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Application of Non-linear Flow Laws in Determining Rock Fissure Geometry From Single Borehole Pumping Tests

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This paper deals with the effect of non-linear flows on the results of constant pressure well tests in single fractures. The non-linearity results from kinetic effects as flow leaves the laminar regime and enters the mixed laminar/turbulent flow regime. Well test designs are considered that use the long term steady flow in addition to designs using the transient period. A multistage steady flow test using as few as three pressure steps can be used to determine the extent of turbulent flow as well as fracture aperture, roughness, and, under some conditions, the distance to a constant pressure boundary. Finite element simulations are used to develop type curves for transient flow analysis where non-linear flow may exist. Unique curves can be developed only for specific values of relative roughness, aperture, and dimensionless well pressure. Non-linear flow strongly affects the magnitude of flow but does not change the form of the transient flow curve greatly, hence neglecting possible non-linear flow may lead to errors as much as an order of magnitude in estimation of transmissivity. Transient tests using a range of pressures, including sufficiently low pressure for laminar flow, can be used to deduce relative roughness, aperture and the distance and nature of fracture boundaries.

NOMENCLATURE

A, B, C, D—Constants for steady state analysis.
K, K_L, K_T—Hydraulic conductivity, laminar fissure conductivity, turbulent fissure conductivity.
K_4, K_5—Hydraulic conductivity coefficients for zone 4 and 5 flow.
R_e—Reynold's number.
S—Storativity.
$\mathbf{T}$—Transmissivity tensor.
a, b—Coefficients in Forchheimer relationship.
b—Mean fissure aperture.
c, m—Coefficient and exponent from Missbach relationship.
g—Gravitational acceleration.
$h, \nabla h, \nabla h_s$—Total hydraulic head, head gradient and maximum hydraulic gradient.
k—Mean fissure roughness double amplitude.
k_n—Fissure normal stiffness.
n—Media porosity.
q, q_c, q_d—Hydraulic discharge rate, critical discharge rate and dimensionless discharge rate.
r_e, r_w—Fissure external and well bore radii.
t, t_d—Time and dimensionless time.
v—Macroscopic fissure flow velocity.
x—Cartesian space (x_1, x_2, x_3).
α'—Media compressibility.
α—Turbulent flow exponent.
β—Fluid compressibility.
∇^T—Linear differential operator $[\partial/\partial x_1, \partial/\partial x_2]$.
γ_w—Unit weight of fluid.
η—Inertial factor.
ν—Fluid kinematic viscosity.

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INTRODUCTION

This paper deals with the influence of non-linear, or turbulent, flow on the interpretation of constant pressure well tests. The emphasis is on single fractures, although the general principles would apply to porous aquifers or test zones with multiple fractures. The potential errors introduced by non-linear flow in constant pressure tests (also known as packer tests or Lugeon tests) has long been recognized in the engineering literature [1]. Multiple pressure stage tests may be used to check for turbulence as well as for fracture opening. In the following text it is suggested how such multistage tests can be put to good use to determine fracture aperture, roughness, and effective radius. A second extension is to quantify the consideration of non-linear flow in transient well testing analysis. Transient analysis, though long overlooked in engineering practice, has been the basis for most petroleum, gas, and water well testing [2].

Nonetheless, turbulent flow in constant pressure well testing has not received great attention in the reservoir engineering literature, with a few exceptions [3] relating to non-linear flow in vertical hydraulic fractures.

The importance of well testing to civil and mining engineering cannot be underestimated with the rapid expansion of work in the hazardous and nuclear waste disposal area. The potential information well testing can yield on the geometry and interconnection of fractures may also help in predicting the mechanical integrity of rock and in excavation design.

Standard type curve analyses are available to aid data reduction for both constant bottom hole pressure and constant discharge rate pumping tests. For these, flow is assumed linear, laminar, free from substantial inertial effects and within idealized fissure aquifers of constant section and conductivity. Either open or closed fissure boundaries may be accommodated with it being possible to obtain fissure aperture, roughness and extent from a single series of tests. The geometric configuration of the type curves are, however, similar for large variations in fissure parameters, the consequence being that ambiguities in interpretation are difficult to avoid. If it were possible to remove these ambiguities, it would be feasible to both increase the degree of confidence in the resulting prediction and extend inferences as to the subsurface configuration of interconnecting fissures. To this end, the use of quantified non-linearities in interpreting the response of the rock mass to hydraulic loading are exploited.

Non-linearities in the behaviour of the jointed mass may be recognized in both the mechanical and hydraulic response of the interconnected fissures. The load–deformation characteristics of rock joints are intrinsically non-linear in both shear and normal displacements. From a hydraulic standpoint, non-linearities arise from the combined effects of inertial and kinetic losses and at higher flow velocities turbulent flow may be present within the open fissures.

Non-linearity, within the context of the following, is restricted to the kinetic effects of laminar and turbulent flow. In all cases, the fissures are considered rigid and the wall rock impermeable. Cases of axisymmetric flow at constant bottom hole pressure are treated for both open and closed external (fissure) boundaries.

HYDRAULIC CHARACTERIZATION OF ROCK FISSURES

It is most convenient to decompose the problem of fluid transmission in fractured rock masses to the hydraulic behaviour of the individual fractures constituting the system. The performance of these individual fractures may be evaluated to determine the hydraulic conductivity and to calculate the aperture. Individual conduits may then be assembled to represent the geometry of the system. An example of linear modelling of rock masses using discrete fracture properties is described in Long [4]. The performance of a fractured rock mass may then be simulated numerically for arbitrary boundary conditions.

