



Evolution of coal permeability from stress-controlled to displacement-controlled swelling conditions

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ARTICLE INFO

Article history:

Received 11 February 2010

Received in revised form 25 May 2010

Accepted 21 April 2011

Available online 18 May 2011

Keywords:

Coal permeability model

Coal–gas interactions

Stress control

Displacement control

Coal swelling

ABSTRACT

When a coal sample is constrained either by displacements or by a confining stress, additional force and resulting stress develop within the coal. A simple “free expansion + push back” approach is developed in this work to determine the magnitude of this stress and its effect on permeability evolution. In this approach, the coal is allowed to expand freely due to gas sorption, and then it is pushed back by the applied effective stress to the original constrained conditions. The total “push-back” strains are used to calculate the change in coal permeability. This free expansion plus push back approach is applied to examine the variety of permeability responses observed in the laboratory and the veracity of their representation by theoretical models linking this behavior to gas sorption-induced swelling/shrinkage. These cases include (1) coal swelling tests under the uniaxial strain condition; (2) coal swelling tests under the displacement controlled condition; (3) coal swelling tests under the stress controlled condition. These responses are verified against other coal permeability models available in the literature and against experimental data and field data where few analytical solutions are currently available. In particular, this approach has led to a new coal permeability model that can be used to explain stress-controlled experimental observations. Stress-controlled swelling tests are normally conducted in the laboratory to characterize the evolution of coal permeability under the influence of gas sorption. Typically reductions in permeability are observed from gas-sorption-induced swelling even where effective stresses remain constant. This behavior remains enigmatic as the permeability of the porous coal is determined by the effective stress only. Our model is capable of replicating this apparently anomalous behavior.

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1. Introduction

Coal swells with gas adsorption and shrinks with desorption, which changes the coal cleat apertures and thus the permeability under reservoir conditions. Therefore, understanding how gas sorption-induced changes in effective stresses affect the permeability of coal is crucially important not only to operations involving the production of natural gas from coalbeds, but also to the design and operation of projects to sequester greenhouse gases in coalbeds [1].

The potential impacts of coal swelling on the evolution of coal permeability have been investigated through experimental and analytical studies. Measurements of the effects of coal shrinkage have been completed for the injection of different gases and the implications for the change in cleat permeability have been evaluated using model representing a matchstick geometry [2]. Similarly, volumetric changes within the coal matrix due to gas desorption

have been evaluated for a variety of gases [3]. These studies identify the change in coal permeability and volumetric strain rate as a result of gas pressure and suggest that sorption-induced deformations dominate over effective-stress-generated deformations at low gas pressures for both carbon dioxide and methane. Similar results are available for measurements on coal samples in a triaxial stress permeameter [4].

Changes in permeability of coal cores confined under isotropic stresses show that desorption of an adsorbing gas, e.g. methane, is accompanied by matrix shrinkage [5,6] and may result in a net permeability increase. More recently, measurements of changes in permeability due to the sorption of carbon dioxide within an induced longitudinal fracture in coal have explored the impact of confining stress [7]. To avoid possible permeability change due to gas adsorption-induced coal swelling, permeabilities measured under constant gas pressure but with variable confining pressure have allowed comparison with Palmer and Mansoori and Shi and Durucan permeability models [8]. Similar work shows an increase in permeability with decreasing effective stress on the sample when a non-sorbing gas is used but a reduction in permeability

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with increasing pore pressure when an adsorbing gas is used [9]. This observed switch in behavior is presumably due to the influence of the swelling on cleat deformation.

Other laboratory experiments have measured the change of coal permeability as a function of pore pressure and injected-gas composition at constant effective stress [8,10]. The effect of CO₂ injection on the permeability of coal samples has been investigated with a high-pressure core flooding apparatus by imposing a constant effective stress on the sample, and injection of CO₂ resulted in an observed increase in permeability even with adsorbable gases including CO₂, CH₄ and N₂ [11,12]. Chemical and thermodynamic aspects of coal swelling due to solvents have been reported in the literature [13–15] together with the potential influence of CO₂ injection to enhance coalbed methane recovery. These studies indicate that coal undergoes simultaneous swelling and shrinkage when carbon dioxide is injected into a coal seam to displace and recover the methane.

Based on experimental observations, a variety of models have been formulated to quantify the evolution of permeability during coal swelling/shrinkage. Gray [16] firstly attempted to quantify the role of stresses on the evolution of coal-reservoir permeability, in which permeability was computed as a function of reservoir pressure-induced coal-matrix shrinkage assumed directly proportional to changes in the equivalent sorption pressure. Since then, a number of theoretical and empirical permeability models have been proposed [2,17–21]. However, most of these studies are under the assumption of either an invariant total stress or uniaxial strain conditions. These critical and limiting assumptions have been relaxed in new models rigorously incorporating in situ stress conditions [22–24] and are extended to rigorously incorporate CO₂–CH₄ coal–gas interaction relevant to CO₂–ECBM [25,26].

When experimental results from these tests were interpreted, a matchstick or cubic coal model was assumed. Under this assumption, matrix swelling would not affect coal permeability because of the complete separation between matrix blocks caused by through-going fractures. In this case, for a given fracture pore pressure, the swelling will result in an increase of fracture spacing, rather than a change in fracture aperture [29]. However, this has not been consistent with laboratory observations that show significant effects of matrix swelling on coal permeability under constant confining stress conditions [5,6,9]. In order to explain these effects, a number of researchers simply applied the uniaxial strain model to match the match the experimental data with little success [27,28]. It is generally believed that the reason for these failures is the inconsistency between the experimental conditions and the model assumptions [29], and developed a new permeability model based on the internal swelling stress concept. However, we believe the reason for these failures may be the internal actions between coal fractures and matrixes have not been taken into consideration.

In this study, a more general approach is developed to characterize the evolution of coal permeability under a full spectrum of mechanical conditions from stress-controlled to displacement-controlled swelling/shrinkage conditions. When a coal sample is constrained either by displacements or by a confining stress, additional force and resulting stresses develop within the coal. A simple “free expansion plus push back” approach is developed to determine the amount of stress. In this approach, the coal is allowed to expand freely due to the gas sorption, and then it is pushed back by the applied effective stress to the original constrained conditions. The total “push-back” strains are used to calculate the coal permeability. This free expansion plus push back approach is applied to a series of cases commonly used in the laboratory tests and theoretical analysis to generate typical response curves of coal permeability to gas sorption-induced swelling/shrinkage. After this, it is assumed that both fracture and matrix systems contribute

to the coal resultant permeability, and then a new permeability model is formulated through elastic modulus reduction ratio, R_m , to evaluate the contribution from each system. The validity of the proposed permeability model is evaluated against three sets of lab experimental data, which represents another new and important contribution to this subject.

2. Free expansion + pushback approach

The following two assumptions are considered in this study: (1) the coal is a homogeneous, isotropic and elastic continuum, and the system is isothermal; (2) strains are infinitesimal.

The gas sorption-induced strain ε_s is assumed to result in only normal strains and these resulting strains are isotropic. The effects of gas sorption on the deformation of coal seams can be treated analogous to the effects of temperature for elastic porous media (e.g. [17]), stress–strain relationships for an isothermal gas adsorbing coalbed may be written as [22]

$$\varepsilon_{ij} = \frac{1}{2G} \sigma_{ij} - \left(\frac{1}{6G} - \frac{1}{9K} \right) \sigma_{kk} \delta_{ij} + \frac{\alpha}{3K} p \delta_{ij} + \frac{\varepsilon_s}{3} \delta_{ij} \quad (1)$$

where $G = \frac{E}{2(1+\nu)}$, $K = \frac{E}{3(1-2\nu)}$, $\alpha = 1 - \frac{K}{K_s}$, $\sigma_{kk} = \sigma_{11} + \sigma_{22} + \sigma_{33}$.

