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Characterization of Swelling Modulus and Effective Stress Coefficient Accommodating Sorption-Induced Swelling in Coal

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ABSTRACT: The effective stress law transforms external stress (σ) and pore pressure (p), into a single equivalent variable ($\sigma_{\text{effective}}$), expressed as $\sigma_{\text{effective}} = \sigma - \alpha p$, where α is the effective stress coefficient. For porous media, every property such as drained deformability, permeability, storage capacity, and acoustic velocity has its own particular effective stress coefficient. We extend the effective stress law for deformation in sorbing porous media (coal and organic-rich shales), accommodating sorption-induced swelling, by introducing the concept of an effective modulus of swelling/shrinkage. This attributes the volumetric strain (ε_v) of the sorbing medium to changes in the effective stress as $\varepsilon_v = (\sigma - \alpha_s p)/K$, with the effective stress coefficient $\alpha_s = 1 - K/K_s + K/Z_p$, in terms of the bulk modulus (K) of the sorbing porous medium, the bulk modulus (K_s) of the solid grains and the swelling modulus (Z_p). Thus, the static problem of deformation in sorbing porous media can be simplified into an elastic problem in nonporous and nonsorbing media with merely one variable: effective stress ($\sigma - \alpha_s p$). Unconstrained experiments on coal define the swelling modulus (Z_p) and its effective stress coefficients for CH_4 and CO_2. At low gas pressures (<7 MPa), the swelling modulus (Z_p) is an order of magnitude lower than the bulk modulus of solid grains (K_s) as $\sim 4 < K_s/Z_p < 30$, depending upon the particular gas (CH_4/CO_2) and gas pressure. This is consistent with the dominant influence of sorption-induced swelling at low gas pressures and its important effect on stress-permeability evolution during depletion of coalbed methane. Where the influence of swelling is included, the effective stress coefficient may exceed the normal bound ($0 < \alpha < 1$) of unity for CO_2 (α_{CO_2}) and for CH_4 (α_{CH_4}). For the stronger affinity of CO_2 to coal, α_{CO_2} is ∼2−3 times larger than α_{CH_4}, varying with gas pressure. As anticipated, for relatively "stiff" sorbing media, α_{CO_2} and α_{CH_4} are much larger than 1 and decline more rapidly with an increase in gas pressure, compared to relatively "soft" sorbing media, where $\alpha_{CO_2} \approx \alpha_{CH_4} \approx 1$, where the decline is less rapid with gas pressure. The effective stress coefficient for non/lightly sorbing helium remains constant at α_{He} less than ∼1 for both "soft" and "stiff" sorbing media. This effective stress law also applies to uniaxial conditions—and, appropriately, is shown to be independent of mechanical boundary conditions. Experiments on coal samples under uniaxial strain conditions validate this. The results indicated that the effective stress, accommodating sorption effects, may be transformed among different mechanical boundary conditions as a unified effective stress coefficient. Under uniaxial strain, the effective stress can be expressed as a function of overburden stress and pore pressure as $\sigma_{\text{effective}} = \sigma_v - \alpha_s p$, which attributes to the changes in volumetric strain as $\varepsilon_v = \sigma_{\text{effective}}/M$, where M is the constrained axial modulus.

1. INTRODUCTION

The effective stress law is widely applied to unconventional gas reservoirs where sorbed gas content may be significant and where deformability and fracture permeability is strongly dependent on both effective stress and gas sorption. This includes describing the evolution of other stress-dependent or strain-dependent properties, such as the evolution of permeability during gas depletion. Some approaches[1−7] use the concept of effective stress for single-porosity media with an assumed uniform pore pressure in matrix and cleat to calculate the effective stress effects. Other approaches[8,9] accommodate the true behavior of dual-porosity[10,11] or triple-porosity media[9,12] where separate (dual or triple) pressure fields are accommodated in both matrix and cleat. All such models reveal mechanisms of deformation and transport during unconventional gas depletion. Unconventional gas reservoirs, such as coalbeds and organic-rich shales, contain coal and kerogen with a significant capacity to sequester the resource gas in adsorbed form. This gas may be directly recovered as a desorbed resource, in the case of CH_4, competitively desorbed by the injection of CO_2 and left with the remnant and sequestered CO_2.[13]

Terzaghi first proposed the effective stress law, and attributed all measurable effects of a change in stress exclusively to changes in this "effective" stress.[14,15] The term "effective" referenced the intergranular stress that was the sole agent "effecting" deformation and failure. Based on observations of one-dimensional consolidation, the Terzaghi effective stress (σ_{Terzaghi}) was defined as the difference between the external stress (σ) and the pore pressure (p) as

$$\sigma_{\text{Terzaghi}} = \sigma - p \tag{1}$$

This particular definition of Terzaghi effective stress has been widely used in the mechanics of soil. However, Terzaghi's treatment is only representative of the special case of saturated soils with incompressible grains and a significantly more

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compressible frame. Biot[16] extended this theory[17] to the three-dimensional case and established equations valid for any arbitrary load variable with time. A rigorous and complete treatment of the theory of elasticity leads to a more general and well-accepted form of the effective stress law for porous media. The effective stress is given by

$$\sigma_{ij}^{e} = \sigma_{ij} - \alpha p \delta_{ij} \tag{2}$$

where σ_{ij}^{e} is the effective stress tensor, σ_{ij} the total stress tensor, p the pore pressure, and δ_{ij} the Kronecker delta ($\delta_{ij} = 1$ when $i = j$ and $\delta_{ij} = 0$ for all other cases). α is the effective stress coefficient, which is also well-known as the Biot coefficient, defined as

$$\alpha = \frac{2(1+\nu)G}{3(1-2\nu)H} \tag{3}$$

where G and ν are the shear modulus and the Poisson ratio, respectively, and H is an additional physical constant representing a new effective modulus.[16]

A physically based definition of the effective stress coefficient requiring no additional arbitrary parameters was advanced, based on heuristic arguments,[18] as

$$\alpha = 1 - \frac{K}{K_s} \tag{4}$$

and its basic correctness is illustrated by experimental observations where K and K_s are the bulk modulus of the dry aggregate and the intrinsic bulk modulus of solid grains, respectively. The expression was first suggested experimentally.[19,20]

