Meteo 466: Homework #6

(due Mon., Mar. 21)

1. Starting from the First Law of thermodynamics, derive the dry adiabatic lapse rate for Earth's atmosphere, and evaluate it numerically. Recall that the First Law may be written as

$$dE = dQ + dW$$

Here, dE is the (molar) change in internal energy of an air parcel, dQ is the heat energy absorbed by the parcel, and $dW = -PdV$ is the work done on the parcel. P is pressure, and V is molar volume. Assume that the gas is ideal, so that $dE = -c_v\, dT$, where c_v is the (molar) specific heat of air at constant volume and T is temperature. Recall that $dQ = 0$ for an adiabatic process. Recall also that the specific heat at constant pressure is given by: $c_p = c_v + R^*$, where R^* is the universal gas constant.

Relevant constants:

$g = 9.81$ m/s^2 (Earth gravity)
$c_p = 1.005 \times 10^3$ J/kg-K

Hint: You will need the ideal gas law and the barometric law.
P.S. If you have not had atmospheric thermodynamics, write that on your assignment, then look up the answer on the Web. Most of you should be able to do this, however.

2. Suppose now that the atmosphere consists of pure water vapor. What is the tropospheric lapse rate for this atmosphere? Evaluate at $T = 300$ K. You may assume that the troposphere remains fully saturated. Start from the integrated form of the Clausius-Clapeyron equation

$$P_v = P_v^0 \exp\left[L_v/R_v(1/T_o - 1/T)\right]$$

Here, P_v is the saturation vapor pressure, L_v (= 2.5×10^6 J/kg) is the latent heat of vaporization per unit mass, and R_v (= 461.5 J/kg/K) is the specific gas constant for water vapor. Assume that the water vapor remains ideal.