



# Power law dependence of unstable slip velocity on decreasing shear loading stiffness

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## ABSTRACT

Shear loading stiffness plays a critical role in conditioning the stability of slip on reactivated faults. However, a relationship linking peak slip velocities and shear loading stiffness is lacking. To explore this, we shear granite faults in double direct shear with shear loading stiffnesses spanning two orders to define the effects of shear loading stiffness in conditioning the transition from stable to unstable slip. Our results show that peak slip velocity and acceleration decrease as power law relationships with respect to the shear loading stiffness ratio, with identical exponent, and with a linear relationship between the peak slip velocity and acceleration. The maximum acceleration occurs during the velocity weakening process that limits the peak slip velocity. The power law exponent increases linearly with increasing normal stress. Experimental results also indicate that slip magnitude, stress drop and recurrence time of unstable slip events increase, and slip durations decrease with decreasing stiffness ratios. The spans of limit cycles of velocity versus shear stress decrease, and their shapes evolve from triangular to semicircular with increasing loading stiffness ratio. Stress drops mostly occur during deceleration. The deceleration phase dominates the unstable slip duration that decreases as peak slip velocity increases. Our results indicate that the average stress drop rates over a slip duration increase as a power law relationship with reducing shear loading stiffness, which also contributes to a lower shear loading stiffness producing increased slip velocities and accelerations. Our findings highlight that loading stiffness ratio is an underlying mechanism defining unstable slip behaviors with the normal stress merely conditioning the exponent. The present relationship of unstable slip velocity with shear loading stiffness suggests a way to evaluate hazard of an impending instability event based on the initial shear loading stiffness ratio that can be calculated through the linear-elastic behaviors at the early quasi-static phase.

## 1. Introduction

Frictional slip on laboratory faults show a spectrum of responses that transit from slow to fast slip (Kaproth and Marone, 2013; Leeman et al., 2018) and mimic the response of natural faults. Slip velocities of natural faults range from several millimeters per year to tens of meters per second. After this instability transition of frictional slip, sliding accelerates unstably before finally decelerating to a new quasi-static state (Gu and Wong, 1991; Sone and Shimamoto, 2009; Im et al., 2017; Gu et al., 2023). Although some mechanisms, such as dilatant hardening (Segall et al., 2010; Samuelson et al., 2011) or a transition in frictional constitutive behavior with increasing slip velocity (Ghosh et al., 2012; Ikari et al., 2013; Kaproth and Marone, 2013; Im et al., 2020), are suggested

to limit slip velocity, controls on the maximum slip velocity or acceleration in unstable sliding remain undefined.

Peak slip velocity and rupture speed are two physical parameters characterizing earthquakes and constraining kinematic model of faults. Faults can attain subshear through supershear propagation speeds (Xia et al., 2004, 2005; Schubnel et al., 2011; Tal et al., 2022). Laboratory experiments (Ohnaka, 1973; Ohnaka et al., 1987) and numerical results (Bizzarri, 2012) indicate the interdependence between the peak slip velocity and rupture speed. Peak slip velocity in unstable slip of faults is a fundamental dynamic parameter defining an earthquake, and is related to the magnitude of the event and the ground velocity (Bizzarri, 2012). Peak slip velocity and its timing provide clues to estimate the critical slip weakening distance and stress breakdown time (Mikumo

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et al., 2003; Tinti et al., 2004, 2009), although mechanisms that govern these are poorly constrained. Despite reports on the significance of peak slip velocity to ground motion prediction and hazard assessment (Bizzarri, 2012; Passelègue et al., 2016; Yao and Yang, 2023), the underlying processes defining the peak slip velocity remain elusive.

The criterion for slip instability depends on the ratio between the elastic stiffness of the loading system and the critical weakening rate (stiffness) of the fault (Ruina, 1983; Rice and Ruina, 1983; Gu et al., 1984). When the shear stress reaches the static frictional strength of the fault, slip initiates and the stored elastic energy surrounding the fault is released. Experimental and numerical results (Goodman and Sundaram, 1978; Gu and Wong, 1991; Kaproth and Marone, 2013; Leeman et al., 2015, 2016; Scuderi et al., 2017) indicate that the stiffness of the loading apparatus controls the development of unstable slip. Observations (Kaproth and Marone, 2013; Leeman et al., 2015, 2016) show that the full spectrum of fault slip behaviors, from slow to fast stick-slip, can be reproduced by varying the shear stiffness ratio of the loading frame to the fault with peak slip velocities changed from 50 to 100  $\mu\text{m}$  per second to several millimeters per second. Although many studies indicate the dependence of peak slip velocities on the shear loading stiffness (Kaproth and Marone, 2013; Leeman et al., 2015; Tinti et al., 2016; Chen, 2023), no clear description of this relationship exist.