In addition to this behavior of inert, nonsorbing media, the effective stress law has been extended to sorbing media[21] applicable to coalbed methane reservoirs. This approach first introduced the technique to convert the three variables, of external stress, pore pressure and matrix shrinkage/swelling into a single variable, "effective stress". The effective stress coefficient is given as

$$\alpha = 1 - \frac{K}{K_s} + \frac{3Ka\rho RT\ln(1+bp)}{E_A V_0 p} \tag{5}$$

where a and b are Langmuir sorption constants, ρ is the coal solid-phase density, R is the universal gas constant, T is the reservoir temperature, E_A is the modulus of solid expansion,[22] and V_0 is the gas molar volume. These parameters were derived from a model of sorption-induced swelling or shrinkage.[23]

For fractured porous materials, significant differences in permeability between fractures and matrix[24] lead to more than a single distinct pressure field. Based on this idea, the concept of effective stress has been extended to a general expression for multiporous media[12] as

$$\alpha_{ij}^{e} = \sigma_{ij} - (\gamma_n p_n + \gamma_{n-1} p_{n-1} + \cdots + \gamma_1 p_1)\delta_{ij} \tag{6}$$

where $\gamma_1, \gamma_2, ..., \gamma_n$ and $p_1, p_2, ..., p_n$ are effective stress coefficients and pore pressures for multiple porous media, respectively. The effective stress law for single porosity media with a single-pressure field is a special case of this.[12] For single porous media, the effective stress coefficient is determined by solid compressibility and bulk compressibility, agreeing with experimental observations[19,20] and analytical characterizations.[18]

In the following, deformation due to sorption-induced swelling is incorporated into the strain−stress relationship to build a new effective stress principle for sorbing porous media with a single pressure field. Experimental observations demonstrate the veracity and ubiquity of the effective stress coefficient under different mechanical boundary conditions where effective stress coefficients for different gases ($He/CH_4/CO_2$) are measured based on mechanically unconstrained experiments on coal.

2. DERIVATION OF EFFECTIVE STRESS LAW FOR DEFORMATION OF SORBING POROUS MEDIA

2.1. The Elastic Stress−Strain Relation for Sorbing Porous Media. For an homogeneous isotropic porous aggregate, adopting the convention that compression is positive, the general tensor expression for strain, in terms of the stress tensor and pore pressure, can be obtained as[18,25]

$$\varepsilon_{ij} = \frac{1}{2G}\left[\sigma_{ij} - \frac{1}{3}(\sigma_{kk}\delta_{ij})\right] + \frac{1}{9K}(\sigma_{kk}\delta_{ij}) - \frac{1}{3H}p\delta_{ij} \tag{7}$$

The first term on the right-hand side of eq 7 is the strain due to the deviatoric stress conditioned by the shear modulus. The second term is the strain due to hydrostatic stress, conditioned by the bulk modulus. The last term is the strain due to pore pressure and a new effective modulus H, introduced by Biot.[16] The deviatoric stress (eq 7) only influences the shear, but not the normal, strains, and has a tendency to distort the medium without a change in the volumetric strain (the sum of the three principal strains, $\varepsilon_v = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}$). Conversely, the hydrostatic stress and pore pressure only influence the normal, but not the shear, strains.

For sorbing media, such as coal and organic-rich shale, the swelling/shrinkage strain due to adsorption/desorption may be non-negligible, in comparison to the total strain. Based on the theory of poroelasticity,[25] an analogy may be made between thermal contraction and swelling/shrinkage[3] to recover a general strain tensor for a sorbing medium. This enables the stress tensor, pore pressure, and the swelling/shrinkage strain ε_s to be linked as (positive in compression)

$$\varepsilon_{ij} = \frac{1}{2G}\left[\sigma_{ij} - \frac{1}{3}(\sigma_{kk}\delta_{ij})\right] + \frac{1}{9K}(\sigma_{kk}\delta_{ij}) - \frac{1}{3H}(p\delta_{ij}) + \frac{1}{3}(\varepsilon_s\delta_{ij}) \tag{8}$$

If we adopt the same concept as Biot,[16,21] the general tensor expression for strain, in terms of the stresses and pore pressure, may be given as

$$\varepsilon_{ij} = \frac{1}{2G}\left[\sigma_{ij} - \frac{1}{3}(\sigma_{kk}\delta_{ij})\right] + \frac{1}{9K}(\sigma_{kk}\delta_{ij}) - \frac{1}{3H}(p\delta_{ij}) - \frac{1}{3Z_p}(p\delta_{ij}) \tag{9}$$

where H is the new effective modulus introduced by Biot[16] and Z_p is the swelling modulus.[21] Z_p is not a constant, since the adsorption/desorption-induced strain follows the typical Langmuir relationship with pore pressure. Comparing eq 9 with eq 8 gives the swelling modulus Z_p of a sorbing porous media as

$$Z_p = -\frac{p}{\varepsilon_s} \tag{10}$$

The swelling/shrinkage of coal may be approximated as the response of a Langmuir solid.[23,26−29] This includes both the adsorptive content for shale and also the related strain−

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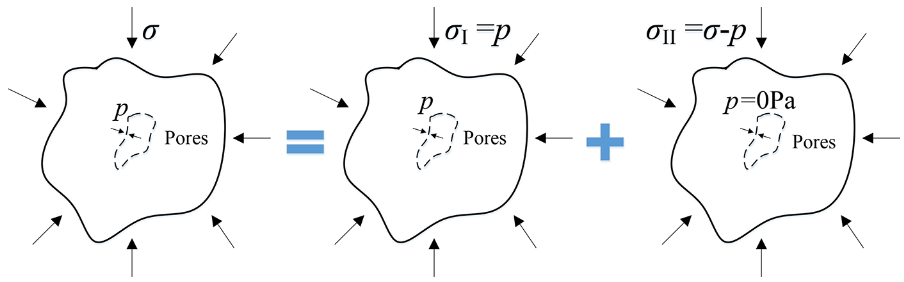

Figure 1. Schematic of the state of stress for a homogeneous aggregate with pores.

