A Modified Gauss-Newton Method for Aquifer Parameter Identification

by Jingsheng Li[a] and Derek Elsworth[b]

Abstract

The accuracy of parameter estimation procedures is evaluated for a modified Gauss-Newton method applied to transient ground-water flow. Three different approaches of evaluating the sensitivity coefficient matrix are examined, including *influence coefficient, sensitivity equation,* and *variational approaches.* The performance of each of the techniques is evaluated by applying a common synthetic data set. The latter two techniques are shown to perform with least sensitivity to starting parameters and extraneous sampling noise. Where either random or systematic noise is added to the time-series data set, the resulting predictions become increasingly more sensitive to the form of the starting transmissivity vector. It is concluded that the modified Gauss-Newton method is attractive because of its simplicity, high rate of convergence, and modest computational demands, especially when the number of the parameters to be identified is not large.

[a]On leave from Central Coal Mining Research Institute, Xi'an, P.R. China, 1993.
[b]Department of Mineral Engineering, Pennsylvania State University, University Park, Pennsylvania 16802-5000, USA.
Received December 1993, revised September 1994, accepted October 1994.

132'' 7 Pages

1. Introduction

Solution methods for parameter identification using inverse models may be classified as either direct or indirect (Neuman, 1973), depending on whether the desired parameters are recovered as dependent variables in the method, or by material characteristics. A parallel categorization may be applied if the models solve the inverse problem with an error criterion applied either to the equations or to the output (Yeh, 1986). The modified Gauss-Newton method developed in this work belongs to the suite of indirect methods and incorporates Rosen's gradient projection method (Rosen, 1960) to ensure that the parameter changes remain in the range of some given constraints. The Gauss-Newton method may be used to define transmissivity distributions directly from measured head distributions or time-series data of heads. An objective function is prescribed to link transmissivity estimates to the differences between measured and calculated head magnitudes. Transmissivity magnitudes are modified systematically and iteratively to minimize this mismatch between predicted and known head magnitudes.

projection

This work compares accuracies of parameter identification results obtained by using the *influence coefficient*, the *sensitivity equation*, and *variational* approaches for the evaluation of the Gauss-Newton direction. Inverse analyses are conducted with various levels of noise added to the head time-series data to determine the sensitivity of predicted transmissivity magnitudes to the choice of method and starting parameters. A Galerkin finite-element method is applied to solve the flow equation where the distributed magnitudes of ~~hydraulic~~ transmissivity and storativity are defined as piecewise constant throughout the region. The aquifer is subdivided into several subregions with each subregion characterized by constant parameter magnitudes of transmissivity and storativity. These values are the parameter unknowns in the model. The ability of the inverse model to reach a stable and bounded solution is demonstrated by using synthetic head time-series data sets, produced by forward analysis, and corrupted with noise of a predefined severity.

2. Numerical Method

The distribution of head that is developed in a confined aquifer may be evaluated in a straightforward manner by application of a forward finite-element method. A Galerkin formulation is used for triangular finite elements comprising linear basis functions and corresponding constant velocity fields within the element. This procedure is used to evaluate synthetic "predicted" head magnitudes and their sensitivity to changes in aquifer transmissivities, as required in the Gauss-Newton analysis. Use of the "synthetic" data set enables the performance of the solution methods to be evaluated in a definitive manner without further complication by unknown domain geometry and material characteristics.

2.1. Modified Gauss-Newton Method

Several authors have evaluated the potential of Gauss-Newton type methods in defining the parameters of both oil reservoirs and aquifer systems (Jacquard and Jain, 1965; Thomas et al., 1972; Gavalas et al., 1976; Yeh and Yoon, 1981; Sun and Yeh, 1985; Willis and Yeh, 1987). The method utilizes the basic optimization dictum as

$$\text{Minimize } E(\mathbf{K}) = \sum_{l} f_l^2(\mathbf{K}) \quad (l = 1, 2, \ldots, L) \qquad (1)$$

$$\text{Subject to } a_i \leq K_i \leq b_i \qquad (i = 1, 2, \ldots, m) \qquad (2)$$

where
$$f_l = (h_l^c - h_l^o) \qquad (3)$$

where L represents the total number of observations; m represents the total number of parameters; **K** are the parameters; E is the objective function; a and b represent lower and upper bounds on the parameters; and h_l^c and h_l^o represent, respectively, calculated and observed heads. The modified Gauss-Newton method generates the parameter sequence,

$$\mathbf{K}^{n+1} = \mathbf{K}^n + \lambda^n \mathbf{P}^n \qquad (4)$$

where $\mathbf{P}^n$ is a search direction; and λ^n is the step size for iteration n. The method starts with an initial estimate of $\mathbf{K}^0$ and generates a sequence of $\mathbf{K}^{n+1}$ until the convergence criterion is satisfied. Given a vector, $\mathbf{K}^n$, a direction vector, $\mathbf{P}^n$, is sought together with a suitable step size, λ^n, yielding a new vector $\mathbf{K}^{n+1}$. The process is then repeated until

