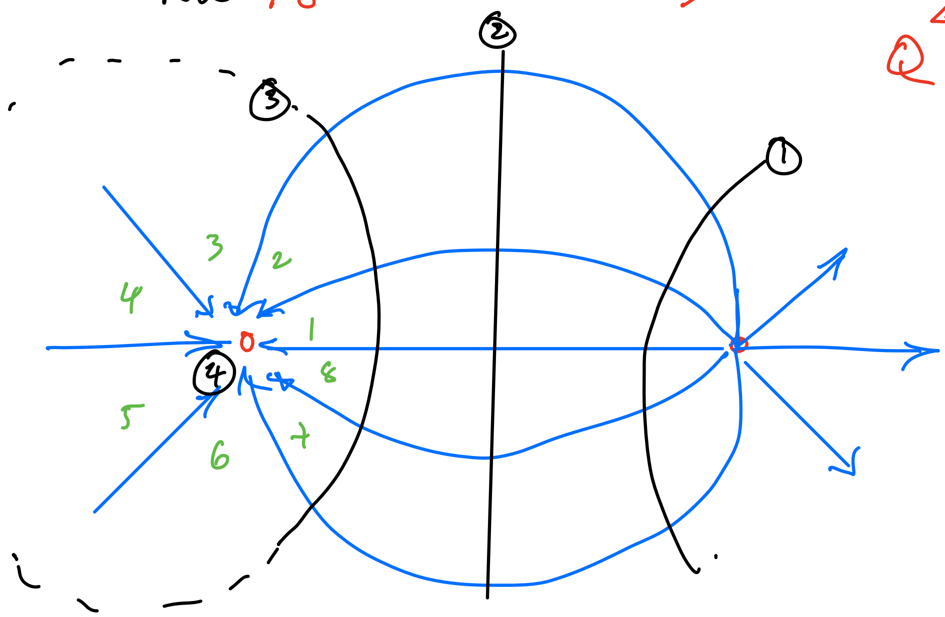
$$Q = \frac{N_f}{N_d} \cdot K \, \Delta h \doteq \frac{-}{-} \, \frac{m}{s} \cdot m \times \left(\begin{array}{l}\text{length} \\ \text{into page}\end{array}\right)$$

m^3/s.



$$Q = \frac{N_f}{N_d} \cdot \frac{K}{\rho g}(\Delta h \, \rho g) = \frac{N_f}{N_d} \cdot \frac{k}{\mu} \Delta p$$





$$\frac{N_f}{N_d} \sim \frac{7}{4} \sim \frac{1}{2}$$

$$\boxed{Q \sim \text{Depth} \times \frac{N_f}{N_d} \times \frac{k}{\mu} \cdot \Delta p}$$

$$\boxed{\text{Power} \sim Q \rho c \Delta T}$$

QED.

Recall economic limit of $Q = 100\ l/s \sim 0.1\ m^3/s$. for ~5 MW.

Check. $0.1 \frac{m^3}{s} \times 1000 \frac{kg}{m^3} \times 4000 \frac{J}{kg.K} \times 100\ K$

~ 40 MW Thermal.

What is limiting k for 100 l/s?