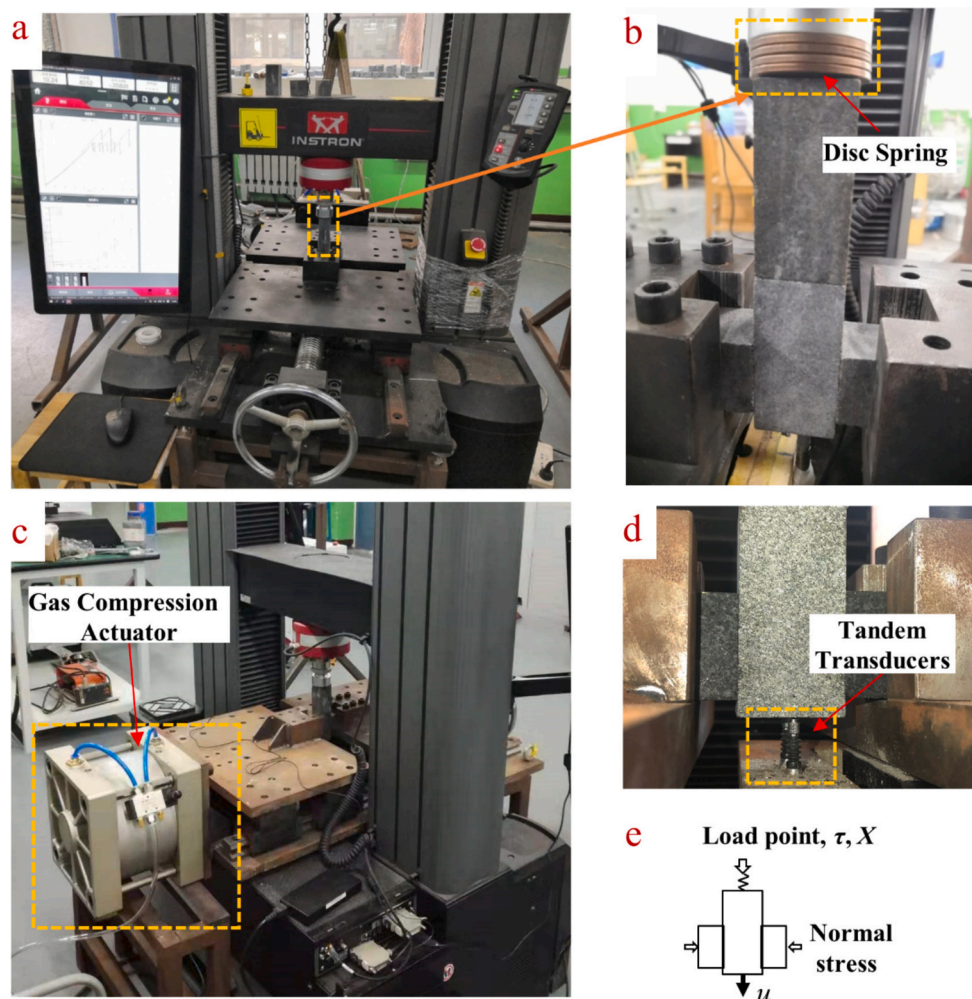
We develop a quantitative description linking the dependence of peak slip velocity and acceleration to this loading stiffness ratio based on

systematic laboratory observations. Double direct shear tests are conducted with varying shear loading stiffnesses that span two orders of magnitude. Limitations on slip velocity are demonstrated by comparing evolutions of shear stresses against sliding velocities for different shear loading stiffnesses. From these observations, we quantify relationships between peak slip velocities and stress drop, slip duration and recurrence times.

## 2. Experimental materials and methods

Fault reactivation experiments are reported on granite interfaces in a servo-controlled double-direct shear apparatus at room temperature and humidity (Fig. 1). As shown in Fig. 1b, the simulated fault system comprises three precision-ground and polished blocks allowing a central forcing block to be sheared between two stationary side blocks. The dimensions of the side blocks are 40 mm  $\times$  36 mm  $\times$  40 mm with a central sliding block 40 mm  $\times$  36 mm  $\times$  80 mm. The nominal frictional contact area is 36 mm  $\times$  40 mm. X-ray diffraction (XRD) analysis shows that the granite sample comprises 46.5% diopside, 19.5% albite, 33.9% anorthite, and 0.2% quartz. The sample has a mean crystal size of  $\sim$ 680  $\mu\text{m}$  (Hao et al., 2017). The uniaxial compressive strength is  $\sim$ 222 MPa, and the longitudinal P-wave velocity is  $\sim$ 6.7 km/s.

Normal stresses are applied horizontally by a gas compression actuator to the side blocks and shear stresses are applied vertically to the



**Fig. 1.** Testing setup. (a) Double-direct shear test system. (b) Zoom-in on the sample and the disc springs. Disc spring (yellow dotted box in panel b) present between the moving platen and the sliding granite block. (c) Gas compression actuator (yellow dotted box) used to impose normal stresses. (d) Two tandem transducers (yellow dotted box) monitoring the slip displacement,  $u$ . (e) Schematic of double-direct shear configuration. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

central blocks in an Instron 5982 testing machine. The current experiments simulate the condition where the energy release from the elastic environment surrounding the fault drives unstable slip. We posit that a decrease in normal stress reduces the critical stiffness of the fault and thus enhances stability (Im et al., 2017). Thus, initial stiffness ratio and normal stress are two key factors that determine the stability of slip. Our previous tests (Gu et al., 2023) demonstrate that for the loading stiffness of this system, a normal stress of 3.4 MPa produces significant stick-slip cycles. Prior experiments by others produce stick slip when normal stress is  $>1.5$  MPa (Scholz et al., 1972) and also in the range 5–7 MPa (Im et al., 2017) and are even more common at higher normal stresses (Scholz et al., 1972; Tullis, 1996). Consequently, Reactivation experiments are performed at three normal stresses,  $\sigma_n = 3.4, 6.8$  and 10.2 MPa with the central block driven at constant load point velocity,  $V_L = 1$  mm/min. In the current experiments, the maximum normal stress of 10.2 MPa is a moderate stress for stick-slip experiments in the lab.