Table 1. Volumetric Strain for Each Constituent Resulting from Stress Applied in Two Separate Stages

constituent	bulk material	pore space	solid phase
volumetric strain caused by the first stage, ε_{I}	$p/K_s - p/Z_p$	$p/K_s - p/Z_p$	$p/K_s - p/Z_p$
volumetric strain caused by the second stage, $\varepsilon_{\mathrm{II}}$	$(\sigma - p)/K$	$(\sigma - p)/K_p$	$(\sigma - p)/K_s$
total volumetric strain, ε_v	$[\sigma - (1 - K/K_s + K/Z_p)p]/K$	$[\sigma - (1 - K_p/K_s + K_p/Z_p)p]/K_p$	$\sigma/K_s - p/Z_p$

pressure response.[30,31] The sorption-induced strain for a sorbing porous medium, can be written as

$$\varepsilon_s = -\frac{\varepsilon_l p}{P_L + p} \tag{11}$$

where ε_s is the swelling strain at gas pressure p, the negative sign denotes "swelling", and ε_l and P_L are Langmuir-type constants for adsorption of gas on the substrate of the sorbing porous medium; ε_l also represents the maximum volumetric strain at infinite pore pressure, and P_L is the pore pressure at which the measured volumetric strain reaches $0.5\varepsilon_l$. Substituting eq 11 into eq 10, we obtain the swelling modulus Z_p as

$$Z_p = \frac{P_L + p}{\varepsilon_l} \tag{12}$$

The pore pressure in the fourth terms of eq 9, relevant to the adsorption/desorption-induced strain, only influences the normal strains—without distortion. This is also controlled by the mean compressive stress, $\sigma = {}^1/_3(\sigma_{11} + \sigma_{22} + \sigma_{33})$, which is also termed the "external stress". Since the deviatoric stress (the first term) has no effect on the volumetric strain, the volumetric strain caused by the hydrostatic stress, pore pressure, and matrix swelling/shrinkage can be obtained from eq 9 as

$$\varepsilon_v = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33} = \frac{1}{K}(\sigma) - \frac{1}{H}(p) - \frac{1}{Z_p}(p) = \frac{1}{K}(\sigma - \alpha_s p) \tag{13}$$

where α_s is the revised effective stress coefficient, which is defined as $\alpha_s = K/H + K/Z_p$. The volumetric strain increases with the external stress and decreases with pore pressure. This strain can thus be expressed as a function of a linear combination of the external stress and the pore pressure. In many static cases, using a single new variable to describe the deformation of the solid in terms of both external stress and pore pressure greatly simplifies the analysis. This single new variable, the link between elastic mechanics and poro-elastic mechanics, is called the effective stress and is defined as

$$\sigma^e = \sigma - \alpha_s p \tag{14}$$

If we use a more general form of the effective stress law as $\sigma_{ij}^e = \sigma_{ij} - \alpha_s p \delta_{ij}$, then eq 9 can be simplified as

$$\varepsilon_{ij} = \frac{1}{2G}\left[\sigma_{ij}^e - \frac{1}{3}\left(\sigma_{kk}^e \delta_{ij}\right)\right] + \frac{1}{9K}\left(\sigma_{kk}^e \delta_{ij}\right) \tag{15}$$

where the strain tensor is defined as usual.[18,21] A comparison between eqs 9 and 15 shows that the static problem of any deformation in a sorbing porous medium with pore pressure and sorption-induced swelling can be simplified into an elastic problem in a nonporous and nonsorbing medium without pore pressure and without "swelling".

The revised effective stress coefficient α_s may be rigorously defined[18] for any isotropic elastic aggregate with connected pores of arbitrary shape and concentration that is subjected to an external stress σ and a uniform pore pressure p ($\sigma \geq p$). The state of stress of Figure 1 is achieved by applying a load in two stages.[19] First, a pore pressure p and an equal external stress σ_{I} ($\sigma_{\mathrm{I}} = p$) is applied as a hydrostatic stress. This results in a volumetric strain of the aggregate equal to $\varepsilon_{\mathrm{I}} = p/K_s$, where K_s is the bulk modulus of the solid grains (the solid devoid of any cavities). Second, a remaining stress $\sigma_{\mathrm{II}} = \sigma - p$ is applied without any change in the pore pressure, resulting in a volumetric strain of the aggregate equal to $\varepsilon_{\mathrm{II}} = (\sigma - p)/K$, where K is the bulk modulus of the dry aggregate. For a sorbing porous medium such as coal or organic-rich shale, gas adsorption causes a swelling of the matrix. Considering the adsorptive behavior and assuming only one solid constituent exists, the volumetric strain of the bulk material, pore space and solid phase may be defined as in the tabulation of Table 1.

Therefore, the total volumetric strain of a sorbing porous medium, such as coal or shale, caused by external stress, pore pressure and sorption-induced swelling can be given by

$$\varepsilon_v = \frac{1}{K}\left[\sigma - \left(1 - \frac{K}{K_s} + \frac{K}{Z_p}\right)p\right] \tag{16}$$

Solving eqs 16, 12, and 13, the new effective modulus H[16] and the revised effective stress coefficient α_s can be obtained as

$$H = \frac{K}{1 - K/K_s} \tag{17}$$

$$\alpha_s = \frac{K}{H} + \frac{K}{Z_p} = 1 - \frac{K}{K_s} + \frac{K\varepsilon_l}{P_L + p} \tag{18}$$

Note that H can also be obtained by solving eqs 3[16] and 4.[18]

2.2. Deformation of Sorbing Porous Media under Uniaxial Strain Conditions. Conditions of uniaxial strain are widely utilized to represent depletion of laterally extensive unconventional reservoirs where the lateral gradients of

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resulting gas pressures remain small.[3,4,7] According to eq 9, the strain–stress tensor can also be expressed by Young's modulus (E) and Poisson's ratio (ν), as

$$\varepsilon_{ij} = \frac{1+\nu}{E}(\sigma_{ij}) - \frac{\nu}{E}(\sigma_{kk}\delta_{ij}) - \frac{p}{3H}(\delta_{ij}) - \frac{p}{3Z_p}(\delta_{ij}) \tag{19}$$

Therefore, the lateral strain can be expressed as

$$\varepsilon_{22} = \varepsilon_{33} = \frac{1+\nu}{E}(\sigma_L) - \frac{\nu}{E}(\sigma_V + 2\sigma_L) - \frac{p}{3H} - \frac{p}{3Z_p} \tag{20}$$

where σ_V is the vertical or overburden stress, and σ_L is the lateral or horizontal stress. For the condition of zero lateral strain, the relationship between overburden stress and horizontal stress can be defined as

$$\sigma_L = \frac{\nu}{1-\nu}(\sigma_V) + \frac{1-2\nu}{1-\nu}(\alpha_s p) \tag{21}$$

From this, it is clear that the lateral stresses are not constant during pore pressure depletion under uniaxial strain-controlled boundary conditions.