Fissure hydraulics

Relations between discharge rate and the hydraulic loading of potential gradient may be deduced from analytical representation of the problem or by empirical means. The transmissivity term relating the discharge rate (q) to the hydraulic gradient (∇h) is linear only for laminar flow at low velocities. At high velocities and beyond the range of laminar flow, the transmissivity term is a non-linear function dependent on the potential gradient. These relations may be identified symbolically as

$$q = [\mathbf{T}]\nabla h \qquad \text{Linear flow} \tag{1}$$

$$q = [\mathbf{T}(\nabla h)]\nabla h \quad \text{Non-linear flow} \tag{2}$$

where $\mathbf{T}$ = transmissivity tensor. Characterization for the linear laminar and the non-linear turbulent regimes must be treated individually.

Linear potential flow

Analytical treatment of fissure flow is possible if the conduit is idealized as parallel plates [5]. This analogy is relatively consistent for saturated, laminar, incompressible flow where fluid transmission within the wall rock is negligible. Wall roughness must be slight in relation to the mean fissure aperture. The parallel plate analogy yields the transmissivity of a fissure as

$$T = \frac{g\,\mathrm{b}^3}{12\nu} \tag{3}$$

where T = transmissivity, g = gravitational acceleration, b = mean fissure aperture and ν = fluid kinematic viscosity [2]. A considerable amount of experimental work has been completed attempting to increase the fidelity of this relation for rock fissures with significant wall roughness. The relations used in this work are from the work of Louis [6] where a correction factor is included to allow for the case where wall roughness becomes significant.

Non-linear potential flow

The non-linearity inferred in this context is that discharge rate is no longer a unique function of hydraulic gradient. The transmissivity tensor for the fissure is consequently a non-linear function of gradient and may be expressed symbolically as

$$q = [\mathbf{T}(\nabla h)]\nabla h. \tag{2}$$

The non-linearity results from the mixed interference of inertial and kinetic effects, both of which may be substantial for high velocity flow.

(*i*) *Inertial effects*. Inertial effects are manifest where high spatial accelerations within the flow domain are apparent. Acceleration results from flow convergence and may be envisaged to occur adjacent to well bores or in fissures tapering in the direction of flow. This effect is not exhibited in plane, non-tapering flow situations.

The results of divergent flow tests by Iwai [7] illustrated the possibility of defining a critical ratio of inertial to viscous head losses within a system. This factor may be stated as

$$\eta = \frac{\left| v \dfrac{\partial v}{\partial x} \right|}{\left| -g \dfrac{\partial h}{\partial x} \right|} \tag{4}$$

where v is macroscopic flow velocity and x corresponds to an increment of length in a cartesian system of co-ordinates. The term may be evaluated for any given flow configuration. For laminar flow, a marked discrepancy between the discharge estimated from linear flow theory and that measured in actual tests is shown to exist only for values of η greater than 0.5. Thus, it is expected that if values of η in any flow system are maintained below this threshold, inertial effects may be neglected.

(*ii*) *Kinetic effects.* Kinetic effects result from head losses at increased flow velocities being dependent on velocity head in addition to the potential head. For low velocity flows, the kinetic head is negligible in comparison to potential head and may be neglected. However, for flow in fissure systems, kinetic losses may account for a significant portion of the total losses.

In addition to kinetic losses, turbulent flow may be manifest at increased flow velocities. The onset of turbulence may be indexed by recourse to the critical Reynolds number. For flow in rock fissures, the Reynolds number may be defined as [6].

$$R_e = \frac{2b \cdot v}{\nu} \tag{5}$$

where v = velocity, b = mean fissure aperture and ν = fluid kinematic viscosity. For rock fissures, the critical Reynolds number commonly lies between 100 and 2300, increasing from the lower datum with decreased joint wall relative roughness.

Two flow laws have been proposed to relate flow velocity to hydraulic gradient in a non-linear manner. These relations are compiled in recent literature [8,9]. The first of these is the Forchheimer law using a polynominal expression to describe the velocity dependant gradient (∇h) as

$$\nabla h = av + bv^2 \tag{6}$$

where a and b are constants and v is the flow velocity in the direction of maximum gradient. Constants a and b are determined experimentally and are properties of the fluid and transmitting medium. The constants are only applicable over a given range of flow velocities, outside which, revised parameters must be substituted. For low velocity flows, constant a is much larger than b and the expression reduces to the normal Darcy law. The validity of this relation has been inferred from both experimental results and from manipulation of the Navier–Stokes equation [10].

A second relation between hydraulic gradient and flow velocity is the Missbach law in the form of a power function

$$\nabla h = cv^m \tag{7}$$

where c is a proportionality constant and m is an exponent ranging between 1 and 2. The magnitude of both c and m vary with flow velocity although may be sensibly constant over a given range. The validity of the law has been confirmed for fissure flow in simulated rock fractures by Louis [6]. These flow laws enable the conductance of a fissure of known geometry to be determined uniquely as a function of flow velocity. The Missbach law in equation (7) is more conveniently rewritten as

$$v = -K(\nabla h)^{\alpha} \tag{8}$$

where K represents the hydraulic conductivity of the fissure and α is equal to unity for laminar flow and 1/2 for turbulent flow in a rough walled fracture. Suitable values of K and the range of validity of the laminar regime are obtained from the work of Louis [6]. Fissure conductivity is a function of aperture and fissure wall relative roughness. A critical Reynolds number is ascribed for any given fissure relative roughness to delineate the transition from laminar to turbulent flow. Empirical results [6] for the constants K and α are shown in Table 1. The range of validity of these parameters as a function of the dimensionless parameters of relative roughness ($k/2b$) and Reynolds number is shown in Fig. 1. The form of non-linear conductivity is illustrated in Fig. 2 as a function of gradient beyond a threshold gradient for a number of representative fissure apertures. It is evident that as the driving gradient is increased, the impedence to flow similarly increases in a non-linear manner.