$$|E(\mathbf{K})| \leq \xi \qquad (5)$$

where ξ is a small positive number, or until a predetermined number of iterations are completed. In order to determine the direction, $\mathbf{P}^n$, a Gauss-Newton direction, $\Delta\mathbf{K}$, and a gradient, g, of the objective function, E, are first evaluated. Correspondingly

bold

$$\mathbf{P}^n = \begin{cases} \Delta \mathbf{K} & \text{if } \mathbf{g} \cdot \Delta \mathbf{K} < 0 \\ -\mathbf{g} & \text{if } \mathbf{g} \cdot \Delta \mathbf{K} \geq 0 \end{cases} \tag{6}$$

with

$$\Delta \mathbf{K} = -(\mathbf{J}^T\mathbf{J})^{-1}\mathbf{J}^T\mathbf{f} \tag{7}$$

where **J** is a matrix of sensitivity coefficients, as later defined in equation (9), and superscripts represent transpose and inverse. In this, revised transmissivity magnitudes are scaled, relative to the mismatch between known and evaluated heads. A projective operator, $\overline{\mathbf{P}}$, is introduced to ensure that $\mathbf{K}^{n+1}$ will be satisfied under the applied constraints. The definition of $\overline{\mathbf{P}}$ is

$$\overline{\mathbf{P}}_i = \begin{cases} 0 & K_i = a_i \text{ and } P_i < 0 \\ 0 & K_i = b_i \text{ and } P_i > 0 \\ P_i & \text{otherwise} \end{cases} \tag{8}$$

where $\mathbf{P}^n = \overline{\mathbf{P}}$. The scalar magnitude of step size, λ^n, defined in equation (4), may be determined by quadratic interpolation. The calculation procedure is illustrated in the flow chart of Figure 1.

2.2. Sensitivity Coefficients

The modified Gauss-Newton method requires the evaluation of a sensitivity coefficient matrix, as defined in equation (7). For unsteady-state flow in a two-dimensional, piecewise-heterogeneous, isotropic, and confined aquifer, the unknown parameter is the transmissivity vector, **T**. This assumes that storage coefficient and source/sink magnitudes are known. ~~or may be estimated with reasonable bounds.~~ Thus the sensitivity coefficient matrix can be expressed as

$$\mathbf{J} = \begin{bmatrix} \frac{\partial h_1}{\partial T_1} & \frac{\partial h_1}{\partial T_2} & \cdots & \frac{\partial h_1}{\partial T_m} \\ \frac{\partial h_2}{\partial T_1} & \frac{\partial h_2}{\partial T_2} & \cdots & \frac{\partial h_2}{\partial T_m} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial h_L}{\partial T_1} & \frac{\partial h_L}{\partial T_2} & \cdots & \frac{\partial h_L}{\partial T_m} \end{bmatrix} \tag{9}$$

where the index L represents the total number of observations; and the index m represents the number of components comprising the transmissivity vector. Three approaches are commonly used to calculate the sensitivity coefficient matrix. These are defined as the *influence coefficient, sensitivity equation,* and *variational* approaches, described briefly in the following.

2.2.1. Influence Coefficient Approach

This approach is based on a finite-difference representation where the sensitivity coefficients can be expressed as (Becker and Yeh, 1972; Bard, 1974),

$$\frac{\partial h_l}{\partial T_i} = [h_l(x, y, t, T_1, T_2, \ldots, T_i + \Delta T_i, \ldots, T_m) - h_l(x, y, t, T_1, T_2, \ldots, T_i, \ldots, T_m)]/\Delta T_i \tag{10}$$

where $h(x, y, t, \underline{T})$ is a solution to the flow equation; and ΔT_i is a small increment of T_i, with

bold

$$\Delta T_i = \alpha T_i \tag{11}$$

where $10^{-5} \leq \alpha \leq 10^{-2}$. This method solves the inverse problem according to the method of Becker and Yeh (1972). In this approach, one difficulty is the choice of the transmissivity increment, ΔT_i. The magnitude of the transmissivity increment, ΔT_i, is selected by a trial-and-error procedure. A total of m + 1 simulation runs are needed to compute the sensitivity coefficient matrix.