Similarly, the state of stress under conditions of uniaxial strain can also be achieved by applying a sequence of loads, as shown in Figure 2. First, equivalent pore pressure (p) and

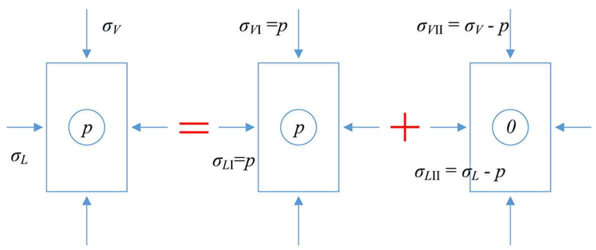

Figure 2. Schematic of state of stress under uniaxial strain conditions.

vertical and lateral stresses ($\sigma_{VI} = \sigma_{LI} = p$) are applied to form a hydrostatic stress state. Second, the remaining vertical stress ($\sigma_{VII} = \sigma_V - p$) and lateral stress ($\sigma_{LII} = \sigma_L - p$) are applied, without a change in pore pressure. According to eq 19, the volumetric strain components resulting from this loading in two stages are ε_I and ε_{II}, given as

$$\varepsilon_I = \frac{p}{K_s} - \frac{p}{Z_p} \tag{22}$$

and

$$\varepsilon_{II} = \varepsilon_{11} + 2\varepsilon_{22} = \frac{1-2\nu}{E}(\sigma_V + 2\sigma_L - 3p) \tag{23}$$

Therefore, the volumetric strain can be obtained as

$$\varepsilon_v = \varepsilon_I + \varepsilon_{II} = \frac{1+\nu}{3K(1-\nu)}(\sigma_V - \alpha_s p) \tag{24}$$

Under uniaxial strain conditions, the mean external stress can also be expressed as $\sigma = {}^1/_3(\sigma_V + 2\sigma_L)$. Substituting this relationship into eq 13 recovers eq 24. Assuming the compression modulus M is

$$M = \frac{E}{1-2\nu}\left(\frac{1-\nu}{1+\nu}\right) \tag{25}$$

where M is also called the constrained axial modulus,[4] then eq 24 can be rewritten as

$$\varepsilon_v = \frac{1}{M}(\sigma_V - \alpha_s p) \tag{26}$$

From eq 26, the effective stress coefficient (α_s), under conditions of uniaxial strain, is identical to that for the general condition in eq 13. Theoretically, the effective stress is controlled by both overburden stress and pore pressure under uniaxial strain conditions, and the volumetric strain (also equal to the vertical component of strain, since lateral strains are null) is controlled by the constrained axial modulus and the effective stress. Therefore, the concept of effective stress, regardless of the mechanical boundary conditions, is universal for all modes of constraint on deformation in sorbing porous/swelling media.

2.3. Discussion of the Moduli *G*, *K*, *H*, Z_p, and *M*. The first term on the right side of eq 9 is the shear strain due to the deviatoric stress ($\sigma_{ij} - \sigma_{kk}\delta_{ij}/3$) conditioned by the shear modulus G. The second term is the normal strain due to hydrostatic stress ($\sigma_{kk}\delta_{ij}$) conditioned by the bulk modulus K. The third term is the normal strain due to pore pressure p and conditioned by the new effective modulus H. The last term is the normal strain due to pore pressure p, conditioned by the swelling modulus Z_p. Based on this and isotropic elasticity,[25] shear modulus (G), bulk modulus (K), and the new effective modulus (H) are elastic constants that measure resistance of a sorbing porous medium to being deformed elastically either in shear mode or in normal mode when a stress (deviatoric or hydrostatic) or pore pressure is applied. The greater these moduli, the smaller the deformations. According to the theory of isotropic elasticity,[25] G and K are related to E and ν. The new effective modulus H in eq 17 is related to the bulk modulus K of a sorbing porous medium and bulk modulus K_s of its solid grains.[16,18]

Analogously, the swelling modulus Z_p measures the resistance of a sorbing porous medium to swelling as a result of sorption when pore pressure is applied by a sorbing gas. However, Z_p is not a constant and increases linearly with pore pressure according to eq 12. For a sorbing gas, the greater the pore pressure, the greater the swelling modulus, and the smaller the incremental swelling deformation due to gas sorption. The concept of swelling modulus explains "mechanistically" why the sorption-induced swelling rate of a sorbing porous medium decreases as the pore pressure increases (viz. directly following the form of Langmuir isotherms).

The constrained axial modulus[4] (M) is defined by eq 25 and conditions the volumetric strain of a sorbing porous medium under uniaxial strain conditions as a function of effective stress ($\sigma_V - \alpha_s p$) (eq 26). Similarly, M is an elastic constant used to measure resistance of a porous medium to being deformed elastically solely in the axial direction when both external stress (σ_V, σ_L) and pore pressure p are applied to it under uniaxial strain conditions. M is also related to E and ν.

To summarize, among the moduli G, K, and M, which are related to E and ν, only two of them are independent. H is related to K and K_s. Generally, $H \approx K$ for porous media, since, typically, $K_s \gg K$. Each type of modulus controls a particular mode of deformation. The moduli G, K, H, and M, which are of the same order of magnitude, reflect the capacity for mechanical deformation. The swelling modulus (Z_p), which is independent of the other moduli and determined by Langmuir constants and sorbing gas pressures, reflects the capacity for sorption-induced swelling.

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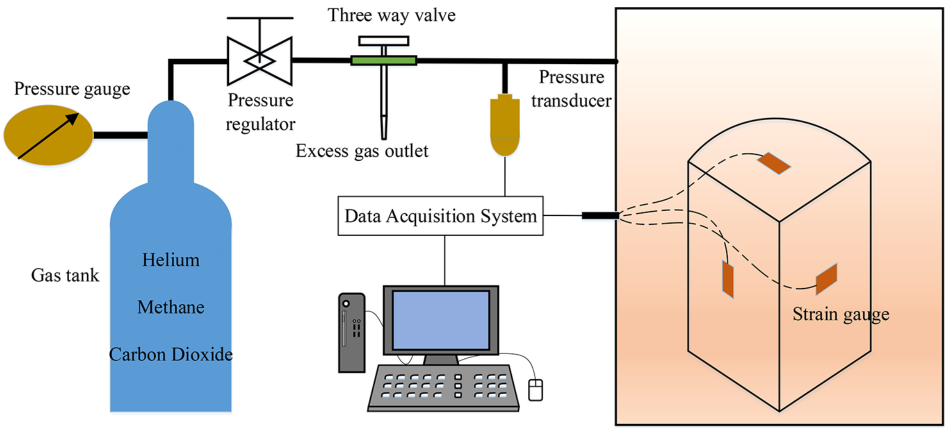

Figure 3. Schematic of the experimental setup for measurement of swelling/shrinkage of the coal matrix.