The two phases of flow in the laminar and turbulent regime give

Laminar $$v = K_L \nabla h; \quad \nabla h = \frac{v}{K_L}. \tag{9}$$

Turbulent $$v = K_T (\nabla h)^{1/2}, \quad \nabla h = \frac{v^2}{K_T^2}. \tag{10}$$

The Missbach relation implies that flow is linear until the onset of turbulence. Beyond this threshold, the effect of kinetic head in contributing to total head loss is considered. The implication of this is that head loss due to velocity effects is only considered beyond the turbulent threshold rather than as a continuous function of velocity as in the Forchheimer relation. The Missbach

Table 1. Equivalent hydraulic conductivities (from Louis [6])

Hydraulic zone	Hydraulic conductivity (L/T)	Exponent (α)
1	$\dfrac{gb^2}{12\nu}$	1.0
2	$\dfrac{1}{b}\left[\dfrac{g}{0.079}\left(\dfrac{2}{\nu}\right)^{1/4} b^3\right]^{4/7}$	4/7
3	$4g^{1/2}\log\left[\dfrac{3.7}{k/2b}\right]b^{1/2}$	1/2
4	$\dfrac{gb^2}{12\nu(1+8.8(k/2b)^{3/2})}$	1.0
5	$4g^{1/2}\log\left[\dfrac{1.9}{(k/2b)}\right]b^{1/2}$	1/2

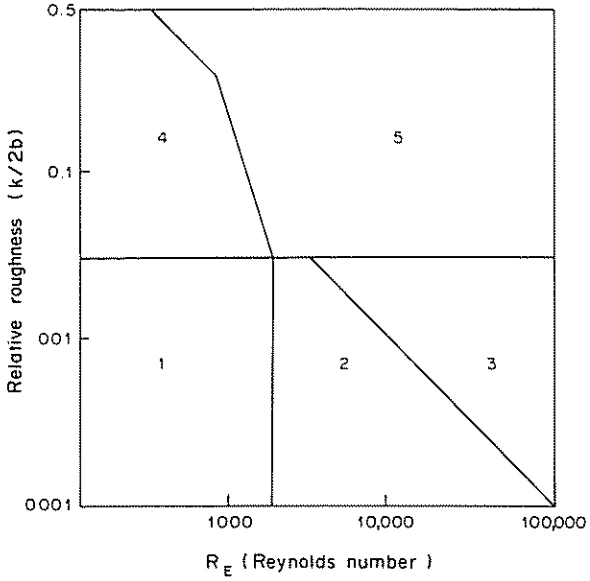


Fig. 1. Flow regions for laminar and turbulent flows. Corresponding flow laws shown in Table 1 (after Louis [6]).

relation is analogous to the Forchheimer relation where the coefficient a and b may be substituted as

Laminar	$a = \dfrac{1}{K_L}$	$b = 0$
Turbulent	$a = 0$	$b = \dfrac{1}{K_T^2}$

Throughout this work, the Missbach flow law is used to characterize the full flow regime. This is the form in which the only authoritative experimental data known to the authors is reported.

Fissure storativity

Evaluation of fissure storativities are required for transient analyses. Fissure storativity is defined as the mass rate of change of fluid consequent of a unit change

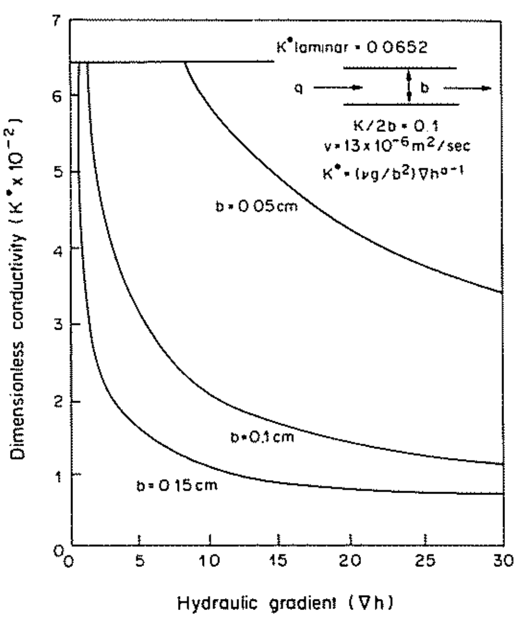


Fig. 2. Variation in hydraulic conductivity with hydraulic gradient.

in head and is a function of both fluid compressibility and the change in void volume of a deforming fissure. In the most general form, storativity (S) may be stated as

$$S = b\gamma_w(\alpha' + n\beta) \tag{11}$$

where b = fissure aperture, γ_w = fluid density, α' = media compressibility, n = porosity and β = fluid compressibility.

Media compressibility is defined as

$$\alpha' = \frac{dV_t/V_t}{d\sigma} \tag{12}$$

where V_t = total void volume and σ = ambient stress. For a fissure comprising non-contacting walls and exhibiting constant normal stiffness, the storativity may be defined as [2]

$$S = \gamma_w(1/k_n + b\beta) \tag{13}$$

where b = mean fissure aperture and k_n = fissure normal stiffness. This expression is sufficient for the case of limited fissure wall contact only with recourse to a different treatment being warranted at higher stress levels [11]. Since the fissures are assumed rigid in this treatment, only cases of constant storativity are examined.

Numerical formulation

The non-linear nature of the constitutive flow law results in numerical solution being the only tractable method of treatment for problems of realistic geometry. A number of very powerful numerical techniques are available to solve such problems. The domain methods of finite difference and finite element analysis may readily accommodate non-linear constitutive behaviour. Of these two methods, the finite element method has been chosen and a model developed to solve the case of mixed laminar/turbulent flow. Any number of arbitrarily oriented and interconnected fissures may be accommodated within the three dimensional framework.