E is the equivalent Young's modulus of the coal-fracture assemblage; K represents the bulk modulus coal-fracture assemblage, and K_s represents the bulk modulus of coal matrixes. G is the shear modulus of coal, ε_s is the sorption-induced strain, and ν is the Poisson's ratio of the coal-fracture assemblage. α represents the Biot's coefficient, p the gas pressure in the pores and δ_{ij} is the Kronecker delta; 1 for $i = j$ and 0 for $i \neq j$.

From Eq. (1), we obtain

$$\varepsilon_v = -\frac{1}{K} (\bar{\sigma} - \alpha p) + \varepsilon_s \quad (2)$$

where $\varepsilon_v = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}$ is the volumetric strain of the coal matrix and $\bar{\sigma} = -\sigma_{kk}/3$ is the mean compressive stress. The effective stress σ_{eij} is also defined as $\sigma_{eij} = \sigma_{ij} + \alpha p \delta_{ij}$.

Considering a porous medium containing solid volume V_s and pore volume V_p , the bulk volume can be defined as $V = V_p + V_s$ and the porosity can be defined as $\phi = V_p/V$. According to Eq. (2), the volumetric evolution of the porous medium loaded by $\bar{\sigma}$ and p can be described in terms of $\Delta V/V$ and $\Delta V_p/V_p$, the volumetric strain of the coal and the volumetric strain of the pore space, respectively. The relations are

$$\frac{\Delta V}{V} = -\frac{1}{K} (\Delta \bar{\sigma} - \alpha \Delta p) + \Delta \varepsilon_s \quad (3)$$

$$\frac{\Delta V_p}{V_p} = -\frac{1}{K_p} (\Delta \bar{\sigma} - \beta \Delta p) + \Delta \varepsilon_s \quad (4)$$

where $\beta = 1 - K_p/K_s$.

We assume that the sorption-induced strain for the coal is the same as for the pore space. Without the gas sorption effect, the volumetric variation of the porous medium satisfies the Betti-Maxwell reciprocal theorem, $\left. \frac{\partial V}{\partial p} \right|_{\bar{\sigma}} = \left. \frac{\partial V_p}{\partial \bar{\sigma}} \right|_p$, [30] and we obtain

$$K_p = \frac{\phi}{\alpha} K \quad (5)$$

where K_p is the bulk modulus for pore system.

Using the definition of porosity, the following expressions can be deduced as

$$\frac{\Delta V}{V} = \frac{\Delta V_s}{V_s} + \frac{\Delta \phi}{1 - \phi} \quad (6)$$

$$\frac{\Delta V_p}{V_p} = \frac{\Delta V_s}{V_s} + \frac{\Delta \phi}{\phi(1 - \phi)} \quad (7)$$

Solving Eqs. (3)–(7), we obtain the relationship as

$$\Delta\phi = \phi \left(\frac{1}{K} - \frac{1}{K_p} \right) (\Delta\bar{\sigma} - \Delta p) \quad (8)$$

Substituting Eq. (5) into the above equation yields

$$\phi - \phi_0 = \phi \left(1 - \frac{\alpha}{\phi} \right) \frac{\Delta\bar{\sigma} - \Delta p}{K} \quad (9)$$

Rearranging Eq. (9) gives

$$\phi = \frac{\phi_0}{\left(1 - \frac{\Delta\bar{\sigma} - \Delta p}{K} \right)} - \frac{\alpha}{\left(1 - \frac{\Delta\bar{\sigma} - \Delta p}{K} \right)} \frac{\Delta\bar{\sigma} - \Delta p}{K} \quad (10)$$

Because generally $(\Delta\bar{\sigma} - \Delta p)/K \ll 1$, the above equation can be simplified into

$$\frac{\phi}{\phi_0} = 1 - \frac{\alpha}{\phi_0} \frac{\Delta\bar{\sigma} - \Delta p}{K} = 1 + \frac{\alpha}{\phi_0} \Delta\epsilon_{et} \quad (11)$$

where $\Delta\epsilon_{et} = -(\Delta\bar{\sigma} - \Delta p)/K$ is defined as the total effective volumetric strain (negative sign represents compressive strain).

Using the cubic relation between permeability and porosity [2] yields

$$\frac{k}{k_0} = \left(\frac{\phi}{\phi_0} \right)^3 = \left(1 + \frac{\alpha}{\phi_0} \Delta\epsilon_{et} \right)^3 \quad (12)$$

Eqs. (11) and (12) are coal porosity model and permeability model that are derived based on the fundamental principles of poroelasticity. They can be applied to the evolution of coal porosity and permeability under different boundary conditions.

Eq. (2) can be written as

$$\Delta\epsilon_v = \Delta\epsilon_{et} - \frac{p}{K_s} + \Delta\epsilon_s \quad (13)$$

If we assume that $K_s \gg K$ or $\alpha = 1$, then the above equation can be simplified into

$$\Delta\epsilon_v = \Delta\epsilon_{et} + \Delta\epsilon_s \quad (14)$$

Comparing Eqs. (12) with (14) suggests that only $\Delta\epsilon_{et}$ is responsible for the change in permeability. To further illustrate the concept, it is assumed that the coal sample consists of four balls as illustrated in Fig. 1. For free expansion, the four balls expand freely in all directions. During this expansion the coal permeability remains unchanged because the coal permeability is independent of the ball radius ($\phi = 1 - \pi/6$), as shown in Fig. 1b. For the push-back process, four balls deform because of stress concentrations as shown in Fig. 1c. Therefore, both coal porosity and permeability changes as a function of the push-back force. Based on the constraint conditions, the directional strains in the x - and y -directions are equal to zero, i.e.,

$$\Delta\epsilon_{ex} + \frac{1}{3}\Delta\epsilon_s = \Delta\epsilon_{ey} + \frac{1}{3}\Delta\epsilon_s = 0 \quad (15)$$

Applying Hooke's law yields

$$\Delta\epsilon_{ex} = \Delta\epsilon_{ey} = \frac{1}{E}(1 - 2\mu)\Delta\sigma_{ex} = -\frac{1}{3}\Delta\epsilon_s \quad (16)$$

Then the effective stress can be expressed as

$$\Delta\sigma_{ex} = -\frac{3E}{(1 - 2\mu)}\Delta\epsilon_s \quad (17)$$

This example illustrates a simple and straightforward way to determine the relation between free expansion strain and the push-back strain and how to determine the amount of stress that develops in the coal. In the following sections, this free expansion plus push-back approach is applied to a series of cases commonly used in the laboratory tests and theoretical analysis to generate typical response curves. These tests include:

Uniaxial strain tests: Uniaxial strain is a strain state for which only one component of principal strain is not zero and is commonly believed to be the conditions applicable to gas reservoirs.

Constant volume tests: Constant volume tests (or displacement-controlled tests) are to constrain the boundary deformation to be zero to keep the total volume constant.

Stress-controlled tests: Stress-controlled tests are widely used for triaxial or hydrostatic tests. The change in effective stress is directly related to the pore pressure change or confining stress variation. It is perhaps the most common way to evaluate fluid flow and core geomechanical properties in laboratory.

3. CST 1 – uniaxial strain tests

When a coal sample is partially constrained by displacements and partially controlled by a confining stress as shown in Fig. 2a, no additional force and no resulting stress develop in the vertical (stress controlled) direction while additional effective stresses develop in all the lateral directions. A simple way to determine the amount of stress is to let the coal expand freely due to the gas sorption as shown in Fig. 2b, then push it back by the applied effective stress in the vertical direction, and by the resulting effective stresses in all the other lateral directions, as shown in Fig. 2c. In this case, the total “push-back” volumetric strain can be calculated as follows.