3. MEASUREMENT OF Z_P AND α_S FOR COAL SAMPLES UNDER UNCONSTRAINED CONDITIONS

As noted previously, a hydrostatic change in stress state promotes only volumetric strain, with no distortion. For mechanically unconstrained experiments on coal samples, the change in external stress is equal to the change in pore pressure. Substituting $\sigma = p$ into eq 16, the changes in volumetric strain can be defined as

$$\varepsilon_v = \frac{p}{K_s} - \frac{p}{Z_p} \tag{27}$$

The swelling modulus and the effective stress coefficients may be measured for different sorbing gases and accommodating sorption-induced swelling. This may be completed by measuring the effective stress and swelling response as both nonsorbing/nonswelling (He) and sorbing/swelling (CH_4/CO_2) gases saturate coal samples under unconstrained conditions. We report such experiments in the following.

3.1. Experimental Setup and Procedure under Unconstrained Conditions. Compression experiments are conducted (Figure 3) in a hydrostatic pressure cell capable of applying gas pressures to ~15 MPa. Strains are measured on the cylindrical sample in both axial and radial modes (Figure 3). The pressure cell is immersed in a water bath to maintain constant temperature and eliminate thermal noise in the strain signal. Hydrostatic stress/unconstrained conditions are applied by gas-pressurizing the sample inside the pressure cell. A more-detailed description of the experimental arrangement and procedures is presented elsewhere.[23,29]

The coal samples for the unconstrained experiments were obtained from the Illinois basin. Core samples with few cleats were split into quadrants, as shown in Figure 4. The four samples were prepared from coal blocks to measure the swelling modulus and the effective stress coefficient accommodating sorption-induced swelling. Gas pressure was applied and strain was monitored continuously via a rosette of three orthogonal strain gauges attached to each sample. The three strain gauges record the three principal strains, with the volume strain defined as their algebraic sum.

The samples were pressurized with a nonsorbing/nonswelling gas (He) in increments of ~1.4 MPa (200 psi) to a final pressure of ~6.9 MPa (1000 psi). To ensure that swelling had completed, the strains were monitored through completion for more than 1 day. Once completed, the helium was bled out from the sample and containers, and the specimen was evacuated for a few days to release any residual helium.

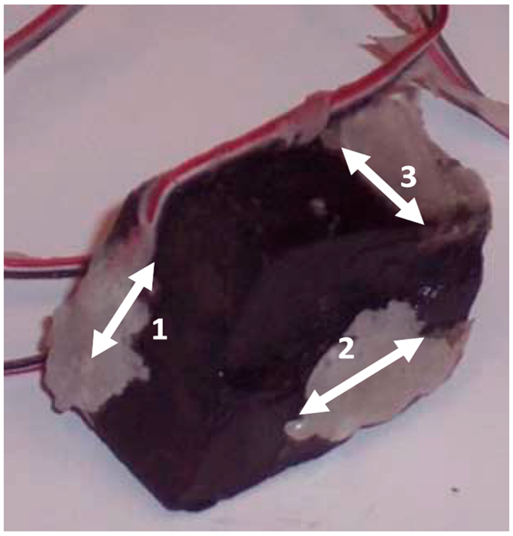

Figure 4. A prism of coal with strain gauges applied in three orthogonal directions.

Subsequently, two samples (Samples 1 and 2) were subjected to the same incremented pressurization but now with methane, and then the two other samples (Samples 3 and 4) were pressurized with carbon dioxide. Since adsorption progresses slowly, each pressurization step was allowed to reach a measured equilibrium of strain (from both pressure and adsorptive effects). One of the four samples (Sample 4, CO_2) failed, so only three samples are presented in this study.

3.2. Experimental Results and Analysis. *3.2.1. Helium Injection.* The purely mechanical response of the solid to changes in external pressure was measured using helium. Since helium is a nonadsorptive gas, the measured volumetric strains for the three samples was purely due to mechanical compression of solid grains. Figure 5 shows the change in volumetric strain with pressure for the unconstrained condition. The volumetric strain is linear with pore pressure for less than ~6.9 MPa (1000 psi), representing a linearly poroelastic solid. The solid, matrix, or grain compressibility of the samples can thus be calculated from the measured volumetric strain. The grain compressibility can be expressed as

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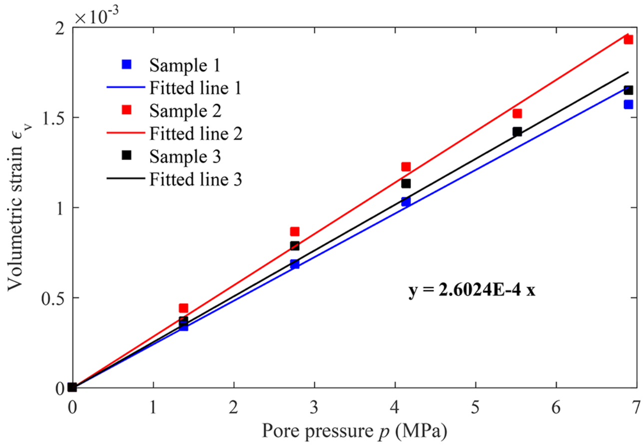

Figure 5. Measured and fitted volumetric strain for saturation with helium.

$$C_s = \frac{1}{K_s} = \frac{\varepsilon_v}{p} \tag{28}$$

(see Table 1). Therefore, the grain compressibility, which is also given by the average slope of the strain-pressure curve of Figure 5, is 2.6×10^{-4} MPa^{-1}, corresponding to a bulk modulus of the solid grains of 3.8 GPa.

3.2.2. Sorption-Induced Swelling with CH_4 and CO_2 Flooding. Sorption-induced changes in swelling strains were similarly measured with exposure to sorptive gases (CH_4/CO_2). Note that we make the convention that compression/shrinkage is positive and dilation/swelling is negative. Figures 6 and 7 show volumetric "swelling" of the coal samples with exposure to CH_4 and CO_2, respectively. A Langmuir isotherm model fits these experimental data well, as noted by other researchers.[23,26−29] Apparent from Figures 6 and 7 is that the absolute "swelling" to CO_2 is greater than that with CH_4. For pressures of less than ∼6.9 MPa (1000 psi), the volumetric swelling with CO_2 reaches ∼1.8%, which is a factor of ∼3 greater than that observed with CH_4 (∼0.6%).