2.2.2. Sensitivity Equation Approach

Taking the partial derivatives with respect to the transmissivity increment, T_i, for the discretized form of the flow equation, yields (Distefano and Rath, 1975; Li et al., 1985)

$$\underline{\underline{B}}\,\underline{h}_{T_i} + \underline{\underline{C}}\,\frac{d\underline{h}_{T_i}}{dt} + \underline{\underline{B}}_{T_i}\underline{h} = 0 \tag{12}$$

bold, see original manuscpt.

where

$$\underline{h}_{T_i} = \frac{\partial \underline{h}}{\partial T_i} \tag{13}$$

ok

$$B_{pq} = \iint_\Omega T\left(\frac{\partial \phi_p}{\partial x}\frac{\partial \phi_q}{\partial x} + \frac{\partial \phi_p}{\partial y}\frac{\partial \phi_q}{\partial y}\right) dx\,dy \tag{14}$$

$$C_{pq} = \iint_\Omega S\phi_p\phi_q\, dx\,dy \tag{15}$$

ok

$$B_{T_i pq} = \iint_\Omega \frac{\partial T}{\partial T_i}\left(\frac{\partial \phi_p}{\partial x}\frac{\partial \phi_q}{\partial x} + \frac{\partial \phi_p}{\partial y}\frac{\partial \phi_q}{\partial y}\right) dx\,dy \tag{16}$$

and ϕ_n are the basis functions of the element. ~~comprising transmissivity, T, and storativity, S.~~ In this, **B** is a "conductance" matrix, **C** is a "storage" matrix, and $\mathbf{B}_{T_i}$ defines rate-of-change of the transmissivity values applied to the system. For the triangular element defined by corner nodes ijk, as illustrated in Figure 2, the basis functions ϕ_i, ϕ_j, and ϕ_k are defined by

$$\phi_n = \frac{1}{2\Delta_e}(\bar{a}_n + \bar{b}_n x + \bar{c}_n y) \quad (n = i, j, k) \tag{17}$$

where

$$\begin{aligned} \bar{a}_i &= x_j y_k - x_k y_i & \bar{b}_i &= y_j - y_k & \bar{c}_i &= x_k - x_j \\ \bar{a}_j &= x_k y_i - x_i y_k & \bar{b}_j &= y_k - y_i & \bar{c}_j &= x_i - x_k \\ \bar{a}_k &= x_i y_j - x_j y_i & \bar{b}_k &= y_i - y_j & \bar{c}_k &= x_j - x_i \end{aligned} \tag{18}$$

and Δ_e is the area of the triangular element with vertices at coordinates (x_i, y_i), (x_j, y_j), and (x_k, y_k). This defines the governing equation which must be evaluated using a total of $m + 1$ simulation runs.

 see copy pg. 8

2.2.3. Variational Approach

For the variational approach the sensitivity coefficients can be written as (Jacquard and Jain, 1965; Carter et al., 1974, 1982; Sun and Yeh, 1985)

$$\frac{\partial h_l}{\partial T_i} = \frac{1}{12}\sum_{m=1}^{k}\sum_{e_i}\frac{1}{\Delta_e}$$
$$[(b_i h_i + b_j h_j + b_k h_k)(b_i\bar{q}_i + b_j\bar{q}_j + b_k\bar{q}_k) +$$
$$(c_i h_i + c_j h_j + c_k h_k)(c_i\bar{q}_i + c_j\bar{q}_j + c_k\bar{q}_k)]\big|_{m\Delta\tau} \tag{19}$$

with

$$\bar{q}_s\big|_{m\Delta\tau} = q_s[(K - m + 1)\,\Delta\tau] - q_s[(K - m)\,\Delta\tau]$$
$$(s = i, j, k) \tag{20}$$

where h is the solution of the flow equations. Σ_{e_i} represents the sum of all elements that have a common node, i; $\Delta\tau$ is the time-step size; K is the number of time steps; and q is the solution of the adjoint problem

$$\frac{\partial}{\partial x}\left(T\frac{\partial q}{\partial x}\right) + \frac{\partial}{\partial y}\left(T\frac{\partial q}{\partial y}\right) = S\frac{\partial q}{\partial t} +$$
$$G_l(x, y)\,H(t) \tag{21}$$

controlled by initial and boundary conditions

$$\begin{aligned} q(x, y, 0) &= 0 && \text{on } (x, y) \in \Omega \\ q(x, y, t) &= 0 && \text{on } (x, y) \in \Gamma_1 \\ \frac{\partial q}{\partial n}(x, y, t) &= 0 && \text{on } (x, y) \in \Gamma_2 \end{aligned}$$

where

$$G_l(x, y) = \begin{cases} 1/p_l & (x, y) \in \Omega_l \\ 0 & \text{otherwise} \end{cases}$$

and

$$H(t) = \begin{cases} 0 & \text{for } t \le 0 \\ 1 & \text{for } t > 0 \end{cases}$$

where p_l is the area of the subdomain, Ω_l, shown in Figure 2. In this, Ω represents the flow region where head and flux boundary conditions are applied on the boundaries Γ_1 and Γ_2, respectively. In this approach, $L' + 1$ simulation runs are needed for a total of L' observation wells.

