

Meteo 466—Homework #4

(due Thurs., Feb. 16)

1. Evidence for low-latitude glaciation is found at both 0.6 Ga and 2.3 Ga. (“Ga” means “giga-aeons”, or billions of years ago.) These are two of the three possible Snowball Earth events mentioned in the text. (We will neglect the event at 0.72 Ga because it is similar to the first one.) Your job is to estimate how thick the ice was at those times.

- a) The variation in solar luminosity with time can be approximated by the following formula (derived by fitting the results of a computer model of the Sun’s evolution, D. O. Gough, *Solar Phys.*, 1981)

$$S = \frac{S_o}{1 + 0.4(t / 4.6)}$$

where S = solar flux at time t

$S_o = 1370 \text{ W/m}^2$ = present solar flux

t = time in Ga (billions of years before present)

Calculate the solar flux at 0.6 Ga and 2.3 Ga both in W/m^2 and as a percentage of its current value.

- b) As we have learned previously, the effective radiating temperature of the Earth can be found from the formula

$$\sigma T_e^4 = \frac{S}{4}(1 - A)$$

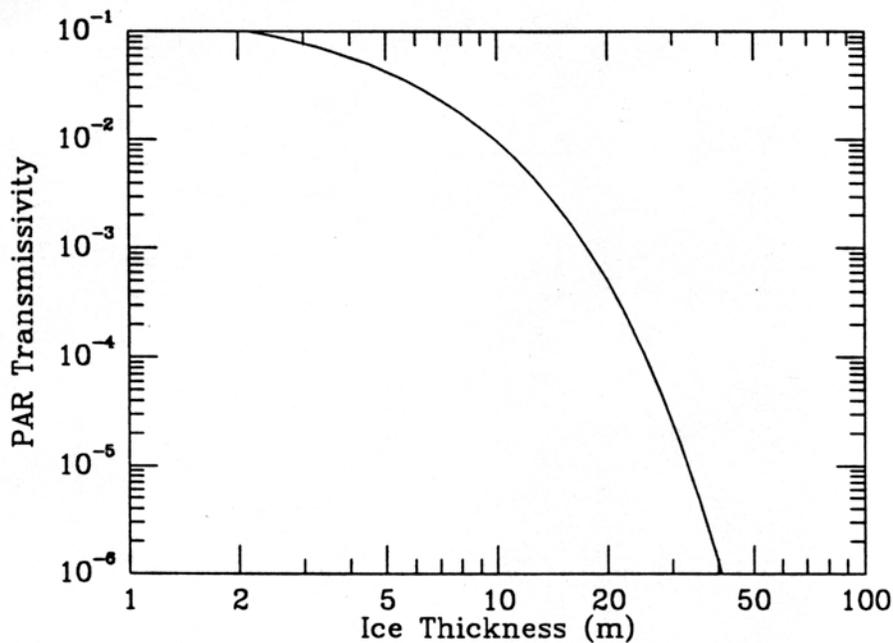
where A is the planetary albedo and σ ($= 5.67 \times 10^{-8} \text{ W/m}^2/\text{K}^4$) is the Stefan-Boltzmann constant. Calculate T_e at 0.6 and 2.3 Ga, assuming an albedo of 0.65 (the value for clean ice and snow). Then, calculate the global average surface temperature, T_s , assuming that the atmospheric greenhouse effect, ΔT_g , was the same as today (33 K). Recall that $T_s = T_e + \Delta T_g$.

- c) The conductive heat flow through ice is given by

$$F = \frac{\lambda \Delta T}{\Delta z}$$

where λ ($= 2 \text{ W/m/K}$) is the thermal conductivity of ice, ΔT is the temperature difference between the top and bottom of the ice layer, and Δz is the thickness of the layer. We know that the current geothermal heat flux, F , is about 0.09 W/m^2 . Assume that F had this same value at 0.6 Ga, but was 3 times higher at 2.3 Ga. Assume also that the top of the ice is at temperature T_s and that the water below the ice has a temperature of 0°C . What is the thickness of the ice at 0.6 Ga and at 2.3 Ga?

2. Suppose now that the ice is transparent enough so that some sunlight makes it through. Let's see how that would change the ice thickness.
- Calculate the solar flux incident at Earth's equator during the Late Precambrian glaciation (0.6 Ga). Assume a local albedo of 0.3, similar to the planetary albedo today. (This is mostly caused by clouds. The ice at the equator would be thin, and hence much darker than thick ice or snow.) Note that the solar flux at the equator is about 20% higher than the planetary average flux.
 - Suppose that 10% of this incident sunlight makes it through the ice. How thin must the ice layer be in order to conduct this heat back out? (Use the formula from question 1c.)
 - Is the ice thickness calculated in part 'b' consistent with the assumption that it would transmit 10% of the sunlight through it? Determine this by consulting Box Fig. 12-2 (below).



Box Fig. 12-2 Diagram showing the transmission of photosynthetically active radiation, $400 \text{ nm} < \lambda < 700 \text{ nm}$, as a function of ice thickness. [From C. P. McKay, *GRL* 27, 2153, 2000]