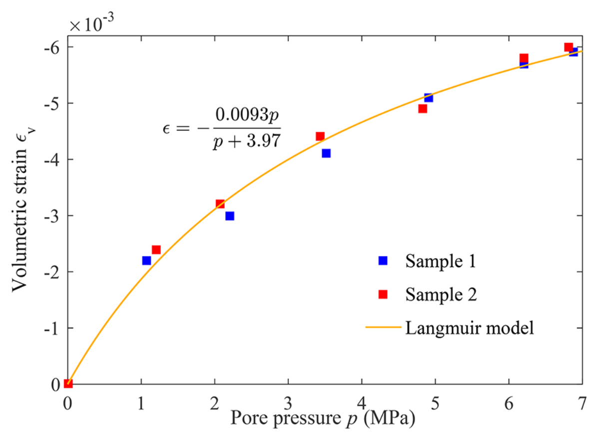

Figure 6. Measured and modeled volumetric strain for saturation with methane.

Comparing Figures 6 and 5, the volumetric swelling to CH_4 at ∼6.9 MPa (1000 psi) is ∼3 times greater than that to He. This shows a slight difference with prior experimental results,[29] where swelling with methane injection at ∼6.9 MPa (1000 psi) was ∼8 times greater than that of He. Therefore, the contribution of mechanical compression to total volumetric strain cannot be considered as negligible. Since the measured volumetric strain is the ensemble/collective effect of sorption-induced swelling and mechanical compression, the actual sorption-induced volumetric strain can be obtained by algebraically subtracting the mechanical effect measured under helium flooding from the measured sorption-induced strain with CH_4/CO_2 flooding. Figure 8 shows the separate sorptive, mechanical, and collective (measured) effects. The Langmuir strains and Langmuir pressures for CH_4 and CO_2 can be recovered as listed in Table 2.

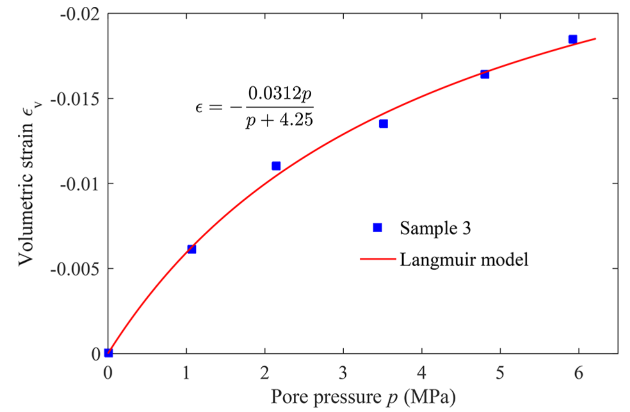

Figure 7. Measured and modeled volumetric strain for saturation with carbon dioxide.

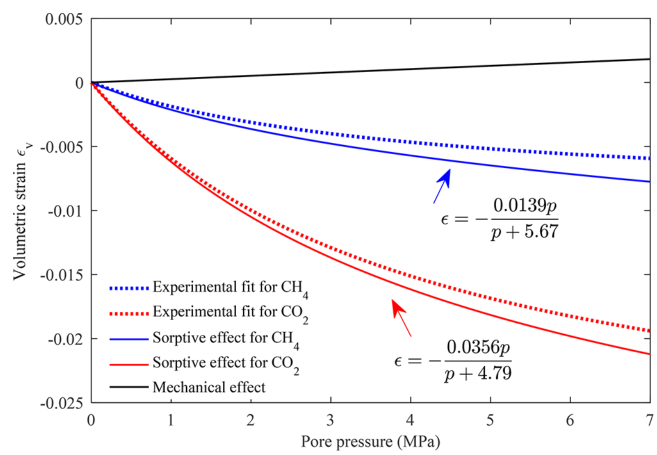

Figure 8. Separate volumetric strains induced by sorptive and mechanical effects.

Table 2. Measured Langmuir-Type Constants for Volumetric Strain

gas	Langmuir strain, ε_l	Langmuir pressure, P_L (MPa)
CH_4	0.0139	5.67
CO_2	0.0356	4.79

3.3. Computation of Z_p and α_s for He/CH_4/CO_2.

Based on the theoretical and experimental analysis above, the swelling modulus Z_p and the effective stress coefficient α_s accommodating sorption-induced swelling can be calculated using the measured Langmuir constants (ε_l and P_L) from Table 2 and estimated E and ν.[28]

For helium, an inert and nonsorbing gas, Z_p approaches infinity and causes zero "swelling". According to eq 12, Z_p for

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the two different sorbing gases (CH_4/CO_2) is only dependent on the swelling capacity of coal to those two gases and can be directly calculated using the measured Langmuir constants listed in Table 2. Figure 9 shows the variation of swelling

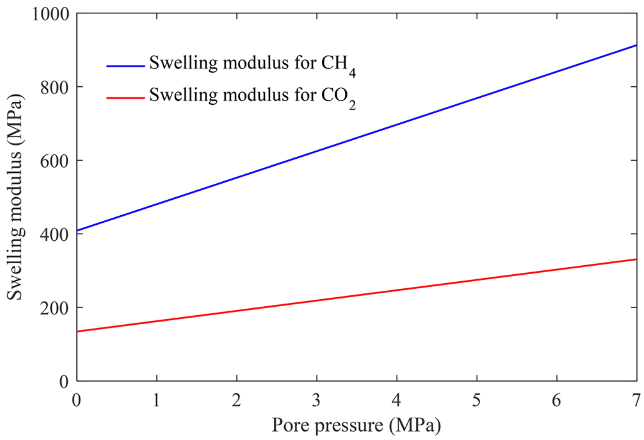

Figure 9. Variation of swelling modulus for saturation with methane and carbon dioxide.

modulus for the two gases. From Figure 9, Z_p is not constant and increases linearly with gas pressure. This infers that the swelling capacity declines with gas pressure. The magnitudes of Z_p for CH_4 and CO_2 are substantially smaller than the bulk modulus of the solid grains K_s for pore pressures less than ∼6.9 MPa (1000 psi). The ratio of K_s to Z_p varies in the range of ∼4−30, in response to the different sorbing gases (CH_4/CO_2) and gas pressures. This suggests that volumetric strain is more sensitive to CH_4/CO_2 sorption-induced swelling effect than to the mechanical effect. Z_p for CH_4 is greater than that for CO_2 varying with gas pressure and is ∼3 times greater at a gas pressure of ∼6.9 MPa (1000 psi), representing the relatively low swelling capacity of coal to CH_4.