3. Evaluation of System Sensitivity

The influence of a variety of factors on the accuracy of the resulting parameter determination is examined. Three methods of calculating the sensitivity coefficient matrix are evaluated, together with the influence of data noise and time-series truncation on the resulting transmissivity estimates.

3.1. Influence of Sensitivity Coefficients

The influence of sensitivity coefficients, evaluated using each of the three approaches described previously, is examined with respect to convergence rate and accuracy of the resulting parameter evaluation. A modified Gauss-Newton algorithm is developed to evaluate these effects for a number of synthetic data sets representing transient flow in a heterogeneous, isotropic, confined aquifer. The aquifer configuration and the finite-element discretization of 59 nodes and 92 elements are illustrated in Figure 3. A constant head of magnitude 100 m is applied to the lower boundary with the remaining portion of the perimeter defined as zero flux, as shown in the figure. Initial conditions are of a uniform head of 100 m. In the generation of the synthetic data set, the storage coefficient, S, applied uniformly over the aquifer, is assumed known and a single well, pumping at constant rate is applied, as shown (Figure 3). The remaining parameters to be identified are the components of the transmissivity vector, T.

In the first model, the flow region is divided into subregions, 1 through 5, of discrete transmissivities of $T_1 = 100$ m^2/day, $T_2 = 500$ m^2/day, $T_3 = 5000$ m^2/day, $T_4 = 20000$ m^2/day, and $T_5 = 1000$ m^2/day. This distribution is considered the "true" values of the parameters to be identified since the head response is determined by running a "forward" analysis of a known system. Observation wells are located in each of the subregions, as illustrated in Figure 4, and the time history of drawdown determined. A total of 20 observations are recorded using a uniform time increment of one day with a prescribed pumping rate of 30000 m^3/day. The basic data related to transmissivity identification are listed in Table 1. The results reported in Table 2 are identified by the modified Gauss-Newton method in which the sensitivity coefficient matrix is evaluated from the *sensitivity equation*, *variational*, and *influence coefficient* approaches. The extreme sensitivity of the influence coefficient approach results to the selected magnitude of the influence parameter, α, in equation (11) is shown in Table 3 defining the broad variability in convergence. The iterative procedures are stable when the sensitivity coefficient matrices are calculated by either the sensitivity equation or the variational approach. This desirable stability is not, however, the norm when the sensitivity matrix is evaluated using the influence coefficient approach, as apparent in Table 3 where large residual errors remain in the transmissivity estimates. This is the case, even though the coefficient α, in equation (11), is selectively and optimally determined. This apparent lack of accuracy is in agreement with the previous observation that an appropriate increment of the identified parameter is difficult to determine (Li et al., 1985; Willis and Yeh, 1987).

3.1.1. Additional Influencing Factors

To enable the influence of auxiliary factors on the accuracy and reliability of the resulting parameter estimation to be deconvolved from other effects, the subdivision of the problem is again reduced. In this second model, the flow region is divided into two subregions, each penetrated by a single observation well, as illustrated in Figure 5. In the following cases, the Gauss-Newton direction is calculated by the sensitivity equation approach.

3.1.2. Case 1

The true values of the transmissivities, $T_1 = 500$ m^2/day and $T_2 = 1000$ m^2/day, are used to generate the "synthetic" time history of observation heads as described previously. Similar boundary conditions and model constraints are used, and the predicted transmissivities are defined for different choices of objective functions, E, in Table 4.

Retaining the boundary conditions and all original data fixed, and changing only the storage coefficient, a new set of head "observations" is generated. This time series is used to invert for the resulting transmissivity distribution. Where storativity is reduced by one order of magnitude to $S = 0.0001$, "true" transmissivity values are again recovered within five iterations of the inverse solution, as documented in Table 5. Apparent from this behavior is that the method successfully evaluates the parameter magnitudes, even when the number of independent observations in the time series is low (in this case only six values at each of two observation wells). Despite this restriction, the resulting predictions remain accurate.

3.1.3. Case 2

The influence of the choice of an initial transmissivity vector on the resulting prediction is also examined. Using the synthetic data applied in Case 1, the results of start-up using a variety of initial transmissivity vectors are given in Table 6. The final parameter estimates are relatively insensitive to the starting values although the rate of convergence is slightly affected. Similarly, changing the solution constraints has little influence on the resulting transmissivity evaluations.