Shear force is monitored by an Instron 5982 Series load cell. This load cell comprises a full strain gauge bridge bonded to a stiff and linear elastic element. The load point displacement  $X$  is the crosshead displacement of the Instron testing machine that is measured by the Instron digital displacement sensor. This is a contacting transducer with low contact force and low friction to allow accurate measurement of

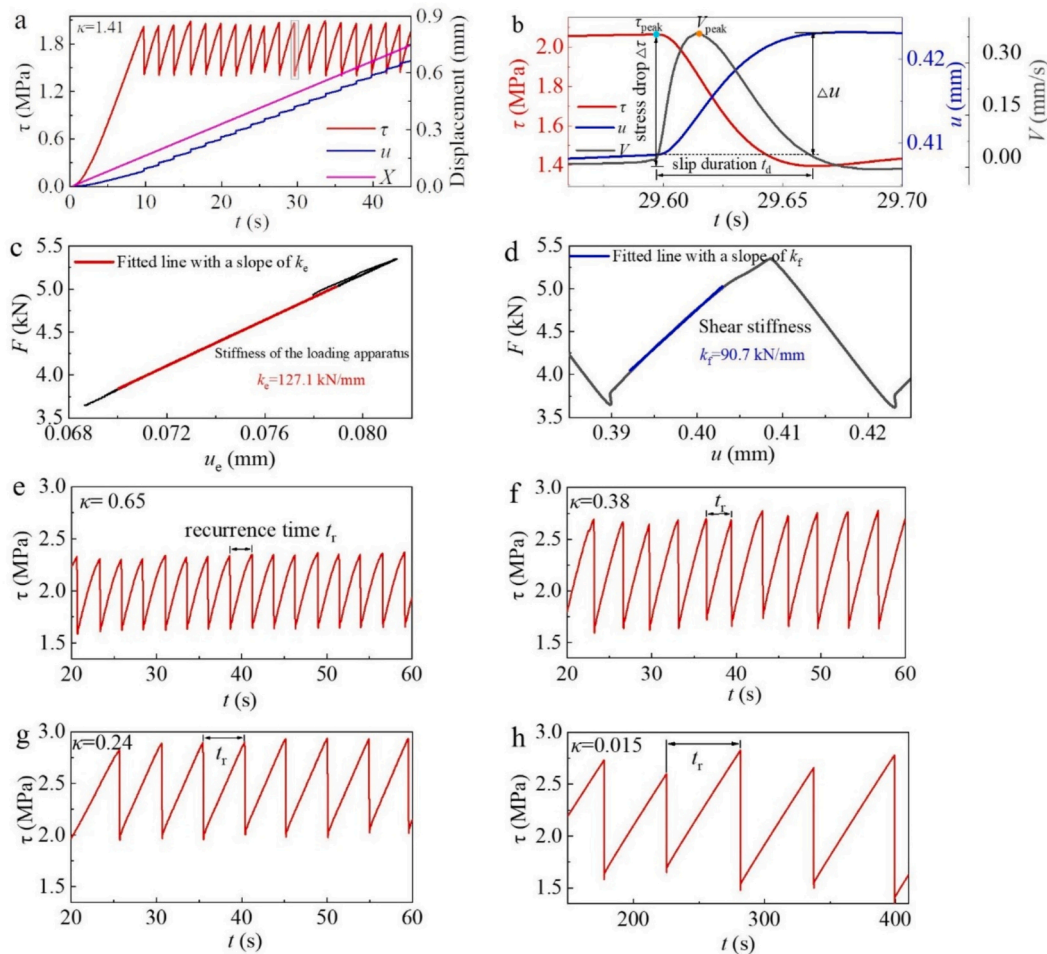
linear displacement with respect to the reference surface. The sliding displacement,  $u$ , is recorded as the average of two tandem transducers (ZX-TDS04T high accuracy contact-type produced by OMRON Corporation) with a resolution of  $0.1 \mu\text{m}$  that are attached under the central sliding block (Fig. 1). Shear force  $F$ , load point displacement  $X$ , and sliding displacement  $u$  are monitored and acquired at a sampling rate of 1000 Hz.

Shear loading stiffness is changed through a variable-height stack of disc springs (yellow dotted box in Fig. 1b) bridging the gap between the moving platen and the sliding granite block. We focus on the peak velocity of unstable slips with the stiffnesses of the loading system less than the critical stiffness required to produce regular stick-slip cycles.

### 3. Experimental results

#### 3.1. Unstable slip behaviors defined by the shear loading stiffness

The shear loading stiffness is a key factor determining the sliding condition as either stable or unstable. Our previous experiments (Gu et al., 2023) illustrate that the present test system, with a stiffness of  $127.1 \text{ kN/mm}$  produces regular stick-slip cycles even under low normal stresses of 3.4 MPa. In order to investigate the relationship between peak



**Fig. 2.** Typical stick-slip results for five stiffness ratios. Slip duration  $t_d$  is the elapsed time between the maximum and minimum stresses within the stick-slip cycle.  $\Delta u$  is the sliding displacement increment between these two points.  $\tau_{\text{peak}}$  is the peak shear stress,  $V_{\text{peak}}$  is the peak slip velocity in an individual stick-slip event.  $t_r$  is recurrence time, defined as the time interval between two successive stick-slip events. (a) Evolution of shear stress  $\tau$ , load point displacement  $X$ , and sliding displacement  $u$  at a stiffness ratio  $\kappa = 1.41$ . (b) Zoom-in on a typical stick-slip event. Stress drop  $\Delta\tau$  is defined as the difference between the maximum and minimum stress. (c) Determination of the stiffness of the loading apparatus,  $k_e$ .  $k_e$  is defined as the slope of the linear-elastic portion of shear force  $F$  versus the deformation of the loading apparatus  $u_e$  in a stick-slip event. (d) Determination of initial shear stiffness  $k_f$  of the frictional surfaces.  $k_f$  is defined as the slope of the linear-elastic portion of  $F$ - $u$  in the loading process of an individual stick-slip event. (e-h) Stick-slip cycles for  $\kappa = 0.65, 0.38, 0.24, 0.015$ .  $\kappa$  is the stiffness ratio defined as the ratio of  $k_e$  to  $k_f$ .

unstable velocities and the loading stiffness, we perform experiments at this system stiffness and below to guarantee stick-slip events for all stiffnesses. Frictional sliding experiments are conducted under a broad range of shear loading stiffnesses ranging from 1.6 to 127.1 kN/mm, spanning two orders of magnitude. Fig. 2 shows plots of typical stick-slip cycles for different loading stiffness ratios. In these experiments, the slip durations of individual unstable slip events range from 0.053 s to 0.093 s. Thus, we use a high sampling rate of 1000 Hz to capture the rapid onset and short duration stick-slip events. Force versus sliding displacement of all stick-slip events in each experiment show similar evolution. Fig. 2 zooms-in on each experiment under different stiffnesses to illustrate the congruence of their evolution.