Since the modulus Z_p varies with gas pressure then the effective stress coefficient also carries that dependency. The effective stress coefficients for different gases accommodating sorption-induced swelling are calculated and shown in Figure 10. The effective stress coefficient for He (α_{He}) is less than unity and remains constant with pressure. This is consistent with the usual concept for an effective stress law for porous

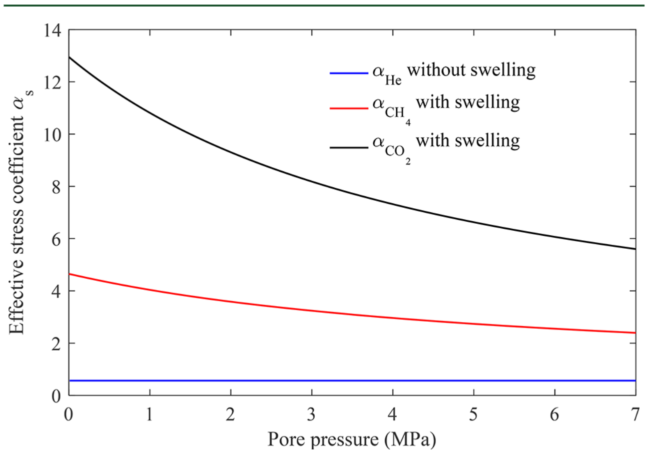

Figure 10. Variation of effective stress coefficients for coal accommodating sorption-induced swelling.

nonsorbing media. However, the values of this effective stress coefficient for CO_2 (α_{CO_2}) and for CH_4 (α_{CH_4}) transit this usual limiting threshold of unity and decrease with gas pressure if sorption-induced swelling is accommodated. The magnitude of this coefficient for CO_2 (α_{CO_2}) is ∼2−3 times larger than that for CH_4 (α_{CH_4}), with both varying with gas pressure. This suggests that the stronger the swelling capacity, the higher the magnitude of the effective stress coefficient, and the more the influence of volumetric swelling. The values of α_{CO2} and α_{CH_4} are larger than unity and also indicate that changes in volumetric strain show more sensitivity to CH_4/CO_2 gas pressure than to confining pressure if other mechanical boundary conditions are employed.

The ratio of the bulk modulus of coal to the bulk modulus of the solid grains (K/K_s) can also influence the variation of its effective stress coefficients with pressure. Figure 10 shows the variation of α_s for the case of $K/K_s = 0.43$, which is calculated according to the measured bulk modulus of solid grains (K_s), and estimated Young's modulus (E), and ν.[28] Two other cases ($K/K_s = 0.1$, $K/K_s = 0.01$) were simulated, as shown in Figure 11 to illustrate the effect of coal stiffness on its effective stress coefficients. A comparison of Figure 10 with Figure 11 shows that the smaller the value of K/K_s (i.e., the less stiff the coal), then the closer the magnitude of effective stress coefficient to 1. Specifically, α_{He} is <1 and increases with a decrease in stiffness, while α_{CH_4} and α_{CO_2} are both >1 and decrease with a decrease in stiffness. Also, for relatively stiff sorbing media with a larger value of K/K_s, α_{CH_4} and α_{CO_2} decline progressively more with gas pressure, compared to relatively "soft" sorbing media.

4. EXPERIMENTAL VALIDATION FOR THE EFFECTIVE STRESS LAW UNDER UNIAXIAL STRAIN CONDITION

To replicate conditions *in situ*, strains were measured with methane depletion for laterally (uniaxial strain) constrained coal samples from the San Juan Basin. Strains were monitored in both horizontal and vertical directions with external stress and gas pressure controlled independently throughout the duration of the experiments. To simulate in situ conditions, the vertical stress was maintained constant at ∼14.6 MPa, also approximately representing the native overburden stress. The initial horizontal stress was ∼9.6 MPa. The pore pressure was stepwise incremented to an initial pressure of ∼7.5 MPa, attaining equilibrium sorption at each step and avoiding mechanical failure. During gas depletion, the horizontal stress decreases from its peak as pore pressure decreases and zero horizontal strain is maintained. Details in the experimental setup and procedure are also described elsewhere.[32]

Elastic and sorptive properties for these coal samples from the San Juan Basin are listed in Table 3. Figure 12 shows both the measured and modeled volumetric strains, as a function of pore pressure during methane depletion and under uniaxial strain conditions. The modeled (eqs 26 and 18) data are in excellent agreement with the experimental observations: as methane pressure declines, the coal shrinks due to the combined mechanical and (de)sorptive effects. In addition, these experiments confirm that the effective stress coefficient is independent of the applied mechanical boundary conditions—implied from the theoretical expression derived under uniaxial conditions. The variation in the volumetric strain under uniaxial strain conditions is controlled by the constrained axial modulus

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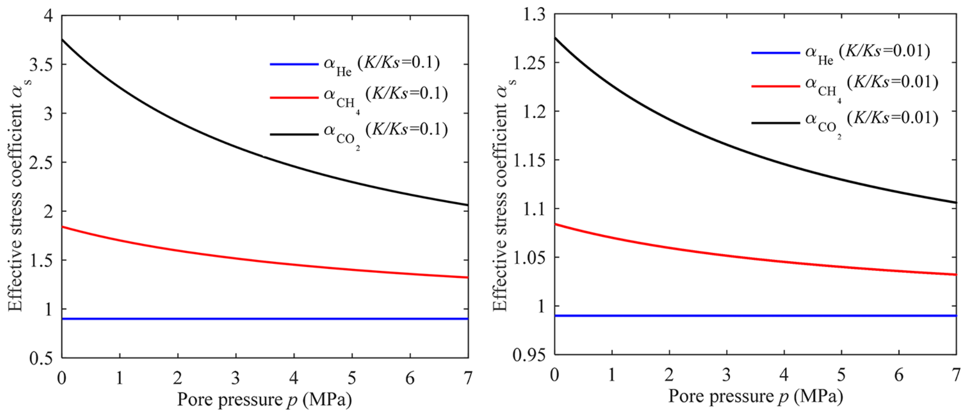

Figure 11. Variation of effective stress coefficient for coal of different stiffnesses.