The total effective strain is defined as

$$\Delta\epsilon_{et} = \Delta\epsilon_{ex} + \Delta\epsilon_{ey} + \Delta\epsilon_{ez} \quad (18)$$

where ϵ_{ei} is the effective strain in the i -direction.

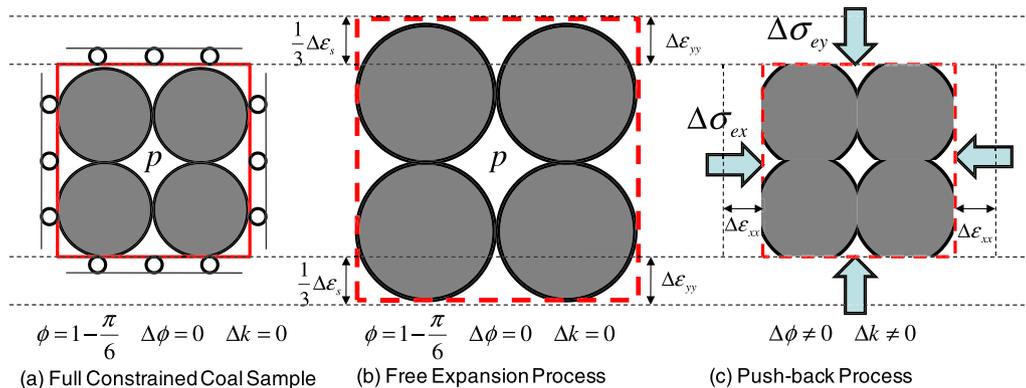


Fig. 1. Illustration of effective stress induced volume strain on permeability change.

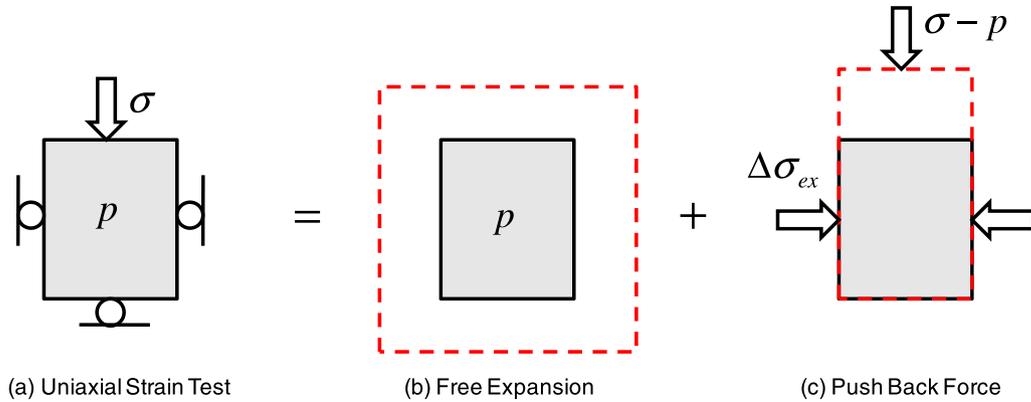


Fig. 2. Swelling tests under the uniaxial strain condition: (a) uniaxial strain condition; (b) free expansion due to gas sorption; (c) push back to original position by the resulting effective stress.

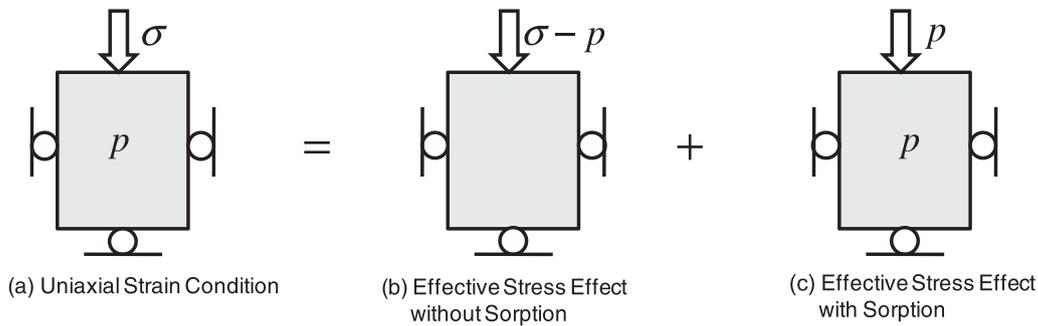


Fig. 3. Swelling tests under the uniaxial strain condition: (a) uniaxial strain condition; (b) effective stress effect without gas sorption; (c) effective stress effect with constrained gas sorption.

In this paper, we define the vertical direction as the z-coordinate with x and y representing the horizontal directions. Applying Hooke's law to Fig. 2c gives

$$\Delta\epsilon_{ex} = \Delta\epsilon_{ey} = \frac{1}{E}[(1 - \mu)\Delta\sigma_{ex} - \mu\Delta\sigma_{ez}] = -\frac{1}{3}\Delta\epsilon_s \tag{19}$$

$$\Delta\epsilon_{ez} = \frac{1}{E}[\Delta\sigma_{ez} - 2\mu\Delta\sigma_{ex}] \tag{20}$$

Solving Eqs. (19) and (20) gives

$$\Delta\epsilon_{et} = \frac{(p - p_0)}{E} \frac{(1 + \mu)(1 - 2\mu)}{1 - \mu} - \frac{2(1 - 2\mu)}{3(1 - \mu)} \Delta\epsilon_s \tag{21}$$

Substituting Eq. (21) into Eqs. (11) and (12) gives

$$\frac{\phi}{\phi_0} = 1 + \frac{\alpha}{\phi_0} \left[\frac{(p - p_0)}{E} \frac{(1 + \mu)(1 - 2\mu)}{1 - \mu} - \frac{2(1 - 2\mu)}{3(1 - \mu)} \Delta\epsilon_s \right] \tag{22}$$

$$\frac{k}{k_0} = \left[1 + \frac{\alpha}{\phi_0} \left(\frac{(p - p_0)}{E} \frac{(1 + \mu)(1 - 2\mu)}{1 - \mu} - \frac{2(1 - 2\mu)}{3(1 - \mu)} \Delta\epsilon_s \right) \right]^3 \tag{23}$$

Eqs. (22) and (23) are the porosity model and the permeability model, respectively, under conditions of uniaxial strain.

For a single gas system the sorption-induced volumetric strain ϵ_s is fit to a Langmuir type curve as verified in experiments [6,21,31]. Therefore, the sorption-induced strain change with pore pressure variation can be expressed as

$$\Delta\epsilon_s = \frac{\epsilon_L p_L (p - p_0)}{(p_L + p_0)(p_L + p)} \tag{24}$$

where the sorption-induced strain constant, ϵ_L , representing the volumetric strain at infinite pore pressure with the Langmuir pres-

sure constant, p_L , representing the pore pressure at which the measured sorption-induced strain is equal to $0.5\epsilon_L$.

Substituting Eq. (24) into (23) gives

$$\frac{k}{k_0} = \left[1 + \frac{\alpha}{\phi_0} (A + B) \right]^3 \tag{25}$$

where $A = \frac{(p - p_0)}{E} \frac{(1 + \mu)(1 - 2\mu)}{1 - \mu}$ and $B = -\frac{2(1 - 2\mu)}{3(1 - \mu)} \frac{\epsilon_L p_L (p - p_0)}{(p_L + p_0)(p_L + p)}$

It has been well accepted that A term represents the effective stress effect while B term represents the coal swelling/shrinkage effect. When p increases, A is positive while B is negative. This is why all current studies claim that A and B are competitive processes. We re-examine this behavior for a coal swelling test under conditions of uniaxial strain as represented in Fig. 3.