For incompressible transient flow within a fissure of piecewise constant aperture, the equation of continuity may be stated as

$$\nabla^T b\mathbf{K}\nabla h = S\frac{\partial h}{\partial t} \tag{14}$$

where $\nabla^T = [\partial/\partial x_1, \partial/\partial x_2]$ is a linear differential operator, h is total potential head, b is fissure aperture and $\mathbf{K}$ is a non-linear conductivity tensor. The domain geometry is illustrated in Fig. 3 where the in-plane cartesian co-ordinates are x_1, x_2 and external flux and head boundary conditions are applied over the surface Γ_q and Γ_h, respectively. Fissure storativity is denoted S and t represents time. The members of $\mathbf{K}$ conform to the previously outlined Missbach laws where

$$\mathbf{K} = K(\nabla h_s)^{\alpha-1} \tag{15}$$

the terms K and α are as given in Table 1 and ∇h_s is the scalar value of maximum gradient.

The continuity relation of equation (14) has been formulated into a finite element model. For full descrip-

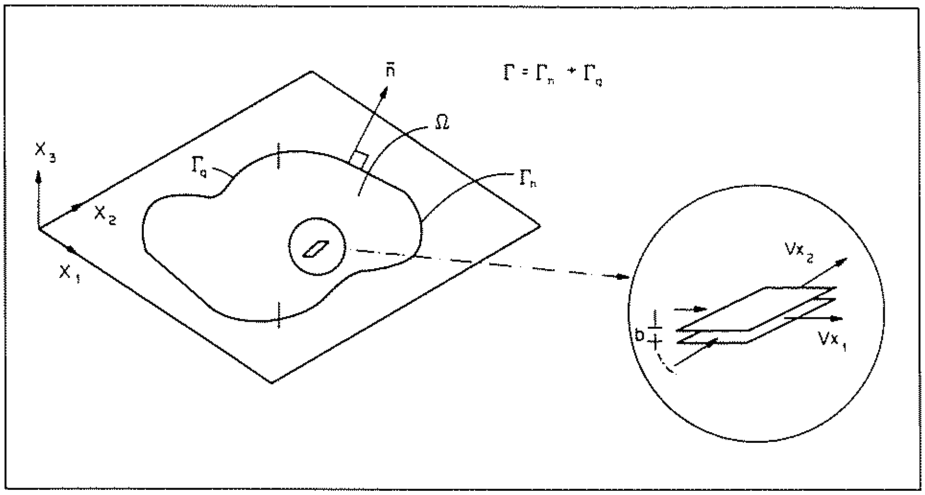


Fig. 3. Flow domain geometry for a single fissure.

tion of the method, the interested reader is referred to a number of excellent general texts [12, 13] with more specific details concerning application to turbulent fissure flow being available in Elsworth [11].

Since the problem is non-linear in terms of the driving gradient, explicit direct iteration is employed whereby the non-linear hydraulic conductivity tensor is updated at each iterative step in time. An original laminar potential distribution is used to determine the conductivity for the first iteration. Successive iterations further refine the solution until a convergence tolerance is met. The sequencing of the algorithm is illustrated in Table 2.

Table 2. Summary of the solution method for transient laminar/turbulent flow in fissure networks

(1) Define input. Geometry, element apertures and storativities, boundary conditions, initial conditions (h_0) and convergence tolerance.
(2) Form initial conductance matrix $\mathbf{T}$ and initial storativity vector $\mathbf{S}$, and modify for boundary conditions.
(3) Compute internal flow due to conductivities at time zero:

$$\mathbf{q}_0 = \mathbf{T}_0 \mathbf{h}_0.$$

(4) Compute the effective system matrices:

$$\mathbf{S}^* = \frac{1}{\Delta t}\mathbf{S}; \quad \mathbf{T}^* = \mathbf{T} + \mathbf{S}^*.$$

(5) Triangularize $\mathbf{T}^*$.

For each time step
(1) Increment time step counter to $t + \Delta t$.
(2) Test for equation reformation:
NO: Advance to step (8);
YES: Continue.
(3) Form new system matrices $\mathbf{T}_t(h)$ and $\mathbf{S}_t$ based on properties $\mathbf{h}_t$.
(4) Modify $\mathbf{T}_t$ for potential boundary conditions.
(5) Compute internal fluid flow due to conductivities: $\mathbf{q}_t = \mathbf{T}_t(h)\mathbf{h}_t$.
(6) Compute new effective system matrixes:

$$\mathbf{S}_t^* = \frac{1}{\Delta t}\mathbf{S}_t; \quad \mathbf{T}_t^*(h) = \mathbf{T}_t(h) + \mathbf{S}_t^*.$$

(7) Triangularize $\mathbf{T}_t^*$.
(8) Define fluid flow vector $\mathbf{q}_{t+\Delta t}$ at time $t + \Delta t$.
(9) Compute effective fluid flow vector: $\mathbf{q}_{t+\Delta t} = \mathbf{q}_{t+\Delta t} - \mathbf{q}_t$.
(10) Solve for nodal point potential increment: $\mathbf{T}_t^*(h)\Delta\mathbf{h}_{t+\Delta t} = \mathbf{q}_{t+\Delta t}$.
(11) Evaluate revised nodal point potentials: $\mathbf{h}_{t+\Delta t} = \mathbf{h}_t + \Delta\mathbf{h}_{t+\Delta t}$.
(12) Check tolerance met over previous cycle:
YES: Continue;
NO: Return to step (3).
(14) Repeat for next time step: Advance to step (1).