In order to clearly illustrate the effects of the shear loading stiffness on stick-slip behaviors, some fundamental physical quantities are defined as shown in Fig. 2. In an individual stick-slip event (Fig. 2b), the shear stress drop  $\Delta\tau$  is the stress difference, the slip duration  $t_d$  is the time difference and  $\Delta u$  is the sliding displacement spanning the displacements at maximum and minimum shear stress. The shear loading stiffness of the apparatus  $k_e$  is defined as the slope of the linear-elastic portion of the curve of shear force ( $F$ ) versus the deformation ( $u_e$ ) of the loading apparatus (Fig. 2c), where  $u_e = X \cdot u$ . The initial shear stiffness  $k_f$  of the frictional surfaces is measured as the slope of the linear-elastic portion of the  $F$ - $u$  load curve (Fig. 2d). The stiffness ratio  $\kappa$  is defined as the ratio of  $k_e$  to  $k_f$  as.

$$\kappa = k_e/k_f. \quad (1)$$

Finally, recurrence time  $t_r$  is calculated as the time interval between two successive stick-slip events in the sequence (Figs. 2e-h).

Fig. 3 shows shear stress, sliding displacement, velocity and acceleration in each individual stick-slip event for different stiffness ratios. In the current experiments, stick-slip events are of slip durations 0.053 to 0.093 s and with average sliding displacements in individual cycles of 0.015 to 1.157 mm, and evolving from small to large events with decreasing stiffness ratios. Shear stress drops are 0.66 to 1.34 MPa and the recurrence time is 2 to 56.4 s in stick-slip events. Resolutions of 0.1  $\mu\text{m}$  for slip displacement and 1 N ( $\sim 6 \times 10^{-4}$  MPa) for the loading are sufficient to record the evolution of the various stick-slip cycles. For all three normal stresses of  $\sigma_n = 3.4, 6.8$  and 10.2 MPa, both shear stress drops and recurrence time increase with decreasing stiffness ratios (Figs. 2 and 3, Appendix Figs. 1 and 2).

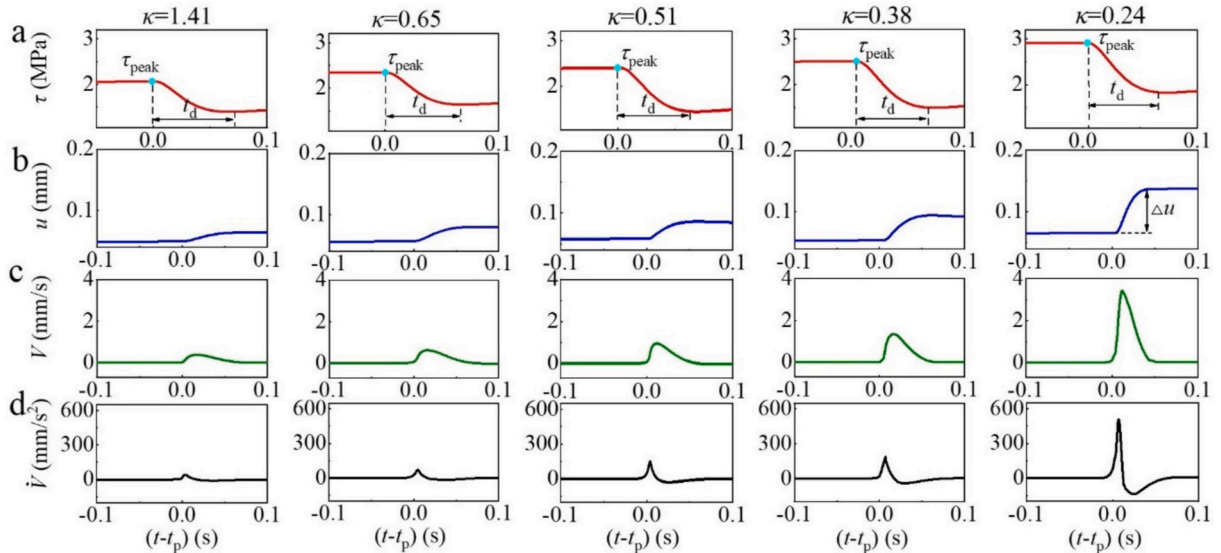


Fig. 3. Representative unstable slip events for five stiffness ratios when the normal stress is 3.4 MPa. (a) Shear stress  $\tau$ . (b) Sliding displacement  $u$ . (c) Slip velocity  $V$ . (d) Slip acceleration  $\dot{V}$ .  $t_p$  is the time of peak shear stress in an individual stick-slip event.

### 3.2. Power law dependence of peak slip velocity and acceleration on shear loading stiffness

For different shear loading stiffnesses, each unstable slip event undergoes a similar sliding process from acceleration to deceleration (Fig. 3, Appendix Figs. 1 and 2). However, the peak slip velocity decreases from 3.43 to 0.38 mm/s and peak acceleration decreases from 508 to 40 mm/s<sup>2</sup> as the loading stiffness ratio is increased from 0.24 to 1.41. Thus, for a change in stiffness ratio spanning two orders, changes in peak slip velocity and acceleration span two orders in magnitude.