3.1.4. Case 3

The effect of noise in the time history of head on the resulting evaluation of transmissivity is examined. Noise is defined as a random or systematic modification of the "real" or "ideal" head response, recovered from the forward analysis. The noise is normally distributed about the mean head and is defined by a standard deviation of noise to drawdown of, $\sigma = 0.2$. This noise is added to the time-series data of Case 1, representing two-zone transmissivities of $T_1 = 500$ m^2/day and $T_2 = 1000$ m^2/day. Changes to the objective function, E, and the transmissivities, T, in the iterative procedure are given in Table 7. The maximum ratio of noise to drawdown, M_R, is about 20%, and the relative error, M_E, of the transmissivities is about 7.5%, under this condition. As noise magnitude is halved, the relative error, M_E, is halved and transmissivity predictions improve, as anticipated.

3.1.5. Case 4

The effect of systematic, rather than random, noise on the identified transmissivity results is also examined. Typical sources of noise may result from inappropriate numerical discretization or through improper representation of the boundary conditions or transmissivity and storage properties. Adding either −0.1 m or −0.9 m noise to each of the time-series data points generates two modified data sets. The transmissivities identified using these data are identified in Table 8. The maximum ratios of noise to drawdown, M_R, are 2.7% and 24.6%, and the maximum relative errors, M_E, are about 5.8% and 60%, respectively. This illustrates the significant sensitivity of the method to the presence of systematic noise, and highlights the practical problems encountered during routine application of these methods to real data.

3.1.6. Case 5

The influece on the resulting parameter evaluations of restricting or truncating the time-series data set is examined by using only the first four data points of the six point data set originally used in Case 1. Even with the truncated time-series record, the "true" magnitudes of the parameters are essentially returned ($T_1 = 499.97$ and $T_2 = 1000.002$ m^2/day). Where random noise of maximum ratio to drawdown of about 20% is added, the resulting parameter estimations are 302 m^2/day for zone 1, and 1031 m^2/day for zone 2. Comparing the results with those of the noise-free system, there is little difference between predictions for the complete (12 observations) and truncated (8 observations) time series. This is not the case where noise is present, and the accuracy of the evaluation is considerably degraded for the truncated time series.

4. Conclusions

The modified Gauss-Newton method is shown to be attractive for aquifer parameter identification due to its simplicity, modest computational demands, and high rate of convergence. Fewer than five iterations were required in all the examples reported in this study. The observed rapid convergence is especially true when the number of the parameters to be identified is not large. In order to guarantee the accuracy of parameter identification by the modified Gauss-Newton method, a key step is selection of the appropriate sensitivity coefficient calculation approach. The sensitivity equation and variational methods are identified as the preferred methods for calculating the sensitivity coefficients. The influence coefficient approach is difficult to apply in practice since appropriate increments of the unknown parameters must be determined from a trial-and-error algorithm. Accurate evaluation of the sensitivity coefficients requires an accurate calculation of heads. Consequently, all factors influencing the evaluation of heads will have an effect on the sensitivity coefficients.

Two factors affecting the accuracy of parameter identification are the magnitude and "type" of noise, and length of the time-series data record. It is apparent from this study that the estimated parameter values are not strongly dependent on the choice of the initial values nor on the parameter constraints. The accuracies of the identified parameters are only slightly related to the length of the data record, in the absence of noise, but are strongly corrupted where significant noise exists.

The relative influence of noise may be indexed by the ratio of noise to the change in drawdown with time since this defines the data content of the signal. Where noise within the signal is systematic, rather than random, a much greater corruption of the resulting parameters ensues. One possibility in reducing this influence of random noise is by prefiltering the head data to remove high frequency components of the drawdown record. If the source of noise is systematic, then filtering may remove valuable information describing the aquifer. Care must be taken in selection of the filter thresholds to ensure that noise, alone, is removed, rather than components representative of the physical system.

The evaluation of transmissivity magnitude is more sensitive to the choice of method than the number of zones defined by the available well data. The influence coefficient approach performs least well, and the sensitivity equation and variational approaches perform much better and at about the same level of accuracy. For the variational and sensitivity equation approaches, there is no marked change in the ability to match the "true" transmissivity values with change in the number of observation wells and corresponding transmissivity zones.

The study details an evaluation of the effect of noise in recorded field data on the evaluation of transmissivity magnitudes recovered from inverse analysis. Although the methods presented appear capable of resolving transmissivity distributions, the conditions of the aquifer used in this study are quite well-defined. Flow is two-dimensional, confined, and "averaged" over the aquifer thickness. No local three-dimensional effects are included, and monitoring locations described in this analysis provide no evaluation of this possible effect. In addition, the external boundary conditions are assumed well-defined, with these data typically being difficult to determine in reality.

5. Acknowledgments

This manuscript benefitted from the thorough and critical review of three anonymous reviewers. The support of the National Science Foundation under grant MSS-9209059 and the National Mine Land Reclamation Center under grant CO388026 is gratefully acknowledged.