APPLICATION OF NON-LINEAR FLOW TO WELL TESTING ANALYSES

As mentioned previously, only well tests utilizing a single borehole held at constant pressure will be discussed. Boundary conditions corresponding to constant well bore pressure apply equally to fissures with open (constant pressure) and closed (no flow) external boundaries. These two special cases will be treated separately, both sets of analyses assuming the well bore to be centrally located within a fissure of idealized circular plan.

Open Boundary Fissure

The discharge pattern corresponding to transient testing of an open fissure is assumed to result in eventual steady discharge at some time following commencement of the test. Thus, both transient and steady state reductions may be conducted with the data to crosscheck parameter evaluation.

(a) Steady state analysis

This form of analysis requires only the long term discharge. An analysis of multistage constant pressure testing is proposed to yield fissure aperture, roughness and effective radius to significant intersection and further reduce ambiguity of interpretation. Of these parameters, the former two are self explanatory. Effective radius to significant intersection denotes the distance from the well bore to an intersection providing a constant pressure boundary condition. A practical example of such a condition would be intersection with another fissure of much greater aperture and hence, greater transmissivity.

A necessary assumption in this case is that an intersection with another fissure over a given length may be transformed to the axisymmetric case with a constant pressure external boundary. This concept is illustrated in Fig. 4. This assumption is observed to be quite reasonable [14] for rigid fissures or at low injection/withdrawal pressures in deformable systems. Similarly, discharge time histories within fissures intersected over as small a

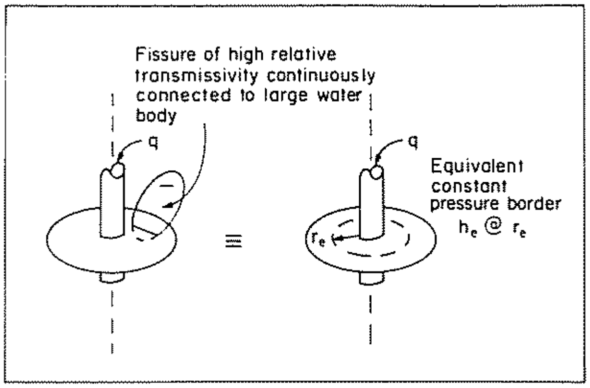


Fig. 4. Equivalent external radius concept for inter-connected fissures.

length as 20% of total circumference are shown to yield similar results to the axisymmetric case.

Testing procedure. The steady state discharge (q) at a minimum of three different pressure head differentials (Δh) are required. Figure 5 illustrates the form of discharge (q) as a function of head differential (Δh) for a multistage test reaching the turbulent regime. The critical discharge (q_c) for the onset of turbulence at the well bore may be observed where the ratio $q/\Delta h$ is no longer constant. The value of q_c, one discharge value prior to turbulence (q_1) and one discharge value for mixed laminar/turbulent flow (q_2) are required.

Analytical development. It is assumed that discharges (q_1, q_2, q_c), pressure head differentials (Δh, Δh_2), well bore radius (r_w) and fluid kinematic viscosity (ν) are all known. The equations for axisymmetric flow for both the laminar and turbulent regimes may be stated as [15,11].

$$\ln r_e = \ln r_w - \frac{2\pi \mathrm{b} K_4 \Delta h_1}{q_1} \tag{16}$$

in the purely laminar regime, and for mixed laminar/turbulent flow

$$\ln r_e = 1 + \ln \frac{q_2 K_4}{2\pi \mathrm{b} K_5^2} - \frac{q_2 K_4}{2\pi \mathrm{b} r_w K_5^2} - \frac{2\pi \mathrm{b} K_4 \Delta h_2}{q_2}. \tag{17}$$

In this, K_4 and K_5 are the fissure hydraulic conductivities for hydraulic zones 4 and 5 respectively as given in Table 1. Equating (16) and (17) and substituting $K_4, K_5 = f(g, \nu, k, \mathrm{b})$ yields

$$\mathrm{b}^3 = \frac{Aq_2}{2\pi g \Delta h_2}\left[1 - \frac{q_2}{q_1}\frac{\Delta h_1}{\Delta h_2}\right]^{-1}[1 + \ln C - C] \tag{18}$$

where

$$A = 12\nu[1 + 8.8(k/2\mathrm{b})^{1.5}] \tag{19}$$

$$B = 16\log^2\left(\frac{1.9}{k/2\mathrm{b}}\right) \tag{20}$$

$$C = \frac{q_2}{2\pi r_w AB} \tag{21}$$

$$\Delta h_1 = h_{w1} - h_{e1}. \tag{22}$$

The heads at the well bore and external boundary are h_{w1} and h_{e1}, respectively for test number one. From knowledge of the critical Reynolds number at which turbulent flow is induced,

$$\frac{q_c}{2\pi r_w} = \mathrm{AB}. \tag{23}$$

In equation (23), the only unknown is the relative roughness ($k/2\mathrm{b}$) which may be evaluated from the dimensionless relation graphed in Fig. 6. With the relative roughness evaluated, the fissure aperture may be calculated from equation (18), more conveniently stated as

$$\mathrm{b}^3 = \frac{Aq_2}{2\pi g \Delta h_2}\left[1 - \frac{q_2}{q_1}\frac{\Delta h_1}{\Delta h_2}\right]^{-1}\left[1 + \ln\frac{q_2}{q_c} - \frac{q_2}{q_c}\right]. \tag{24}$$

With the fissure aperture evaluated, r_e may be obtained from

$$r_e = r_w \cdot \exp[-D] \tag{25}$$

where

$$D = \frac{2\pi g \mathrm{b}^3 \Delta h_1}{Aq_1}. \tag{26}$$

Dimensionless plots for relations (19) and (23) in Fig. 6 enables evaluation of $k/2\mathrm{b}$, b and r_e in a straight-

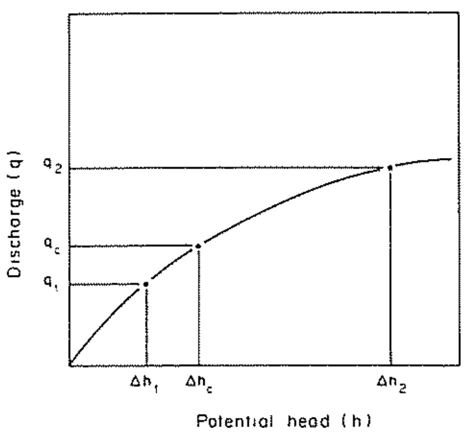


Fig. 5. Response curve for a multistage pumping test.

