Meteo 466 -- Homework #5

(Due: Thursday, October 4)

1. The Jeans' escape rate is determined by multiplying the particle velocity distribution by the upward component of the velocity ($v \cos \theta$) and integrating over the upward hemisphere and over velocities exceeding the escape velocity. The expression that results after the angular integration is

$$\phi_{esc} = \frac{n_c}{\sqrt{\pi}}\left(\frac{m}{2kT}\right)^{3/2}\int_{v_e}^{\infty} v^3 \exp\left(\frac{-mv^2}{2kT}\right)dv$$

Evaluate the velocity integral using integration by parts and verify the result given in class

$$\phi_{esc} = \frac{1}{2\sqrt{\pi}} n_c v_s (1 + \lambda_c) e^{-\lambda_c}$$

where

$$v_s \equiv \left(\frac{2kT}{m}\right)^{1/2} \qquad \lambda_c \equiv \frac{GMm}{kTr_c}$$

(<u>Hint</u>: Substitute $x = v^2$ before you integrate!)

2. We can write the Jeans' escape flux, ϕ_J, as the product of a velocity, v_J and the density of the escaping species at the critical level, n_c. a) Evaluate ϕ_J and v_J for Earth under solar minimum (T_∞ = 700 K) and solar maximum (T_∞ = 1200 K) conditions. Assume that atomic H (not H_2) is the constituent that escapes and that the exobase is located at an altitude of 500 km. The number density of atomic hydrogen at this level, n_c, is about 4×10^4 cm^{-3}. b) Evaluate v_J for Venus. (Don't evaluate the escape rate itself, because n_c is different from the value on Earth.) Assume that H is the escaping constituent, T_∞ = 400 K, and the exobase is at 250 km. How does the Jeans' velocity on Venus compare with that on Earth?

<u>Useful data</u>: In SI units, $k = 1.38 \times 10^{-23}$ J deg^{-1}, $G = 6.67\times10^{-11}$ m^3 kg^{-1} s^{-2}, the mass of an H atom is 1.67×10^{-27} kg, Earth's mass and radius are 5.976×10^{24} kg and 6.371×10^6 m, and Venus' mass and radius are 4.87×10^{24} kg and 6.052×10^6 m.

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3. Hydrogen escape can also be limited by diffusion through the lower atmosphere. We can write the diffusion-limited escape rate as

$$\varphi_l \cong \frac{b_i f_{tot}}{H_a}$$

Here, f_{tot} represents the total hydrogen mixing ratio in all of its chemical forms. The effective diffusion constant b_i (actually, the weighted average of the diffusion constants for H and H_2) is 2.5×10^{19} $cm^{-1}s^{-1}$. H_a is the atmospheric (or pressure) scale height, $kT/(mg)$ (not the scale height for atomic H!). The temperature at this altitude is ~200 K, and the total number density, n, is 6×10^{13} cm^{-3}. a) Calculate the diffusion-limited flux, ϕ_l, at the homopause, assuming that the lower stratosphere contains 3 ppm of H_2O and 1.6 ppm of CH_4. (Recall that the total hydrogen mixing ratio remains constant from the tropopause up to the homopause.) b) Compare with ϕ_J evaluated at the exobase for solar minimum and solar maximum conditions. To account for the spherical geometry, multiply the Jeans escape flux by $(r_h/r_e)^2$, where r_h and r_e are the radial distances to the homopause and exobase, respectively. Don't forget that I've given you data in both MKS and CGS units! Would hydrogen escape be diffusion-limited on Earth if Jeans' escape were the only escape mechanism?

(For those struggling with aeronomy notation, the scale height H_a can also be written as RT/g, where R is the specific gas constant. Thus, $R = k/m$.)

4. Earth's oceans, if spread out evenly over the surface of the planet, would be approximately 3 km deep. What is the lifetime of the ocean with respect to hydrogen loss, *i.e.*, how long will it take for them to disappear at the current H escape rate? (Hint: Calculate the number of H_2O molecules in a vertical column, remembering that the density of water is 1 g/cm^3.)