Fig. 4 shows how peak slip velocities ( $V_{\text{peak}}$ ) and accelerations ( $\dot{V}_{\text{peak}}$ ) respond to loading stiffness ratios at normal stresses of 3.4, 6.8 and 10.2 MPa. These experimental results indicate that both peak slip velocity and acceleration show power law dependences to loading stiffness ratios as.

$$\nu = C_v \kappa^{-\gamma_v} \quad (2)$$

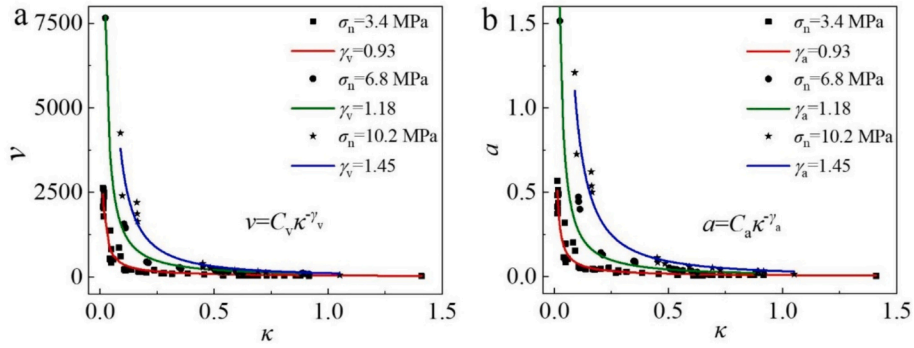
and

$$a = C_a \kappa^{-\gamma_a}. \quad (3)$$

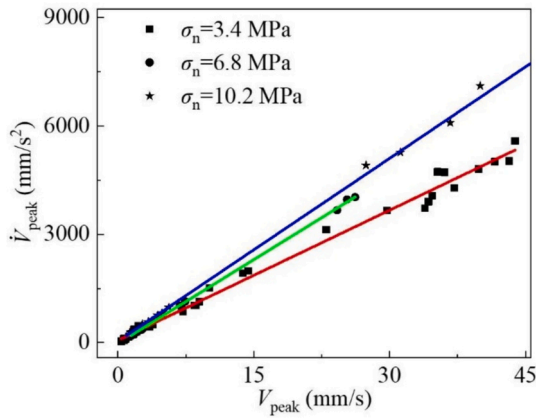
Where,  $\nu = V_{\text{peak}}/V_L$  is the normalized peak slip velocity with  $V_L$  and  $a = \dot{V}_{\text{peak}}/g$  is the normalized peak slip acceleration with gravitational acceleration,  $g = 9.8 \text{ m/s}^2$ .  $C_v$  and  $C_a$  are two constants. Interestingly, dependences of peak slip velocity and acceleration on the loading stiffness ratio share the same power law exponent for the same normal stress:  $\gamma_v = \gamma_a = 0.93$  for  $\sigma_n = 3.4$  MPa, 1.18 for  $\sigma_n = 6.8$  MPa, and 1.45 for  $\sigma_n = 10.2$  MPa, respectively (Fig. 4). This is confirmed by the linear relationship of peak slip velocities and accelerations shown in Fig. 5. Values of exponents ( $\gamma_v$  and  $\gamma_a$ ) increase linearly with normal stresses (Appendix Fig. 3).

### 3.3. Stress drop with velocity and acceleration

Fig. 6 illustrates the evolution of sliding velocities and accelerations with shear stresses under different shear loading stiffness ratios. The spans of limit cycles of velocity versus shear stress decrease, and their shapes evolve from triangular to semicircular with increasing stiffness ratio (Figs. 6a-c). These indicate that during each stick-slip cycle in unstable slip, the shear stress originally drops as convex upwards as slip velocity increases, before drops as convex-down as slip velocity decreases (Figs. 6a-c). Stress drops mostly occur during deceleration



**Fig. 4.** Power law relationships between normalized peak (a) slip velocity and (b) acceleration with stiffness ratios under different normal stresses. Peak slip velocity and acceleration share the same exponent for the same normal stress. Values of exponents increase with normal stresses. Scattered experimental data (symbols) fitted by best-fit solid lines.



**Fig. 5.** Peak slip acceleration versus peak slip velocity. Scattered experimental data (symbols) fitted by best-fit solid lines.

(Figs. 6d-f).

### 3.4. Stick-slip period and unstable slip duration

With decreasing stiffness ratio, both stress drops and recurrence time increase, but slip duration decreases (Figs. 7a-c). Slip duration decreases as  $V_{\text{peak}}$  increases (Fig. 7d). The slip duration is much shorter than the recurrence time in each stick-slip event. Thus, differences in stick-slip

periods under different stiffness ratios are dominated by differences in healing times.