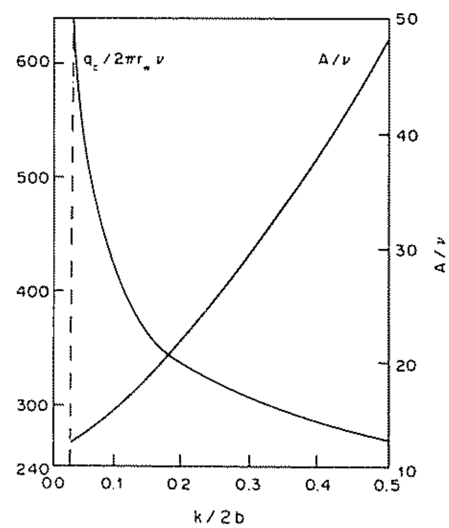
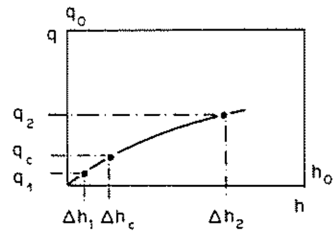


Fig. 6. Dimensionless parameters for steady state, laminar/turbulent data reduction.

Example

A pump test intersecting a single horizontal fissure yields the following results :



Δh (m)	q (m^3/sec)	Data point
-61.540×10^{-3}	0.497×10^{-3}	2
-67.69×10^{-6}	6.790×10^{-6}	Critical
-30.77×10^{-6}	3.210×10^{-6}	1

$r_w = 0.3077$m
$g = 9.81$ m/sec
$\nu = 1.325\times10^{-8}$m^2/sec

$q_c/2\pi r_w \nu = 265.327$ Using figure
$k/2b = 0.5$; $A/\nu = 49.335$

Substituting into equation $b = 0.00185$m

From equation $r_e = 92.497$m

N.B. The problem was originally worked forward using
$b = 0.00185$m ; $r_e = 92.308$ m

Fig. 7. Steady state analysis example.

forward manner. Only three simple steps are required in the entire analysis. An illustrative worked example is included in Fig. 7.

The current analysis is valid for rough fissures when $0.033 < k/2b < 0.5$ since these are considered most important from an engineering standpoint.

(b) *Transient analysis*

Type curve matching is a common procedure for reduction of transient pump test response. For constant pressure (open) external boundaries, semi-analytical solutions are available for laminar flow. Summarized details of such procedures may be found in Doe and Osnes [14]. Extension of this method to mixed laminar/turbulent response during multistage pump testing may, however, provide a convenient method of remotely deducing fissure apertures, roughnesses, and extents, *in situ*.

For laminar analyses, dimensionless discharge (q_d) and dimensionless time (t_d) are uniquely related. The behaviour of any infinite aquifer or fracture is consequently characterized by a single type curve. Using previous nomenclature, these dimensionless parameters are respectively

$$q_d = \frac{q}{2\pi K b \Delta h}, \quad t_d = \frac{tKb}{Sr_w^2} \tag{27}$$

where t = time, S = storativity and r_w = well bore radius. Departure from this system type curve occurs only when the influence of an external boundary is felt. For laminar flow, this departure may be evaluated to occur at [16],

$$t_d = 0.149\,(r_e/r_w)^{2.032} \tag{28}$$

This departure expression is valid regardless of whether the fissure boundary is a constant pressure or no flow (such as the edge of a finite fracture) boundary. Therefore the form of the type curve is shown equally well in Figs 8 and 11 as the uppermost curves. It should be noted from (28) that although the external radius (r_e) is uniquely related to dimensionless time (t_d), some knowledge of the inter-relation between fissure aperture, roughness and normal stiffness is required to fully define the system from the testing results.

If well tests are conducted at pressure head differentials high enough to cause significant early response within the turbulent domain, much of the ambiguity remaining from the laminar analysis may be eliminated. In the turbulent case, no unique relations exists to link the dimensionless parameters of discharge (q_d) and time (t_d). Indeed, these parameters are only meaningful if the fissure transmissivity (Kb) is given a constant arbitrary value. In this work, Kb is defined as the laminar transmissivity.

Retaining use of the dimensionless parameters in evaluation of the true response of the system, a unique response curve is obtained for each pressure head differential

$$q_d, t_d = f\left(\frac{\Delta h}{r_w}, \frac{b}{r_w}, \frac{r_e}{r_w}, \frac{k}{2b}\right). \tag{29}$$

The response curves have been obtained numerically using the finite element formulation for rigid fissure flow outlined previously and are illustrated in Fig. 8. The salient points to be discerned from these results are that:

- the breakaway time and resultant steady discharge at this time are no longer independent of the dimensionless groups of equation (29);
- transient curves for low, moderate and high levels of turbulent flow [$q = f(\Delta h/r_e)$] are similar in overall form but are displaced vertically with respect to discharge (q_d).