Table 3. Elastic and Sorptive Properties for Coal

elastic properties	data source	range	selected value
Young's modulus, E (MPa)	ref 28	2070−4140	2500
Poisson's ratio, (ν)	ref 28	0.23−0.4	0.25
grain modulus, K_s (MPa)	ref 21	5075	5075
Langmuir volumetric strain, ε_l	ref 29	0.01075	0.01075
Langmuir pressure, P_L (MPa)	ref 29	4.15	4.15

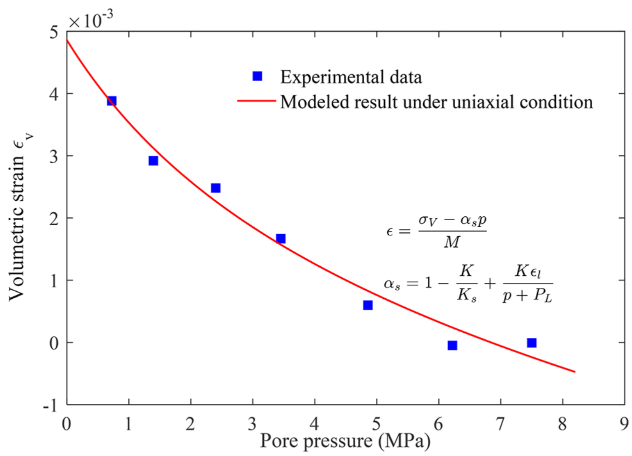

Figure 12. Experimental validation of effective stress law accommodating sorption-induced swelling under uniaxial strain conditions.

and its effective stress, which, in turn, is determined by the pore pressure, overburden stress, and the unified effective stress coefficient. The effective stress relation accommodating the sorption effect under unconstrained conditions may be straightforwardly transformed from that under uniaxial strain conditions, and vice versa.

5. CONCLUSION

This study defines the form and magnitude of a revised effective stress coefficient accommodating sorption-induced swelling, using the concept of a "swelling modulus". This enables the stress−strain relations to be uniquely defined for sorbing porous media in a manner directly analogous to that for linear nonsorbing media. The key single environmental and dependent variable is effective stress. This extended effective stress law accommodates sorption-induced swelling under different mechanical boundary conditions and is rigorously derived and validated via experimental observations. Some main conclusions are described as follows.

The swelling modulus and pore pressure determine sorption-induced swelling. Therefore, the three variables of external stress, pore pressure, and sorption-induced swelling are transformed to a single key variable representing a true "effective stress". The volumetric deformation of the sorbing medium is attributed exclusively to the change of this effective stress and follows the proposed effective stress law as $\varepsilon_v = \sigma_{effective}/K$, with $\sigma_{effective} = \sigma - \alpha_s p$ and $\alpha_s = 1 - K/K_s + K/Z_p$.

The effective stress law is independent of mechanical boundary conditions. In other words, the "effective stresses" may be transformed between different mechanical boundary conditions with a unified effective stress coefficient. Thus, for example, effective stress under uniaxial strain can be expressed as a function of overburden stress and pore pressure as $\sigma_{effective} = \sigma_V - \alpha_s p$, defining changes in volumetric strain as $\varepsilon_v = \sigma_{effective}/M$. Importantly, the effective stress coefficient is a true material property and independent of system dimensionality (1D, 2D, 3D) and mechanical constraint (full-lateral-constraint, plane strain, unconstrained).

At low gas pressures (<7 MPa), the swelling modulus (Z_p) is dependent on the particular gas (CH_4/CO_2) and gas pressure and is an order of magnitude smaller than the bulk modulus of the solid grains (K_s). Importantly, this explains why sorption-induced swelling dominates the volumetric strain response at low reservoir pressures. Because of the lower swelling modulus, CO_2 generates larger swelling strains than CH_4. It is clear from this analysis that the swelling modulus directly reflects the capacity for swelling/shrinkage during the injection/depletion of sorbing gases.

The revised effective stress coefficient reflects the sensitivity of the volumetric strain to external stress and pore pressure. This volumetric strain is sensitive to pore pressure for $\alpha > 1$ and sensitive to external stress for $\alpha < 1$. The effective stress coefficient for nonsorbing/lightly sorbing helium remains constant at α_{He} less than ∼1 for both "soft" and "stiff" sorbing media. Effective stress coefficients accommodating sorption-induced swelling for methane and carbon dioxide are >1 and decrease with gas pressure. For relatively "stiff" sorbing media, α_{CO_2} and α_{CH_4} are much larger than unity and decline more rapidly with an increase in gas pressure, compared to relatively "soft" sorbing media, where $\alpha_{CO_2} \approx \alpha_{CH_4} \approx 1$, where the decline is less rapid with gas pressure.

Overall, this study develops a new means to codify the role of sorption-induced swelling into a material-dependent and

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geometry and deformation constraint-independent model of effective stress. This is achieved by defining a "swelling modulus" and its contribution into a revised effective stress coefficient for sorbing media. The validity of this approach is confirmed through a series of experimental observations over a limited range of confining pressures and pore pressures. This range may be extended in the future to explore the validity of the revised effective stress law at elevated pressures and stresses, and directly linked to the thermodynamics of sorption/deformation interactions.

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Notes

The authors declare no competing financial interest.

■ NOMENCLATURE

σ_{ij} = component of the total stress tensor, MPa
σ_{ij}^{e} = component of the effective stress tensor, MPa
δ_{ij} = the Kronecker delta, dimensionless
σ = external stress, MPa
p = pore pressure, MPa
α = effective stress coefficient, dimensionless
E = Young's modulus, MPa
ν = Poisson's ratio, dimensionless
K = bulk modulus, MPa
G = shear modulus, MPa
H = new effective modulus, defined in this work, MPa
K_s = bulk modulus of solid grains, MPa
K_p = bulk modulus for the pore volumetric strain, MPa
Z_p = swelling modulus, MPa
M = constrained axial modulus, MPa
ε_{ij} = component of the strain tensor, dimensionless
ε_v = volumetric strain, dimensionless
ε_s = sorption-induced volumetric strain, dimensionless
ε_l = Langmuir volumetric strain, dimensionless
P_L = Langmuir pressure, MPa
σ_V = overburden stress, MPa
σ_L = lateral (horizontal) stresses, MPa
C_s = grain compressibility, MPa^{-1}
α_s = effective stress coefficient accommodating sorption-induced swelling, dimensionless
α_{CH_4} = effective stress coefficient for methane, dimensionless
α_{CO_2} = effective stress coefficient for carbon dioxide, dimensionless
α_{He} = effective stress coefficient for helium, respectively, dimensionless

Subscripts

i, j, and k = coordinate indices, with values of 1−3
I = first stage of applied stress
II = second stage of applied stress

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