## 4. Discussion

### 4.1. Energy release from the loading apparatus accelerates slip

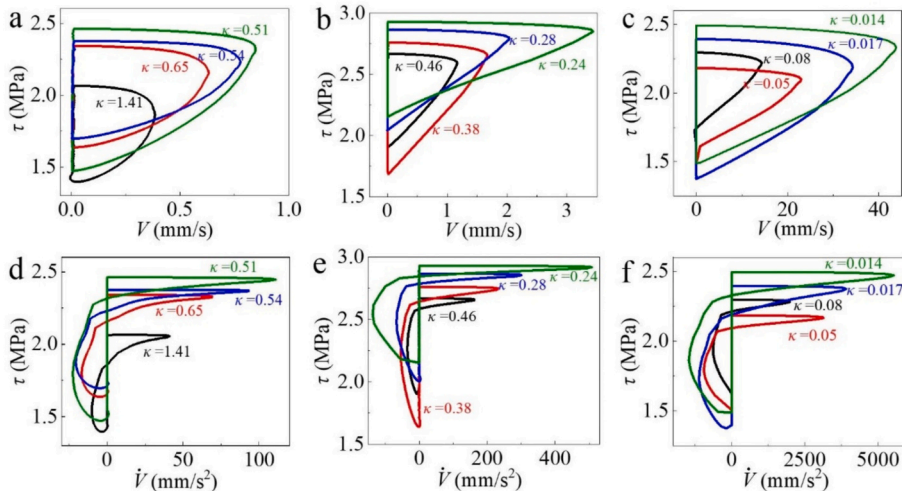
Post-peak shear force, the shear loading apparatus releases its stored strain energy to accelerate the sliding. The sliding displacement,  $u$ , is the difference between the load point displacement ( $X$ ) and the deformation ( $u_e$ ) of the loading system as.

$$u = X - u_e. \quad (4)$$

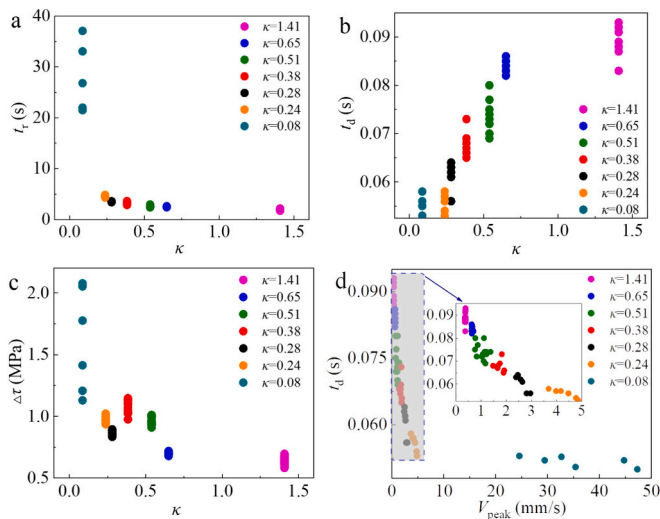
Differentiating Eq. (4) with respect to time gives

$$V = V_L - V_e = V_L + (-\dot{F})/k_e. \quad (5)$$

Where,  $V_e$  is deformation release velocity,  $\dot{F}$  is shear force rate as a proxy for stress drop rate.  $V_L$  is constant in the current experiments. Post-peak force, both  $V_e$  and  $\dot{F}$  are negative since the stored strain energy is released from the loading apparatus. Eq. (5) indicates that reducing  $k_e$  results in a faster slip for the same stress drop. This is the underlying mechanism that results in a higher slip velocity (or acceleration) for a lower shear loading stiffness. Fig. 8 illustrates evolutions of average stress drop rates ( $\Delta\tau/t_d$ ) and average slip velocities ( $\Delta u/t_d$ ) versus loading stiffness ratios when the normal stress is 3.4 MPa. The ratio  $\Delta\tau/t_d$  increases faster when loading stiffness ratio is reduced (Fig. 8a)



**Fig. 6.** Phase diagram showing shear stress evolution against sliding (a-c) velocity and (d-f) acceleration during stick-slip cycles under different stiffness ratios.



**Fig. 7.** Parameters defining unstable slips. (a) Recurrence time, (b) slip duration and (c) stress drop as a function of stiffness ratios. (d) Plots of slip duration as a function of peak slip velocity. The inset shows a zoom-in of the shaded area.

also contributing to a lower shear loading stiffness producing increased slip velocities and accelerations. Both normalized average stress drop rates  $((\Delta\tau/t_d)/(\Delta\tau/t_d)_{\max})$  and average slip velocities  $((\Delta u/t_d)/V_L)$  decrease as a power law relationship with the shear loading stiffness ratio (Fig. 8).

#### 4.2. End-member response at high and low shear loading stiffness ratios

We observe an empirical relationship between unstable slip velocity and shear loading stiffness ratio. Our stiffness ratios cover a limit range. However, it is evident that when the stiffness ratio is very large, so that no unstable slip occurs, then the slider will move at the load point velocity. Thus, the unstable slip velocity is zero. Our experiments are restricted to the unstable slip regime – thus, the slip velocity during unstable slip phase is much greater than prescribed load point velocity. The power law relationship of Eq. (2) describes the dependence of unstable slip velocity on the loading stiffness ratio, which is insensitive to load point velocity. Prediction of the relationship of Eq. (2) also accommodates the case where the unstable slip velocity is zero when the stiffness ratio is very large.

The current experimental results do not probe the velocity response for very low stiffness ratios – as these were unreachable in our current experimental configuration. However, these much approach the load point velocity, in the limit, as a consequence of stable sliding. Also, when the stiffness ratio is very low, the unstable slip velocity or acceleration

cannot tend to infinity as predicted by Eqs. (2) and (3) when the stiffness tends to zero – as this is limited by the inertial response. The slip velocity should tend to a limiting value under very low stiffness ratios.