Applying the Hooke's law to Fig. 3b gives

$$\Delta\epsilon_{ex} = \Delta\epsilon_{ey} = \frac{1}{E}[(1 - \mu)\Delta\sigma_{ex} - \mu\Delta\sigma_{ez}] = 0 \tag{26}$$

$$\Delta\epsilon_{ez} = \frac{1}{E}(\Delta\sigma_{ez} - 2\mu\Delta\sigma_{ex}) \tag{27}$$

Solving Eqs. (26) and (27) gives

$$\Delta\epsilon_{et} = \Delta\epsilon_{ez} = \frac{(p - p_0)}{E} \frac{(1 + \mu)(1 - 2\mu)}{1 - \mu} = A \tag{28}$$

Similarly, applying the Hooke's law to Fig. 3c gives

$$\Delta\epsilon_{ex} = \Delta\epsilon_{ey} = \frac{1}{E}[(1 - \mu)\Delta\sigma_{ex} - \mu\Delta\sigma_{ez}] = -\frac{1}{3}\epsilon_s \tag{29}$$

$$\Delta\epsilon_{ez} = \frac{1}{E}(-2\mu\Delta\sigma_{ex}) \tag{30}$$

Solving Eqs. (29) and (30) gives

$$\Delta \varepsilon_{et} = -\frac{2(1-2\mu)}{3(1-\mu)} \frac{\varepsilon_L p_L (p-p_0)}{(p_L+p_0)(p_L+p)} = B \quad (31)$$

These derivations prove that both *A* and *B* terms are effective stress effects. Therefore, *A* and *B* are additive processes coordinated by the pore pressure.

Following the above ideas, two comparisons are presented here to illustrate the validity of this proposed porosity and permeability models under uniaxial conditions. The first one is to compare our model with the Advanced Resources International (ARI) permeability model for CO₂ injection into a single well in the San Juan Basin [19]. The second explores the compatibility between our new permeability model and three other widely used models, viz. the Palmer and Mansoori (PM) model, Shi and Durucan (SD) model and the Cui and Bustin (CB) model [17,20,21]. The values of parameters used for this comparison are listed in Table 1 (*g* = 0.95 for PM model). Both comparisons use the same parameters and the typical response curves are shown in Figs. 4 and 5.

Fig. 4 shows that the proposed permeability model is almost coincident with the ARI model under conditions of uniaxial strain, and the correspondence with other models can also be seen from Fig. 5. The reason for the difference among these models is due to the difference of assumptions for each model. For instance, PM model is a strain-based model developed by substituting a matrix shrinkage analog to thermal expansion, which is identical to our model by adjusting the coefficient *g* in this model [23]. Conversely the CB model considers the total mean stress as the variable that controls permeability change [21]. The SD model is derived by considering the directional (or horizontal) stress change in the constrained directions as the permeability change force [20].

4. CST 2: constant volume tests

When a coal sample is completely constrained as shown in Fig. 6a, a force and resulting stress develop within the coal. A simple way to determine the amount of stress is to let the coal expand freely due to the gas sorption as shown in Fig. 6b, then push it back to its original positions, as shown in Fig. 6c. In this case, the total strains in all directions are equal to zero, i.e.,

$$\Delta \varepsilon_{ex} + \frac{1}{3} \varepsilon_s = \Delta \varepsilon_{ey} + \frac{1}{3} \varepsilon_s = \Delta \varepsilon_{ez} + \frac{1}{3} \varepsilon_s = 0 \quad (32)$$

Solving Eq. (32) gives

$$\Delta \varepsilon_{et} = \Delta \varepsilon_{ex} + \Delta \varepsilon_{ey} + \Delta \varepsilon_{ez} = -\varepsilon_s \quad (33)$$

Substituting Eq. (33) into Eq. (12) gives

$$\frac{k}{k_0} = \left[1 + \frac{\alpha}{\phi_0} (-\varepsilon_s) \right]^3 \quad (34)$$

Typical response curves for CH₄ gas are shown in Figs. 7 and 8. Parameter values are the same as in Table 1 except for the sorption-induced strain constant and magnitudes of initial porosity.

Table 1
Parameter magnitudes used in the comparison.

Parameter	Value
Boit coefficient	1.0
Young's modulus, psi	1.74 × 10 ⁵
Poisson's ratio	0.39
<i>ε_L</i> for CH ₄	0.052
<i>ε_L</i> for CO ₂	0.084
<i>p_L</i> for CH ₄ , psi	1134
<i>p_L</i> for CO ₂ , psi	231
Porosity	1.0%

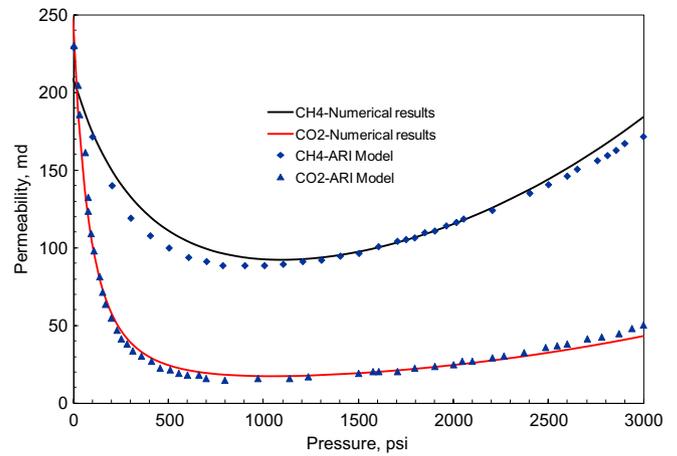


Fig. 4. Comparison with ARI model for CO₂ injection into a well in the San Juan Basin.

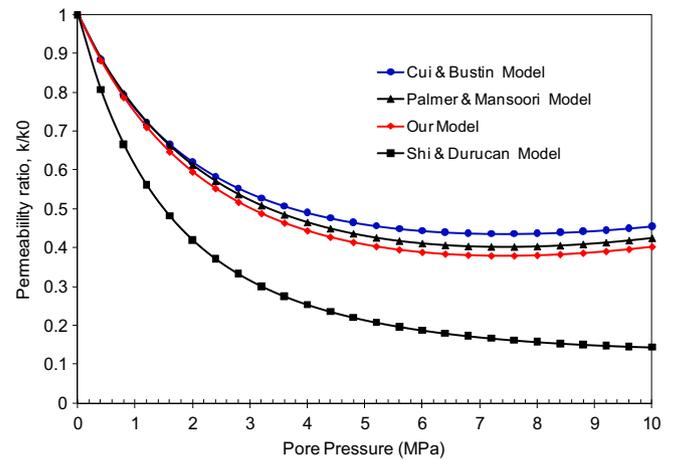


Fig. 5. Comparison between the proposed permeability model and three alternate models.

Fig. 7 shows that permeability change is very sensitive to gas sorption-induced strain constant. Gas with a larger sorption-induced strain constant, like CO₂ relative to CH₄, can induce greater coal matrix swelling, which in turn causes more fracture aperture closure and more permeability reduction. Porosity also plays a significant role on permeability change, especially within lower porosity value ranges, as shown in Fig. 8. That is because permeability change is modulated by porosity change, so within extremely low porosity reservoirs the small absolute decrease in pore volume results in a significant reduction of the cross section of the flow pathways, in turn causing a dramatic reduction in the permeability ratio.

