Meteo 466: Homework #6

(due Thursday, Oct. 18)

1. The observed enrichment, E, of deuterium (D) to hydrogen (H) in Venus' atmosphere compared to Earth is a factor of ~150. Meanwhile, Venus is deficient in water compared to Earth by a factor of 4×10^{-6} (from Homework 2). Assume that Venus and Earth acquired their hydrogen (mostly as water) from the same initial volatile reservoir, e.g., carbonaceous asteroids. Assume that the deuterium escape rate to space is related to the hydrogen escape rate by an efficiency, or *fractionation factor*, f. The factor f depends on the particular escape process involved. It is thought to be low ($f \cong 0.13$) for present, nonthermal escape (D. Grinspoon, Nature, 1993), but it could have been much higher (say, $f \cong 0.6$) for hydrodynamic escape early in Venus' history (Kasting and Pollack, Icarus, 1983). Assume also that no water has been added to Venus' atmosphere since the planet was formed. What was the original water inventory of Venus in units of Earth oceans: a) if no deuterium escaped, i.e., if $f = 0$? b) if f was always equal to 0.13, as it is today? c) if f was always equal to 0.6?

Hint: This type of mathematical problem goes by the name *Rayleigh distillation*. Start by writing time-dependent differential equations for Venus' atmospheric hydrogen reservoir, H, and the deuterium reservoir, D:

  1) $dH/dt = -k \cdot H$

  2) $dD/dt = -f \cdot k \cdot D$

Eliminate the time variable (and the undefined constant, k) by combining eqs. (1) and (2). Then integrate from time zero (reservoir sizes H_o and D_o) to time t (reservoir sizes H and D). Finally, rearrange your result to find Venus' initial water inventory in terms of the observed enrichment, E, and fractionation factor, f.