

Meteo 466 -- Homework #1

(due Thurs., Aug. 30)

Review of basic atmospheric science concepts:

1. What is the mean molecular weight of Earth's atmosphere? Assume that it consists of 78% N₂, 21% O₂, and 1% ⁴⁰Ar, by volume.
2. What is the mass mixing ratio of O₂?
3. Earth's atmospheric surface pressure at sea level is 1.013 bar. What is the surface number density (molecules per unit volume) in MKS and CGS units? Assume that the surface temperature is 288 K.
4. What is the approximate pressure (in bar) at 100 km altitude? To find this, integrate the barometric law, $dp = -\rho g dz$, assuming an isothermal atmosphere with a temperature, $T = 250$ K. Here, p is pressure, ρ is mass density, g is gravitational acceleration, and z is altitude. Use the ideal gas law to relate pressure to density. Take the mean molecular weight from Problem 1.
5. What is the mass and (column-integrated) number density of a vertical column of Earth's atmosphere? Use MKS units. Note that the surface pressure is equal to the column mass times the gravitational acceleration.
6. The ocean mass is 1.4×10^{21} kg. a) What would be its thickness in km if it were spread evenly over the entire Earth's surface? b) What is the pressure at the bottom (in bar)? c) What would be its column number density?

Useful data

$$r_E = 6371 \text{ km (Earth's mean radius)}$$

$$g = 9.81 \text{ m s}^{-2} \text{ (Earth gravity)}$$

$$R = 8.314 \text{ J mol}^{-1} \text{ K}^{-1}$$

Unit conversions

$$1 \text{ bar} = 0.1 \text{ MPa (MKS system)}$$

$$= 10^6 \text{ dynes/cm}^2 \text{ (CGS system)}$$

(over)

Some useful definitions

Volume mixing ratio: $f_i \equiv \frac{n_i}{n}$

where n_i = number density of species i (molecules/cm³, or just cm⁻³)
 n = total number density

Mass mixing ratio: $f_i^m \equiv \frac{\rho_i}{\rho} = \frac{n_i m_i}{nm}$

where ρ_i = mass density of species i (g/cm³)
 ρ = total mass density

$$m = \text{mean molecular mass} = \frac{\sum n_i m_i}{n} = \sum f_i m_i$$

Finally, let: M_i = molecular weight of species i

$$M = \text{mean molecular weight} = \frac{m}{m_H}$$

where m_H = mass of a hydrogen atom (1.67×10^{-24} g)

Then

$$f_i^m = f_i \frac{M_i}{M}$$

with

$$M = \sum f_i M_i$$