6. References

Bard, Y. 1974. Nonlinear Parameter Estimation. Academic Press, New York.

Becker, L. and W. W-G. Yeh. 1972. Identification of parameters in unsteady open channel flow. Water Resources Research. v. 8, no. 4, pp. 956-965.

Carter, R. D., L. F. Kemp, A. C. Pierce, and D. L. Williams. 1974. Performance matching with constraints. Soc. Pet. Eng. J. v. 14, no. 2, pp. 187-196.

Carter, R. D., L. F. Kemp, and A. C. Pierce. 1982. Discussion of comparison of sensitivity coefficient calculation methods in automatic history matching. Soc. Pet. Eng. J. v. 22, no. 2, pp. 205-208.

Distefano, N. and A. Rath. 1975. An identification approach to subsurface hydrological systems. Water Resources Research. v. 11, no. 6, pp. 1005-1012.

Gavalas, G. R., P. C. Shah, and J. H. Seinfeld. 1976. Reservoir history matching by Bayesian estimation. Soc. Pet. Eng. J. v. 16, no. 6, pp. 337-350.

Jacquard, P. and C. Jain. 1965. Permeability distribution from field pressure data. Soc. Petrol. Engr. J. v. 5, no. 6, pp. 281-294.

Li, J. S., A. Lu, N. Z. Sun, and W. W-G. Yeh. 1985. A comparative study of sensitivity coefficient calculation methods in groundwater flow. Working Paper, Dept. of Civil Engineering, Univ. of California, Los Angeles.

Neuman, S. P. 1973. Calibration of distributed parameter groundwater flow models viewed as a multiple-objective decision process under uncertainty. Water Resources Research. v. 9, no. 4, pp. 1006-1021.

Rosen, J. B. 1960. The gradient projection method for nonlinear programming, part I. J. Soc. Ind. Appl. Math. v. 8, pp. 181-184.

Shah, P. C., G. R. G~~e~~avalas, and J. H. Seinfeld. 1978. Error analysis in history matching: The optimum level of parameterization. Soc. Pet. Eng. J. v. 18, pp. 219-228.

Sun, N. Z. and W. W-G. Yeh. 1985. Identification of parameter structure in groundwater inverse problem. Water Resources Research. v. 21, no. 6, pp. 869-883.

Thomas, L. K., L. J. Hellums, and G. M. Reheis. 1972. A nonlines automatic history matching technique for reservoir simulation models. Soc. Pet. Eng. J. v. 12, no. 6, pp. 508-514.

Willis, R. and W. W-G. Yeh. 1987. Groundwater Systems Planning & Management. Prentice-Hall, Inc.

Yeh, W. W-G. 1986. Review of parameter identification procedures in groundwater hydrology: The inverse problem. Water Resources Research. v. 22, no. 1, pp. 95-108.

Yeh, W. W-G. and Y. S. Yoon. 1981. Parameter identification with optimum dimension in parameterization. Water Resources Research. v. 17, no. 4, pp. 664-672.

Table 1. Data Used in Transmissivity Evaluation for the Five Zone Geometry of Figure 4

Transmissivity zone	*Transmissivity magnitudes*			
	True value	*Initial value*	*Lower bound*	*Upper bound*
1	100	50	40	150
2	500	250	200	700
3	5000	7500	3000	8000
4	20000	25000	1700	30000
5	1000	1400	750	1700

(All values in m^2/day.)

Table 2. Parameter Estimation Results for the Fifth Iteration Using Three Solution Approaches

Solution method	*Evaluated transmissivity magnitudes by zone*				
	1	*2*	*3*	*4*	*5*
True magnitudes	100.	500.	5000.	20000.	1000.
Sensitivity equation	99.989	500.000	5000.686	19998.170	1000.001
Variational	99.989	500.000	5000.686	19998.170	1000.001
Influence coefficient	67.373	392.541	5332.794	17000.000	986.248

(All values in m^2/day. Influence coefficient method uses $\alpha = 0.001$.)

Table 3. Parameter Estimation Results from the Influence Coefficient Approach Using $0.0001 \leq \alpha \leq 0.001$

Sensitivity coefficient, α	*Evaluated transmissivity magnitudes by zone*				
	1	*2*	*3*	*4*	*5*
True values	100.	500.	5000.	20000.	1000.
0.0001	60.223	700.000	3000.000	17000.000	750.000
0.001	67.373	392.541	5332.794	17000.000	986.248
0.0015	55.382	272.641	4763.701	30000.000	750.000
0.002	40.036	699.056	3000.000	19485.220	992.673
0.005	40.000	200.000	8000.000	17000.000	750.000

(All values in m^2/day and determined after five iterations.)