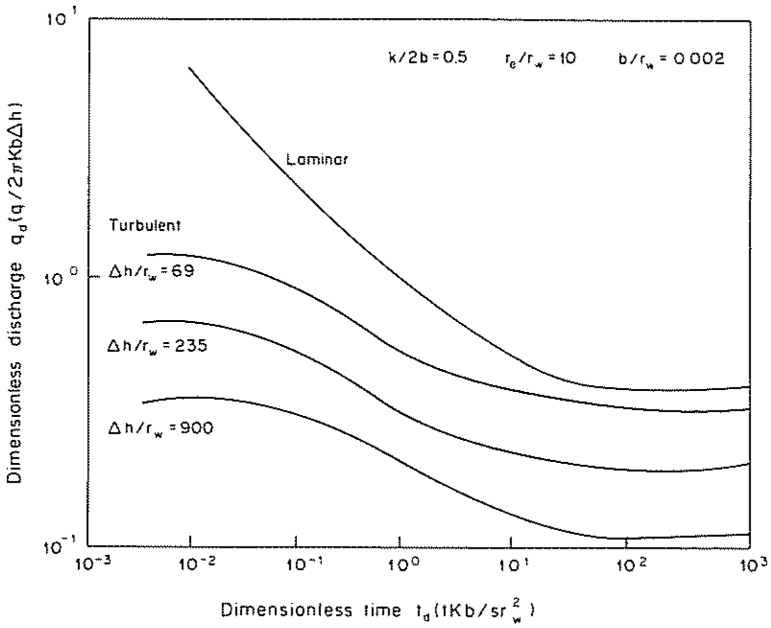


Fig. 8. Transient response of a constant pressure (open) external boundary fissure.

Figure 9 illustrates this second observation with all four curves of Fig. 8 translated to a common vertical datum at steady discharge (q_{steady}). The three curves for the turbulent flow at low, medium and high levels represent turbulent collars surrounding the well bore to dimensionless radii (r/r_w) of 0.1, 5.0 and 10.0 respectively, at steady conditions. It appears, therefore, that accounting for turbulence is a significant factor although the discharge history is insensitive to the level of turbulence.

Closed Boundary Fissure

For the case of a closed boundary fissure with no leakoff, the steady state condition corresponds to uniform fissure fluid pressure and zero discharge. The only useful parameter available from a steady state test, therefore, is the time integral of discharge to give fissure volume.

Transient analyses may, however, be used for such systems. The response in the laminar domain is well documented [16] although the work reported herein represents significant extension of this to the mixed laminar/turbulent regime.

Similar to open boundary fissure pumping test analysis, dimensionless parameters of discharge (q_d) and time (t_d) may be used to facilitate type curve matching. As shown in Fig. 10, the initial and intermediate response to injection/withdrawal is identical to the open boundary case prior to the recharge/depletion front reaching the fissure outer boundary. This is true for laminar and mixed laminar/turbulent flow alike. The significant difference between the open and closed systems is, however, the long term response. As illustrated in Fig. 10, the closed fissure decreases sharply to zero ultimate discharge. Increasing levels of turbulence (increasing $\Delta h/r_w$) yield lower initial discharge and retain increased fluid storage. This excess fluid is available for discharge at large dimensionless times (t_d). This behaviour is intuitively correct and is more pronounced for increased levels of turbulence (Fig. 10).

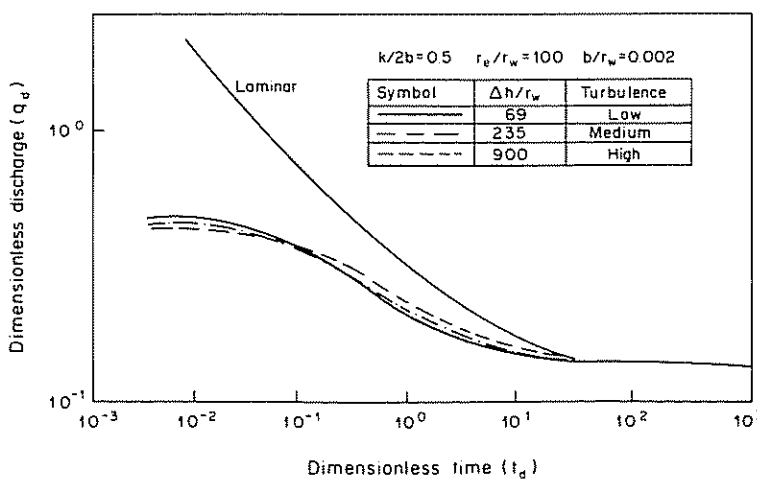


Fig. 9. Shifted transient response of Fig. 8.

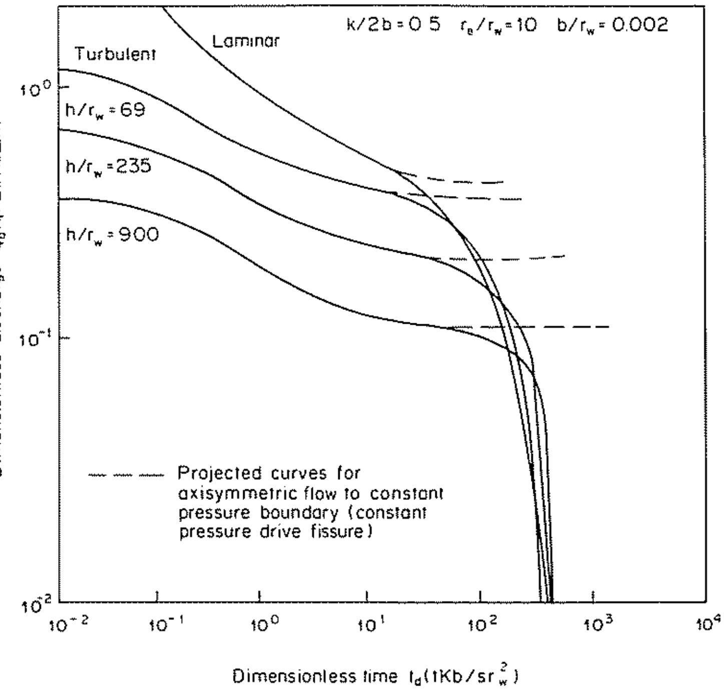


Fig. 10. Transient response of a closed (no flow) boundary fissure.