5. CST 3: stress-controlled tests

When the response is controlled completely by stress alone, as shown in Fig. 9a, no additional force and no resulting stress develops within the coal. A simple way to determine the amount of stress is to let the coal expand freely due to the gas sorption as shown in Fig. 9b, then push it back by the applied effective stress, as shown in Fig. 9c. In this case, the total “push-back” volumetric strain is calculated as

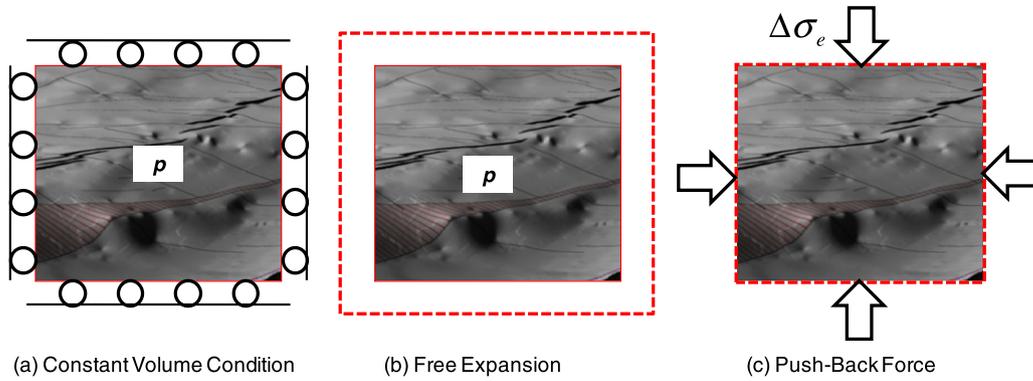


Fig. 6. Swelling tests under the constant volume condition: (a) constant volume condition; (b) free expansion due to gas sorption; (c) push back to original positions by the resulting effective stress.

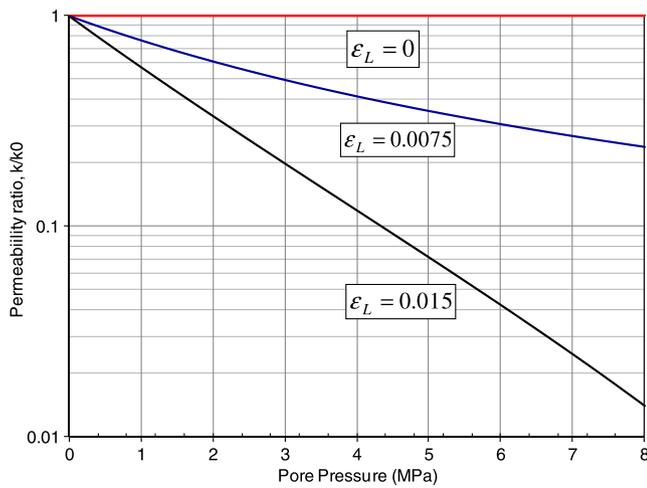


Fig. 7. Influence of swelling strain constant on permeability change under constant volume conditions.

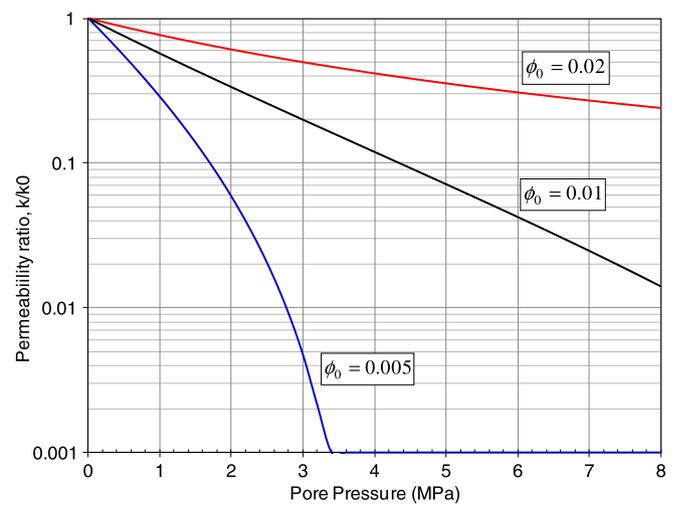


Fig. 8. Influence of porosity on permeability change under constant volume conditions.

$$\Delta \varepsilon_{et} = -\frac{\sigma - p}{K} \tag{35}$$

Substituting Eq. (35) into Eq. (12) gives

$$\frac{k}{k_0} = \left[1 + \frac{\alpha}{\phi_0} \left(-\frac{\sigma - p}{K} \right) \right]^3 \tag{36}$$

In order to investigate the influence of effective stress on permeability change under stress-controlled condition, three cases are tested as shown in Fig. 10. The first two cases are conducted with constant effective stresses (3.0 MPa and 5.0 MPa), and the third involves a comparison between effective stress induced by pore pressure change and confining stress variation. The variable effective stress can be obtained by changing either confining pressure with invariant pore pressure or the pore pressure with invariant confining stress. Parameters listed in Table 1 are adapted for this comparison.

Fig. 10 illustrates that effective stress changes induced by pore pressure or confining pressure have opposite consequence on permeability change. It also shows matrix swelling/shrinkage induced strain has no effects on coal permeability. The permeability remains constant if the effective stress does not change, and the swelling/shrinkage induced strain has no impact on permeability change. In other words, the permeability change is mechanical effective stress dependent only under the stress-controlled conditions. However, this finding is obviously not consistent with laboratory observations, which show significant effects of matrix

swelling on coal permeability under the same conditions [5–6,8,9]. A number of researchers have tried to apply the uniaxial strain models to match the experimental data recovered from stress-controlled conditions, with little success [27,28]. It is generally believed that the reason for these failures is the inconsistency between the experimental conditions and the model assumptions [29]. However, we believe the true reason for these failures may be the internal actions between coal fractures and matrixes have not been taken into consideration. In the following section, we combined the interaction between coal fractures and matrixes through an elastic reduction ratio to qualify the contributions from each part.

6. Impact of fractures

6.1. Conceptual model

In this study, it is assumed that coal matrix blocks are connected to each other by coal-matrix bridges, as illustrated in Fig. 11a. Both matrix and bridges swell during gas adsorption. Matrix swelling tends to narrow the fracture opening while the swelling of the coal-matrix bridge tends to widen the fracture. The net change in fracture opening could be positive (increase) or negative (reduction), as illustrated in Fig. 11b and c. Therefore, the permeability could decrease or increase.

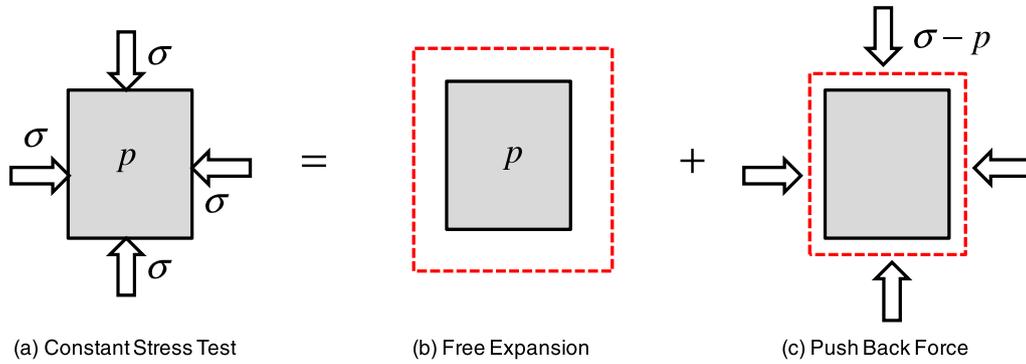


Fig. 9. Swelling tests under the stress controlled condition: (a) stress controlled condition; (b) free expansion due to gas sorption; (c) push back by the applied effective stress.