Table 4. Changes in the Objective Function, E, and Transmissivity Values, T, with Iteration for the Two Zone Geometry

		Evaluated transmissivity	
Iteration no.	*Objective function, E*	*Zone 1*	*Zone 2*
True values	—	500.	1000.
1	24.31203	384.575	1002.445
2	0.19627	475.418	1004.877
3	0.00529	499.395	1000.087
4	0.00000	499.997	999.998
5	0.00000	500.001	999.999

(Storativity magnitude of S = 0.001. Transmissivity values in m^2/day.)

Table 5. Changes in the Objective Function, E, and Transmissivity Values, T, with Iteration for the Two Zone Geometry

		Evaluated transmissivity	
Iteration no.	*Objective function, E*	*Zone 1*	*Zone 2*
True values	—	500.	1000.
1	22.01094	394.431	1020.620
2	0.02263	493.927	1000.276
3	0.00062	499.944	1000.003
4	0.00000	499.936	1000.000
5	0.00000	499.951	1000.002

(Storativity magnitude of S = 0.0001. Transmissivity values in m^2/day and determined after five iterations.)

Table 7. Changes in the Objective Function, E, and Transmissivity Values, T, with Iteration for the Two Zone Geometry Where Random Noise Is Applied to the Data

		Evaluated transmissivity	
Iteration no.	*Objective function, E*	*Zone 1*	*Zone 2*
True values	—	500.	1000.
1	28.28925	368.093	1004.230
2	4.14652	443.004	1007.997
3	3.99459	460.610	1004.261
4	3.99018	460.355	1004.302
5	3.99021	461.792	1004.072
6	3.99010	462.486	1003.979

(Storativity magnitude of S = 0.001. Transmissivity values in m^2/day.)

Table 6. The Influence of Starting Transmissivity Magnitude on the Resulting Parameter Estimate, Two Zones

Initial values		*Final values*	
Zone 1	*Zone 2*	*Zone 1*	*Zone 2*
True values		500.	1000.
250	1300	500.001	999.999
750	700	499.684	1000.002
300	800	500.012	999.999
780	1450	499.989	1000.000

(Transmissivity values in m^2/day.)

Table 8. The Influence of Starting Transmissivity Magnitude on the Resulting Parameter Estimate Where Systematic Noise Is Incorporated into the Data Record; Systematic Noise Is Defined as a Percentage of Head; Two Zones

	Initial values		*Final values*	
Systematic noise	*Zone 1*	*Zone 2*	*Zone 1*	*Zone 2*
—	True values		500.	1000.
-0.1	250	1300	528.927	978.290
-0.9	250	1300	800.000	819.002

(Transmissivity values in m^2/day.)

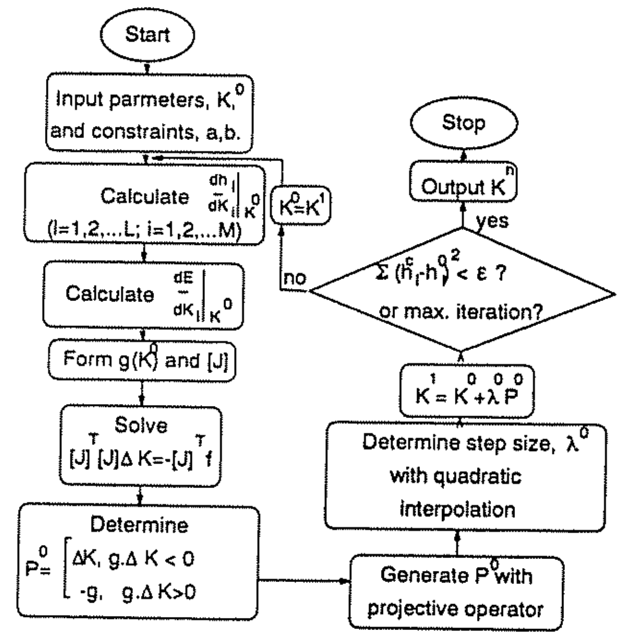

Fig. 1. Flow chart illustrating optimization procedure.

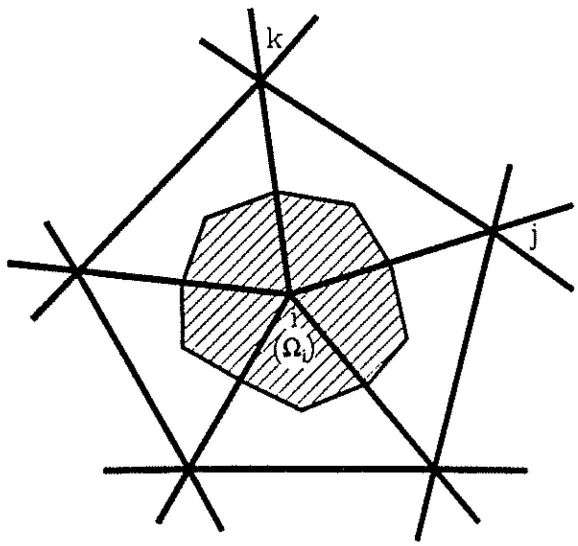

Fig. 2. The triangular element ijk and the exclusive subdomain of node i.

