Meteo 466: Homework #8

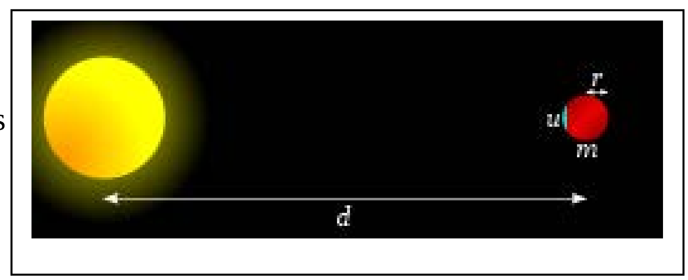
(due Thurs., Dec. 6)

1. Jupiter is emitting about twice as much energy as it absorbs from the Sun. That energy is thought to be coming from gravitational contraction. Let's see if we can estimate how fast Jupiter must be shrinking in order to produce the emitted energy. Steps are as follows:
   a. Assume a constant density planet. (This is not realistic, but it makes the math much more tractable.) Verify that the potential energy of a constant density sphere of mass M and radius R is $\frac{3}{5}\frac{GM^2}{R}$. You will need to perform an integration to do this. Here

$G = 6.67\times10^{-11}\ m^3kg^{-1}s^{-2}$	(universal gravitational constant)
$M = 1.90\times10^{27}$ kg	(mass of Jupiter)
$R = 6.8\times10^{7}$ m	(radius of Jupiter)

   b) Set the rate of potential energy change with time equal to the observed excess heat flux, ~8 W/m^2. Calculate the rate of change of the planet's radius, dR/dt, needed to produce this heat flux.

2. Saturn's rings do not accrete to form a moon because they lie within the planet's Roche limit (the distance at which tidal forces from the larger body disrupt the smaller one). Let's derive the Roche limit for a rigid moon. Steps:



   a. Let the mass and radius of the large body (Saturn, in this case) be *M* and *R*, respectively. Let the mass and radius of the smaller body (the moon) be *m* and *r*. The distance between their centers is *d*. Calculate the gravitational force, F_G, from the moon on a small parcel of mass *u* sitting on the inside edge of the moon (see diagram).
   b. Calculate the tidal force from the large body (i.e., from Saturn). The tidal force is the *difference* between the force exerted on the mass parcel *u* at its current location and the force that would be exerted if it were located at the center of the moon.
   c. Set these two forces equal to each other and verify that the (approximate) Roche limit can be written as $d = R\left(\frac{2\rho_M}{\rho_m}\right)^{\frac{1}{3}}$, where ρ_M and ρ_m are the densities of the large and small bodies.
   d. The actual Roche limit for a fluid body (i.e., a deformable moon) differs from this result by a factor of ~2. Should it be larger or smaller than the value calculated above? Why?