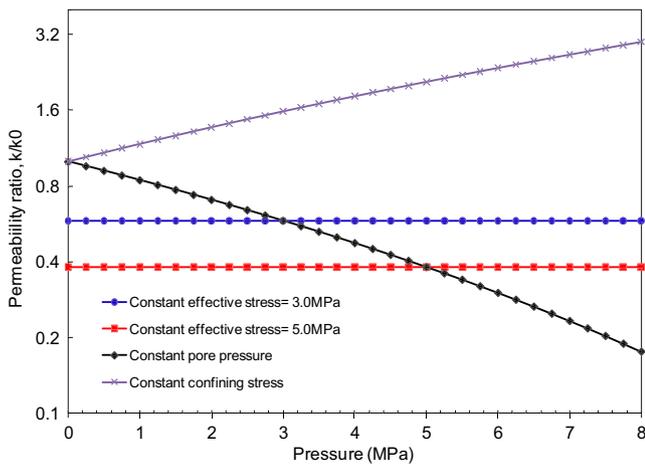


Fig. 10. Influence of effective stress on permeability change.

The change in fracture opening due to the swelling of a coal-matrix bridge may be neglected because the fracture width (the coal bridge height) is too small compared with fracture spacing. Therefore, the net change in fracture opening due to free expansion would be negative. When the coal swelling is constrained by coal bridges and external confining conditions, the overall permeability change will be determined by the change in total effective stress. Because swelling strain does not impact the coal matrix permeability as shown in Fig. 1b, the change in matrix permeability is determined by the mechanical effective stress only, i.e., $\sigma - p_m$. If the confining stress σ does not change, then the change in mechanical effective stress can be defined as $-\Delta p_m$. This change in mechanical effective stress can be used to define the net change in coal matrix permeability.

For fracture system, the permeability change is related to two factors: (1) change in mechanical effective stress; and (2) sorption-induced strain. The ‘free expansion + push back’ approach will be used to determine the total effective strain value for the fracture permeability calculation.

Permeability values for the coal matrix are typically several orders of magnitude smaller than fracture permeability values [31].

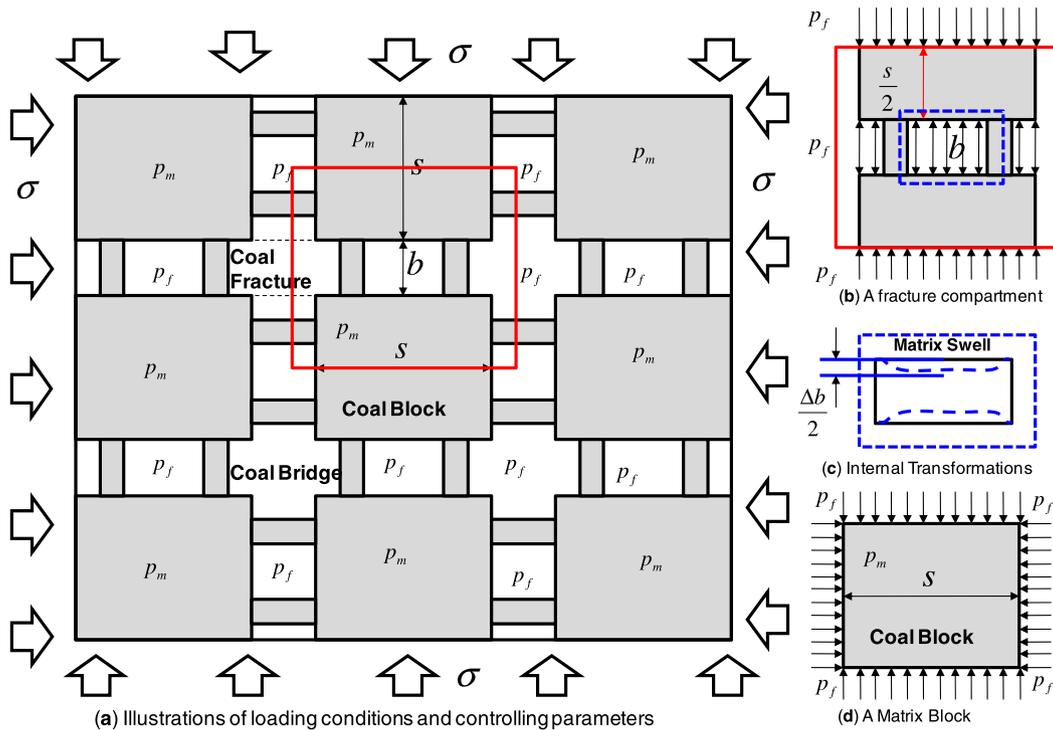


Fig. 11. Conceptual model for coal-matrix bridges.

Therefore, most researchers generally ignore coal-matrix permeability and attribute coal permeability directly to fracture permeability. However, relevant previous studies concluded that multiphase flow processes within a coal matrix may have considerable effects on coalbed methane recovery processes, while these effects are largely ignored in current modeling practice [29,32]. Therefore, it may be important to investigate the coal permeability change by combining the effects both matrix and fracture systems. In other words, the resultant change in coal permeability can be defined to be a combined outcome of the reduction in fracture opening due to coal matrix swelling and effective stress, and the variation in effective stress due to gas diffusion.

6.2. Permeability of fracture compartments

The evolution of the fracture permeability change is related to both change in mechanical effective stress; and sorption-induced strain. Using the ‘free expansion + push back’ approach and combining Eq. (14), the change in fracture opening can be defined as [33]

$$\frac{\Delta b}{b} = \left(\frac{\Delta \varepsilon_v}{3} - \frac{\Delta \varepsilon_s}{3} \right) s \times \frac{1}{b} \tag{37}$$

Simplifying the above equation yields

$$\frac{\Delta b}{b} = \frac{1}{\phi_{f0}} (\Delta \varepsilon_v - \Delta \varepsilon_s) \tag{38}$$

where $\Delta \varepsilon_v$ is the volumetric strain, and ϕ_{f0} is initial fracture porosity, defined as $\phi_{f0} = 3b/s$.

Then the fracture permeability can be expressed as

$$\frac{k_f}{k_{f0}} = \left(1 + \frac{\Delta b}{b} \right)^3 = \left[1 + \frac{1}{\phi_{f0}} (\Delta \varepsilon_v - \Delta \varepsilon_s) \right]^3 \tag{39}$$

In the lab tests, the effective stress could be kept as invariable. In this case, Eq. (39) can be simplified into

$$\frac{k_f}{k_{f0}} = \left(1 + \frac{\Delta b}{b} \right)^3 = \left(1 - \frac{1}{\phi_{f0}} \Delta \varepsilon_s \right)^3 \tag{40}$$

6.3. Resultant permeability

In order to explain the interaction between matrix and the connected bridges, the schematic diagram regarding the fracture aperture change and the effective stress alteration is shown in Fig. 12.