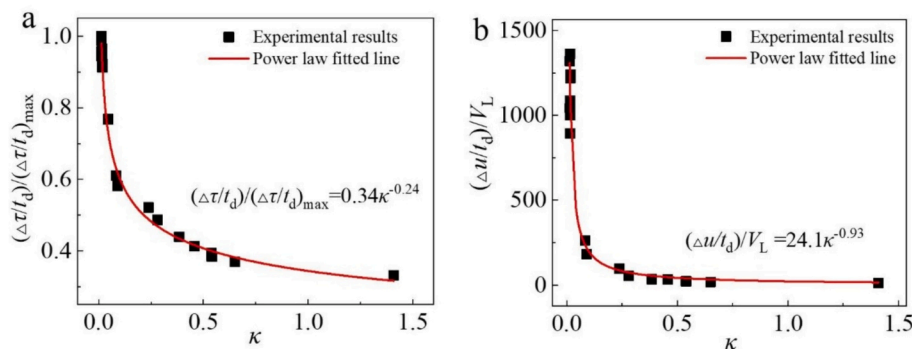
#### 4.3. Linear relationship between peak slip velocities and accelerations

Our results show that parameters characterizing earthquake magnitude, such as stress drop, slip displacement, slip duration and stick-slip recurrence time are directly related to peak slip velocity. Our findings, that peak slip velocities have a linear relationship with peak accelerations, suggest the limitation of peak acceleration is a mechanism that limits the unstable slip velocity and the magnitude of earthquake events. Acceleration is defined by the inertial force that is also conditioned by an effective mass (of the slider). By definition, the inertial force is the difference between the applied shear and reactive frictional forces. Evolutions of slip accelerations (Figs. 6d-f) imply that the inertial force undergoes an evolution from first increasing to then decreasing during the unstable slip cycle. Some mechanisms have been proposed to explain this limitation on slip velocity (Segall et al., 2010; Knaproth and Marone, 2013; Ikari et al., 2013; Im et al., 2020). Our results show that the maximum acceleration occurs during the velocity weakening process (Fig. 6) that limits the peak slip velocity. The slip acceleration is determined by interactions between the decreasing shear force and frictional weakening. Our findings suggest that the maximum difference between shear and frictional forces in unstable slip determines the peak slip velocity. This is a fundamental cause limiting slip velocity.

#### 4.4. Implications of experiments results

Our experiments explore the dependence of parameters characterizing the occurrence of slip instability events. These include the influence of magnitude, slip duration, recurrence time, stress drop, average stress rate, peak velocity and peak acceleration as conditioned by the shear loading stiffness. These systematic and constrained experiments illuminate the mechanisms responsible for unstable slip. The observed events span a wide range of fault stiffnesses and thus are applicable to a wide variety of natural fault conditions.

The current experiments show that shear loading stiffness exerts an essential control on the velocity of the unstable slips and on the stress drop magnitude of these events. The influence of the shear loading stiffness on peak slip velocity has been previously observed (Knaproth and Marone, 2013; Leeman et al., 2015, 2016). However, we provide a description of the dependence of peak slip velocity on the shear loading stiffness and thus further define the actual relationship. These findings thus illuminate the mechanisms controlling the slip velocity and stress drop magnitude of the unstable events. In this study, peak slip velocities range from 0.38 to 43.85 mm/s, spanning two orders of magnitude. The experimental results are consistent with stick-slip velocities inferred for regular, fast earthquakes (Tinti et al., 2016; Mai and Thingbaijam,



**Fig. 8.** Power law relationships of (a) normalized average stress drop rates  $(\Delta\tau/t_d)/(\Delta\tau/t_d)_{\max}$ , and (b) normalized average slip velocities  $(\Delta u/t_d)/V_L$  with stiffness ratios when the normal stress is 3.4 MPa.

2014). This means that the key processes that generate stick-slip in laboratory experiments may be operational in natural faults. Thus, these results provide insight in understanding and interpreting why natural earthquakes occur with a broad spectrum of slip velocities and show different stress drop magnitudes, which remain enigmatic.

We define a relationship linking peak slip velocity and shear loading stiffness. This may serve as a guidance in designing experiments, numerical simulations and model development to explore the nature of unstable slip. Theoretical and experimental results (Kaproth and Marone, 2013; Leeman et al., 2015, 2016; Kilgore et al., 2017) show the coupled effects of the shear loading stiffness and normal stress in defining the sliding behavior. Our results define the different effects of normal stress and the shear loading stiffness on unstable slip.

## 5. Conclusions

Experiments of unstable slip on laboratory faults exhibit a significant dependence on the shear loading stiffness. With decreasing stiffness ratios, all of slip magnitude, stress drop and recurrence time of unstable slip events increase, but slip duration decreases. Slip durations show smaller differences than recurrence time and are much shorter than static-stick duration, and thus static-stick duration dominates the recurrence time. Average stress drop rates and average slip velocities increase with decreased shear loading stiffness. Notably, peak slip velocity and acceleration increase with decreased loading stiffness ratio as a power law relationship. The power law exponent is identical for both peak slip velocities and accelerations, but increases linearly with increased normal stress. Peak slip velocities are linearly related to peak accelerations. The maximum acceleration occurs during the velocity weakening process.

Each unstable slip event undergoes a similar sliding process from acceleration to deceleration, and its duration is dominated by the deceleration phase. The spans of limit cycles of velocity versus shear stress decrease, and their shapes evolve from triangular to semicircular with increasing loading stiffness ratio. Phase diagrams of shear stress evolution against sliding acceleration in stick-slip cycles show that stress

drops mostly occur in the deceleration process. A general energy analysis indicates that reducing shear loading stiffness results in a faster slip – for the same stress drop, which is the underlying mechanism that results in a higher slip velocity (or acceleration) for a lower shear loading stiffness. The average stress drop rates over a slip duration increase as a power law relationship with reducing shear loading stiffness, which also contributes to a lower shear loading stiffness producing increased slip velocities and accelerations.

## CRedit authorship contribution statement

**Lu Gu:** Writing – original draft, Methodology, Investigation, Formal analysis, Data curation. **Shengwang Hao:** Writing – review & editing, Supervision, Resources, Project administration, Funding acquisition. **Derek Elsworth:** Writing – review & editing, Supervision.

## Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

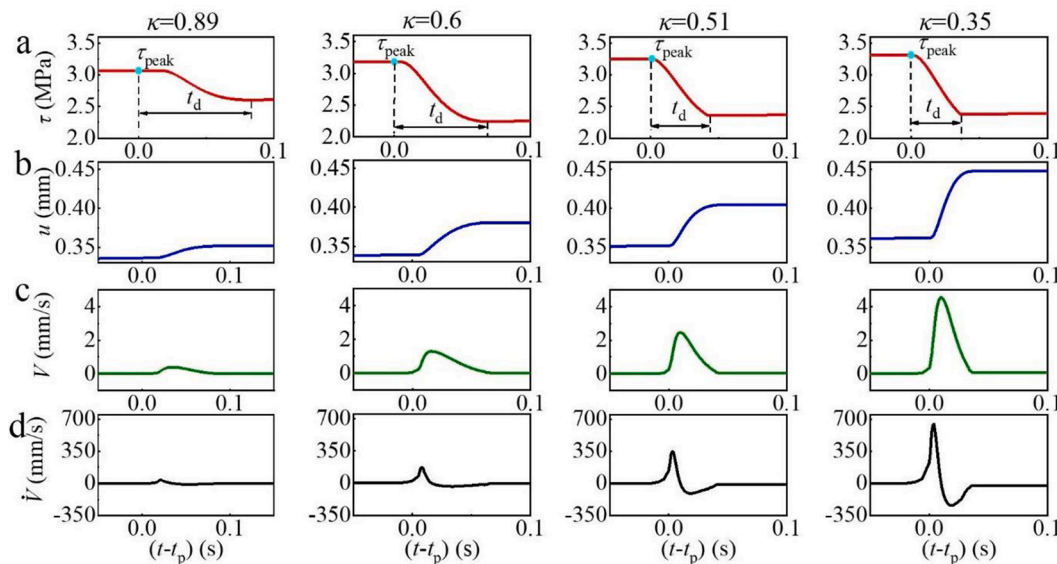
## Data availability

Data will be made available on request.

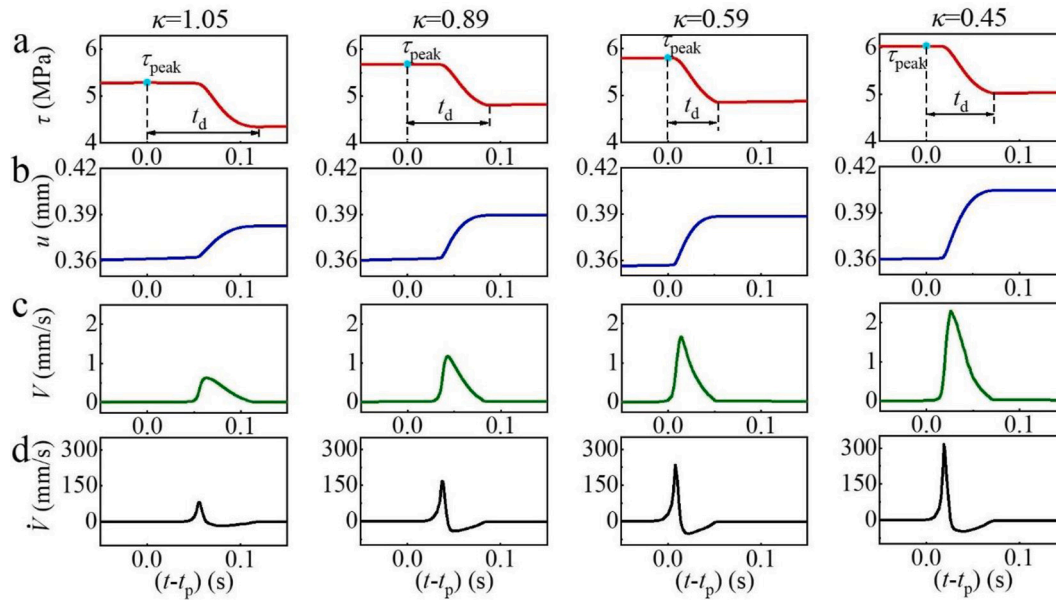
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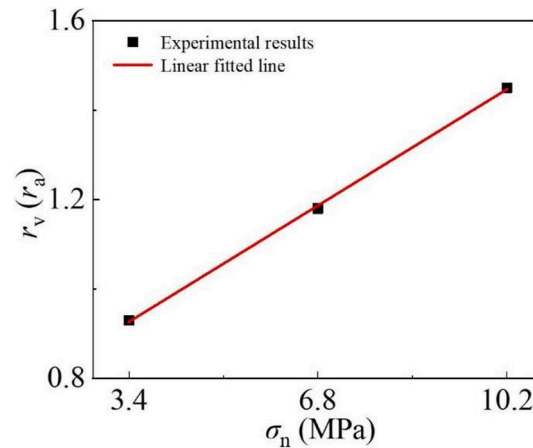
## Appendix A. Unstable slip results of variable shear loading stiffness when normal stresses were 6.8 and 10.2 MPa, and the relationship of power law exponents versus normal stresses



**Appendix Fig. 1.** Representative unstable slip events for four stiffness ratios when the normal stress was 6.8 MPa. (a) Shear stress  $\tau$ . (b) Sliding displacement  $u$ . (c) Slip velocity  $V$ . (d) Slip acceleration  $\dot{V}$ .  $t_p$  is the time of peak shear stress in an individual stick-slip event



**Appendix Fig. 2.** Representative unstable slip events for four stiffness ratios when the normal stress was 10.2 MPa. (a) Shear stress  $\tau$ . (b) Sliding displacement  $u$ . (c) Slip velocity  $V$ . (d) Slip acceleration  $\dot{V}$ .  $t_p$  is the time of peak shear stress in an individual stick-slip event



**Appendix Fig. 3.** Linear relationship between power law exponents and normal stresses.

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