As fissure radius is increased relative to the well bore, the effect of initial turbulence on the long term response is diminished. This factor is illustrated in Fig. 11. For this particular example, the response of the assumed laminar and true laminar/turbulent systems are identical at large dimensionless times for, $r_e/r_w > 100$. This plot is, however, for a low initial degree of turbulence. Increased testing pressure differentials cause increased mismatch in the results at large dimensionless times.

DISCUSSION AND CONCLUSIONS

Extension of single borehole well test analyses to the turbulent regime is demonstrated to provide a potentially effective tool in evaluation of both geometric and hydraulic properties of fissures (fissure aquifer characteristics). Subject to certain assumptions regarding the geometry of the fissure or fissures, quantitative evaluations of fissure aperture, roughness, extent and degree of interconnection are possible.

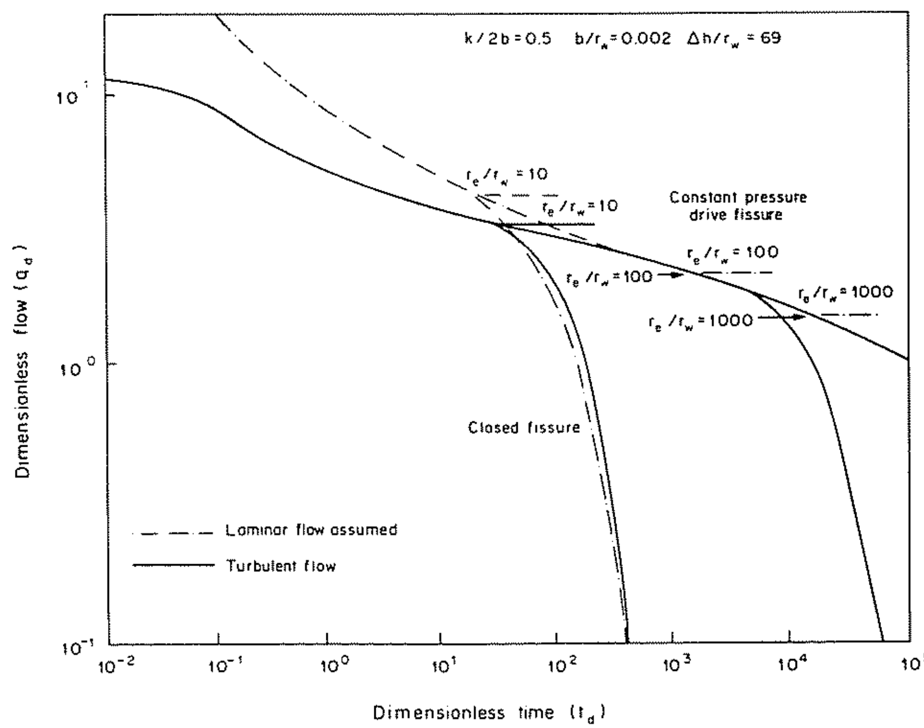


Fig. 11. Transient response of a closed (no flow) boundary fissures of varying radius.

For steady flow analysis, a simple method is proposed to facilitate determination of the component parameters of fissure geometry and intersection. For transient flow analyses, use is made of the fact that, for turbulent flow, the dimensionless ratio of discharge to pressure head differential is no longer constant. Thus dimensionless discharge and time may be expressed as

$$q_d, t_d = f\left(\frac{b}{\Delta h}, \frac{r_e}{\Delta h}, \frac{k}{\Delta h}, \frac{\Delta h}{r_w}\right)$$

Cognizance of the interplay between all these factors and appropriate multistage testing techniques have the potential of enabling full remote determination of fissure parameters. Although the transient flow curves under turbulent conditions are shown to be of similar form for variable roughness, well bore pressure, and boundary conditions, the curves may be appropriately fixed in q_d, t_d log space. Fixing in q_d, t_d space is possible by first performing laminar testing to define the portion of the dimensionless flow curve appropriate to the response. With this calibration completed, further non-linear testing at successively higher pressure differentials is necessary. For these tests, therefore, the form of the response curves are fixed in $\log q_d$, $\log t_d$ space by the initial laminar response. Further testing and appropriate curve fitting would be expected to increase the fidelity of the resultant fissure characterization.

Commonly well testing programmes do not consider the possibility of non-linear flow. For steady analyses this can lead to underestimation of permeability by factors of 4 or more. Similar errors can be made where transient methods are used, as the shapes of the transient flow curves are similar to those of the laminar case.

Although a relatively simple, axisymmetric flow geometry is treated in the above, the implication of the results are that, with the ambiguities removed, a more complete picture of the subsurface fracture geometry may be established. In this respect, analyses of the transient flow may be used to make inferences as to fissure intersections and the hydraulic nature of the inter-connecting fissures. Data reduction techniques would be most appropriately managed, in this regard, if real time numerical simulation techniques complement the pumping test while in progress. In this manner, the "signature" of an individual rock fissure or series of fissures may be used to yield geometric data that would otherwise be impossible to obtain from the limited visual penetration of a single borehole.

The implications of such analysis techniques are far reaching in that complex geometric data pertaining to the rock mass structure may be deduced in addition to the hydraulic behaviour of the constituent fissures. Information of this nature can only enhance understanding of the rock mass prior, say, to any construction or excavation and consequently improve the standard of design and degree of readiness to deal with anticipated construction difficulties.

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