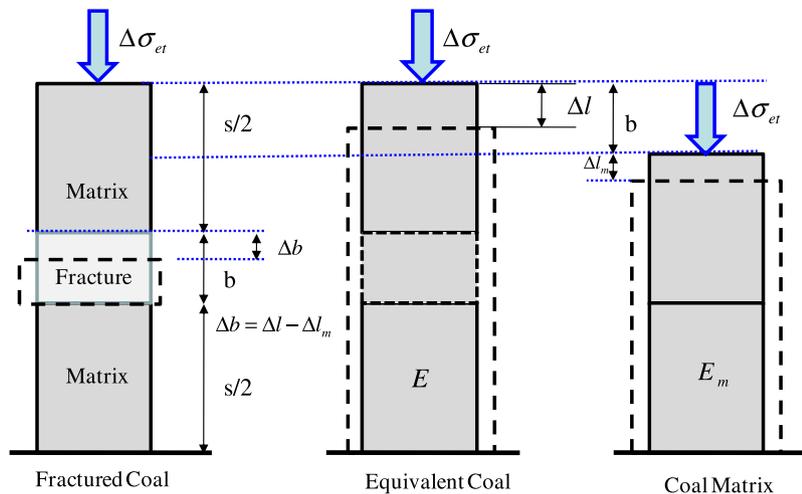


Fig. 12. Schematic diagram of fracture aperture interaction with effective stress.

In the analysis of coal deformation, the fractured coal mass is replaced as an equivalent continuous medium [34].

For fracture system, the aperture closure induced by the effective stress change can be calculated by

$$\Delta b = (b + s) \cdot \frac{\Delta \sigma_{et}}{E} - s \cdot \frac{\Delta \sigma_{et}}{E_m} \tag{41}$$

Simplifying this equation, gives

$$\Delta b = s \cdot \left(1 - \frac{E}{E_m} \right) \frac{\Delta \sigma_{et}}{E} + b \cdot \frac{\Delta \sigma_{et}}{E} \tag{42}$$

If assuming $R_m = E/E_m$, $\Delta \varepsilon_{et} = \Delta \sigma_{et}/E$, then the above equation can be derived

$$\Delta \varepsilon_f = \frac{\Delta b}{b} = \left[\frac{s \cdot (1 - R_m)}{b} + 1 \right] \cdot \Delta \varepsilon_{et} \tag{43}$$

Because $b \ll s$, Eq. (13) can be simplified into

$$\Delta \varepsilon_f = \frac{\Delta b}{b} = \frac{s \cdot (1 - R_m)}{b} \Delta \varepsilon_{et} \tag{44}$$

where R_m is the elastic modulus reduction ratio, $\Delta \varepsilon_{et}$ is the total effective strain change, s is the fracture spacing and b_0 is the initial fracture aperture.

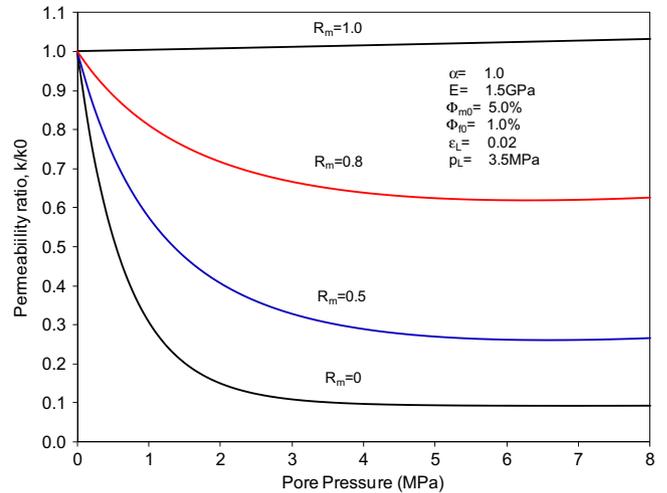


Fig. 13. Impact of partition coefficient on permeability change.

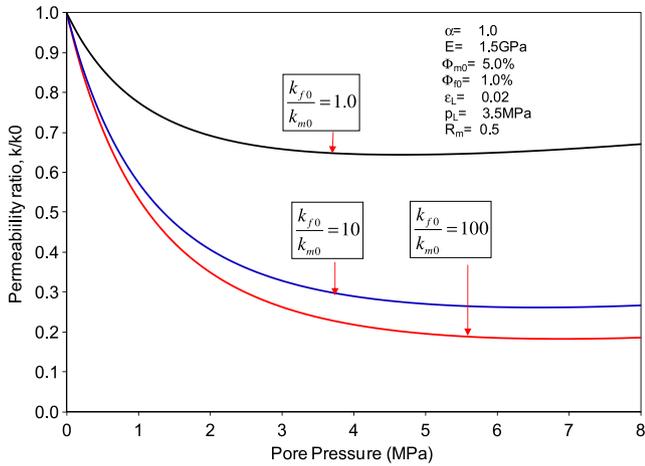


Fig. 14. Impact of the fracture permeability to matrix permeability ratio on permeability change.

Similarly, the porosity change for the matrix system can be expressed as

$$\Delta\phi_m = \frac{\Delta\sigma - \Delta p_m}{K_m} = R_m \frac{\Delta\sigma - \Delta p_m}{K} \quad (45)$$

where σ_{eff} is the mechanical effective stress.

Therefore, the permeability expressions of Eqs. (12) and (39) for both matrix and fracture systems can be given as

$$\frac{k_m}{k_{m0}} = \left(1 + \frac{\alpha}{\phi_{m0}} R_m \frac{\Delta\sigma - \Delta p_m}{K} \right)^3 \quad (46)$$

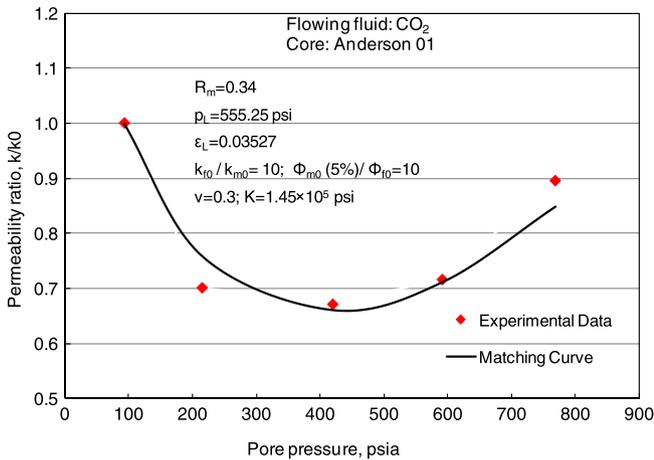
$$\frac{k_f}{k_{f0}} = \left[1 + \frac{(1 - R_m)}{\phi_{f0}} (\Delta\epsilon_v - \Delta\epsilon_s) \right]^3 \quad (47)$$

In this study, we assume that fracture and matrix deformation are both linear and fully recoverable, and deformations in normal closure or opening are the predominant permeability alteration mode. Combining the effect from both systems, the resultant change in coal permeability is resultant outcome of the reduction in fracture opening due to coal matrix swelling and effective stress change, as defined in Eq. (47) and the decrease in effective stress due change in fluid pressure and confining stress, as defined in Eq. (46). Under these assumptions, the resultant coal permeability is defined as [35]

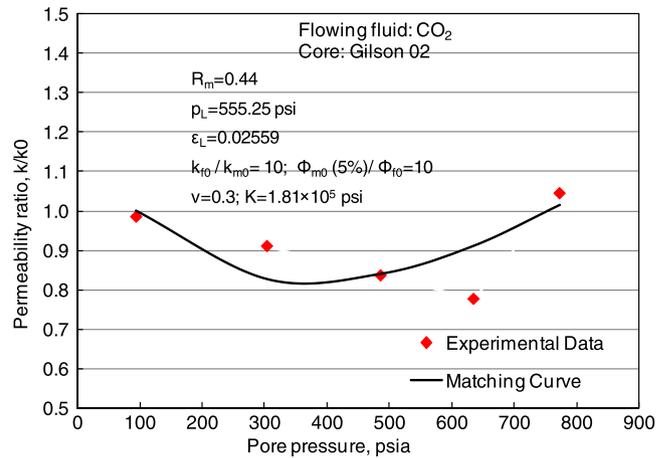
$$k = k_m + k_f \quad (48)$$

$$\frac{k}{k_0} = \frac{k_{m0}}{k_{m0} + k_{f0}} \frac{k_m}{k_{m0}} + \frac{k_{f0}}{k_{m0} + k_{f0}} \frac{k_f}{k_{f0}} \quad (49)$$