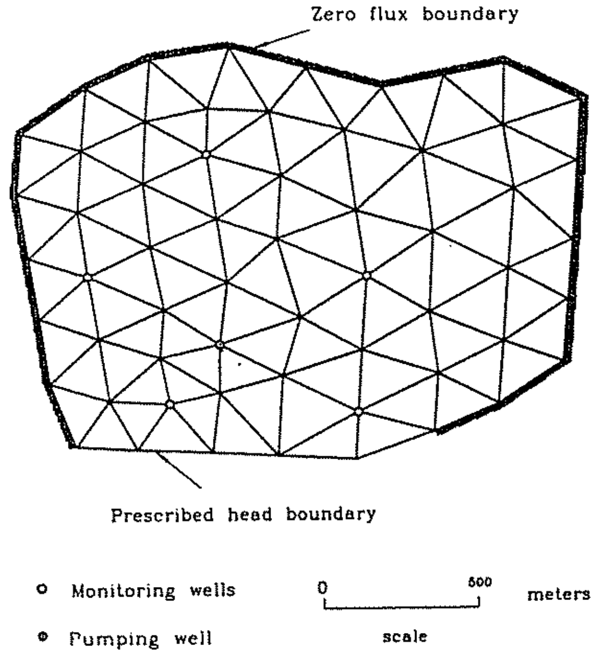

Fig. 3. Finite-element mesh used in the analysis.

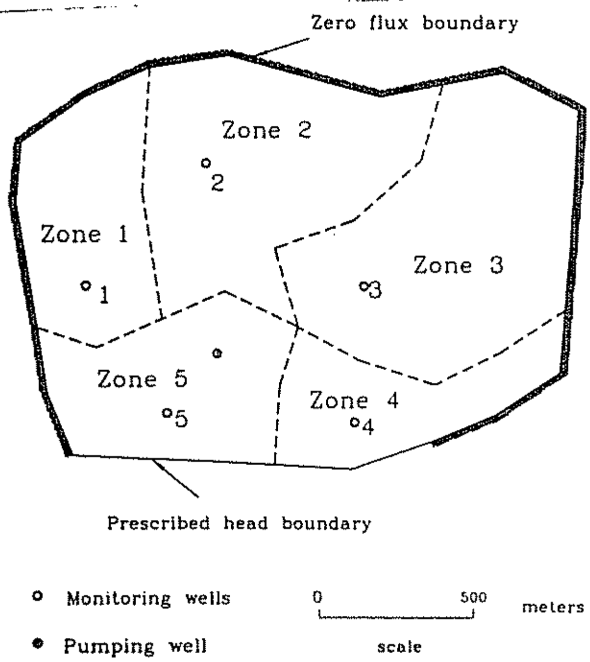

Fig. 4. Zonation used in the determination of transmissivity magnitudes. Five zones.

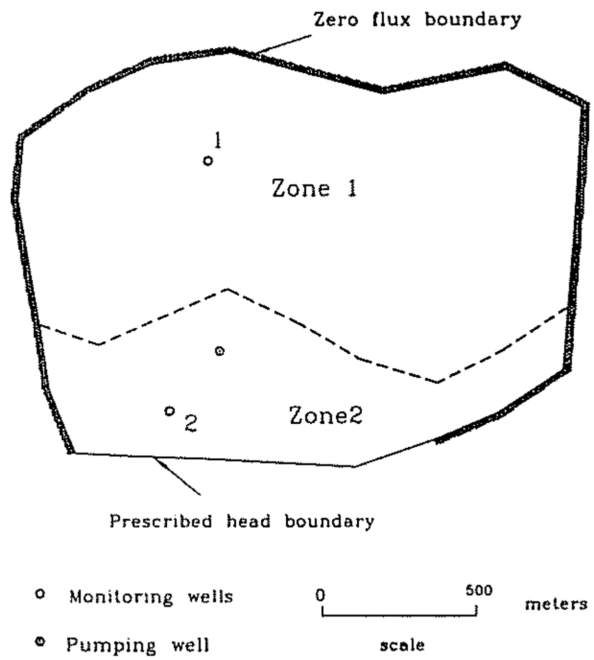

Fig. 5. Zonation used in the determination of transmissivity magnitudes. Two zones.

Li, Elsworth