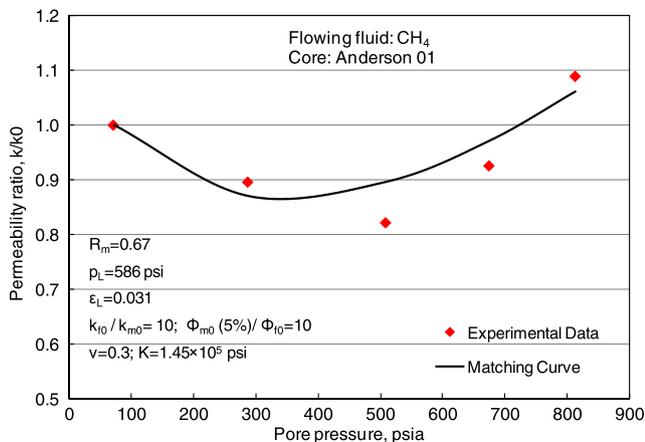
Substituting Eqs. (46) and (47) into (49) gives



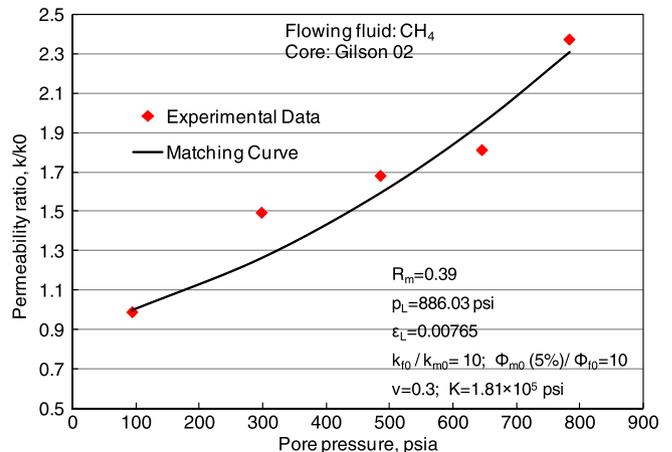
(a) Experimental data match for CO₂ for core Anderson 01.



(c) Experimental data match for CO₂ for core Gilson 02.



(b) Experimental data match for CH₄ for Anderson 01.



(d) Experimental data match for CH₄ for core Gilson 02.

Fig. 15. Experimental data matching for one coal core with 1000 psia confining pressure.

$$\frac{k}{k_0} = \frac{k_{m0}}{k_{m0} + k_{f0}} \left(1 + \frac{\alpha}{\phi_{m0}} R_m \frac{\Delta\sigma - \Delta p_m}{K} \right)^3 + \frac{k_{f0}}{k_{m0} + k_{f0}} \left[1 + \frac{(1 - R_m)}{\phi_{f0}} (\Delta\varepsilon_v - \Delta\varepsilon_s) \right]^3 \quad (50)$$

where R_m is the modulus reduction ratio between 0 and 1. Its impact on the coal permeability is illustrated in Fig. 13. When the rock mass reduction ratio is unity, i.e. $R_m = 1$ then the equivalent modulus of the fractured medium is equal to that of the coal matrix. In other words the coal mass may be considered as unfractured or the fractures are infinitely small. Conversely, in the limit as $R_m = 0$ then the coal matrix is infinitely stiff and the observed deformational response is equivalent to that of the fractures alone. Therefore the parameter $1 - R_m$ represents the ratio of the partitioned strain for the fracture system to the total equivalent strain. If $R_m = 1$, the partitioned strain for the fracture system is due to that of the matrix modulus, therefore the permeability change is due to the deformation of the matrix results. If $R_m = 0$ then the partitioned strain is predominantly due to the fracture deformation, then a maximum permeability change results.

The impact of k_{f0}/k_{m0} ratio on permeability change with $R_m = 0.5$ is shown in Fig. 14. This ratio represents the absolute fracture permeability to absolute matrix permeability. As the permeability ratio increases, the permeability contribution from the matrix diminishes.

Based on this proposed permeability model, one example is conducted to match the experimental data monitored under stress controlled conditions where few models are successful in matching response [28]. In that experiment the coal core was collected from the Anderson seam from an open-pit coal mine near Gillette, Wyoming. The confining stress was 1000 psia (6.8 MPa) for all experiments and the injection gases are CO_2 and CH_4 respectively. The matching curve is as shown in Fig. 15. The matching result shows that our model matches the experimental data reasonably well, proving the validity of our model under stress-controlled conditions.

One more match with experimental data was conducted. In this experiment, the injection gas is CO_2 and three different effective stresses were chosen [8]. For this match, the physical properties of coal sample are identical to reference Pan et al. [8], and only the fracture porosity and elastic modulus reduction ratio R_m are adjustable. The matching results are shown in Fig. 16, and good agreement is achieved.

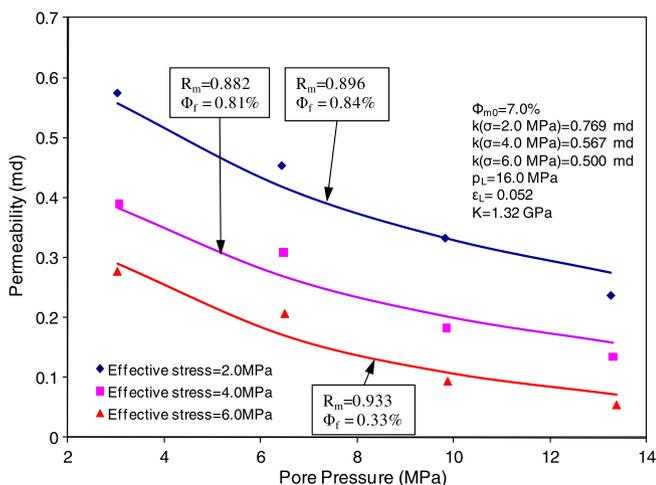


Fig. 16. Experimental data matching for constant effective stress case with CO_2 injection.

7. Conclusions

A simple “free expansion + push back” approach has been developed to generate typical response curves of coal permeability to gas sorption-induced swelling/shrinkage. In this approach, the coal is allowed to expand freely due to the gas sorption, and then it is pushed back by the applied effective stress to the original constrained conditions. The total “push-back” strains are used to calculate coal permeability.

Successful application of this approach has generated a series of coal permeability models. For the uniaxial strain condition, our approach has resulted in a coal permeability model consistent with the ARI model, PM model, SD model and CB model. The same approach is then applied to displacement-controlled and stress-controlled sorption conditions. Stress-controlled sorption tests are normally conducted in the laboratory to characterize the evolution of coal permeability under the influence of gas sorption. Typically reductions in permeability are observed from gas-sorption-induced swelling even where effective stresses remain constant. This behavior remains enigmatic as the permeability of the porous coal is determined by the effective stress only. Our new proposed permeability model is capable of replicating this apparently anomalous behavior.

Acknowledgements

This work was supported by the Western Australia CSIRO-University Postgraduate Research Scholarship, National Research Flagship Energy Transformed Top-up Scholarship, by NIOSH under contract 200-2008-25702, and by State Key Laboratory for Geomechanics and Underground Geomechanics, China University of Mining and Technology. These various sources of support are gratefully acknowledged.

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