Supporting Information Appendix

Inner Edge of the Habitable Zone for Dry Planets

A climate system adjusts to a new equilibrium state in response to a change in the amount of energy it receives. Energy balance must be achieved at both the top of the atmosphere and at the surface. Surface energy balance dictates that the net radiative flux into the surface (F_{net}) must be equal to the sum of the latent (F_L) and sensible (F_S) heat fluxes, which represent the energy carried upwards by convection [1,2,3,4,5,6]. F_{net} is the sum of the net absorbed solar, F_{sol}, plus net outgoing infrared, F_{IR}, surface fluxes: $F_{net} = F_{sol} + F_{IR}$. Therefore,

$$F_{sol} + F_{IR} = F_L + F_S \tag{1}$$

It should be remembered that the latent heat is associated with the phase change of a substance, for example, from liquid to vapor state, whereas sensible heat is associated with the change in the temperature of the substance with no phase change. This temperature change can either arise from conduction (through contact) or absorption of solar radiation by the surface. We can write the sum of the latent and sensible heat fluxes as the total convective heat flux: $F_C = F_L + F_S = F_L(1 + B)$. Here, B is the Bowen ratio, defined as F_S/F_L. An estimation for how B varies with surface temperature is given by [7], and is reproduced in Fig.1. Following Eq.(1), the total convective heat flux is equal to F_{net}. Thus, we can write:

$$F_C = F_{net} = F_L(1 + B) \tag{2}$$

The latent heat flux, F_L, appearing in Eq. (2) depends on the surface relative humidity, RH. Our surface RH parameterization was derived starting with the following commonly used expression, ([8], Eq.(4.27)):

$$F_L = LC_D u\rho(q_0 - q_1) \tag{3}$$

Here, L is the latent heat of vaporization of water (2.5×10^6 J/kg), C_D is the drag coefficient at layer 1 just above the surface, u is the mean horizontal wind speed near the surface in m/s, ρ is the atmospheric sea level mass density in kg/m^3, q_0 is the surface H_2O saturation mixing ratio, and q_1 is the H_2O mixing ratio at layer 1. Layer 1 is then chosen to be at an infinitesimally small distance above the surface such that $q_1 = R_{surf} * q_0$, where R_{surf} is the surface RH. Substituting this expression into Eq.(3) yields:

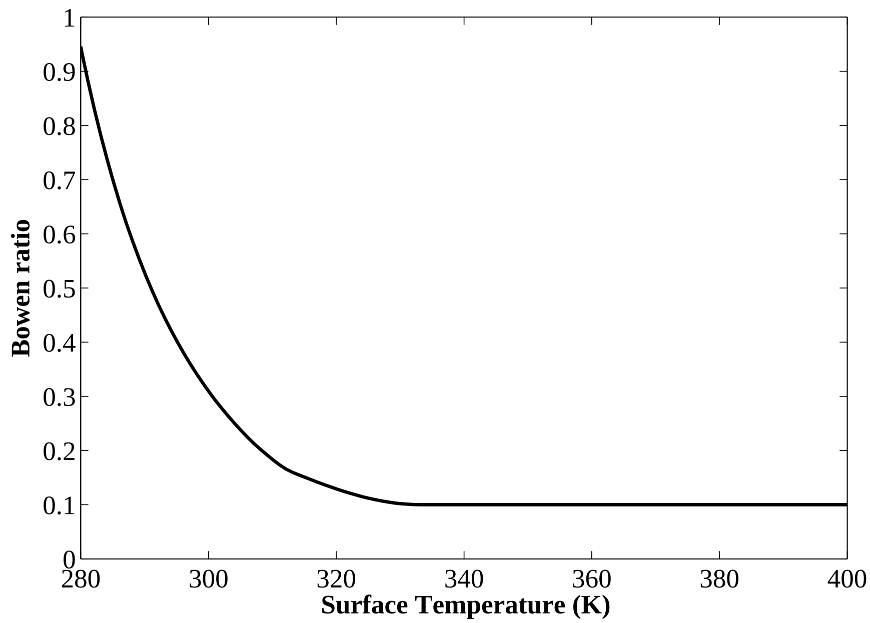

Fig. 1.— Bowen's ratio (F_S/F_L) as a function of surface temperature [7].

$$F_L = LC_D u\rho q_0(1 - R_{surf}) \tag{4}$$

Thus, as R_{surf} increases, so does q_1, moderating the increase of the latent heat flux, F_L, which would otherwise increase exponentially because of the temperature dependence of q_0. To calculate R_{surf} from the surface energy balance, we need the Bowen ratio, B.

Substituting Eq.(4) in Eq.(2) gives

$$F_{net} = LC_D u\rho q_0(1 - R_{surf})(1 + B) \tag{5}$$

The drag coefficient C_D and wind speed u are both unknowns in our expression and cannot be cannot calculated self-consistently in our 1-D model. Thus, we make the assumption that the product $C_D.u$ remains constant at all surface temperatures. This assumption is reasonable for a 1-D climate model, as C_D does not vary strongly over water surfaces [9]. To improve on this assumption, one would need to move to a 3-D model that included horizontal winds and boundary layer physics.

From Eq.(5), one can see the problem if the surface RH is fixed while the surface temperature is varied. As the surface temperature rises, q_0 rises exponentially, following the Clausius-Clapeyron equation, leading to a large increase in the right-hand side that cannot be sustained, and which cannot remain in balance with the net absorbed flux (F_{net}). To moderate this increase in F_L, R_{surf} must increase as surface temperature increases[1].

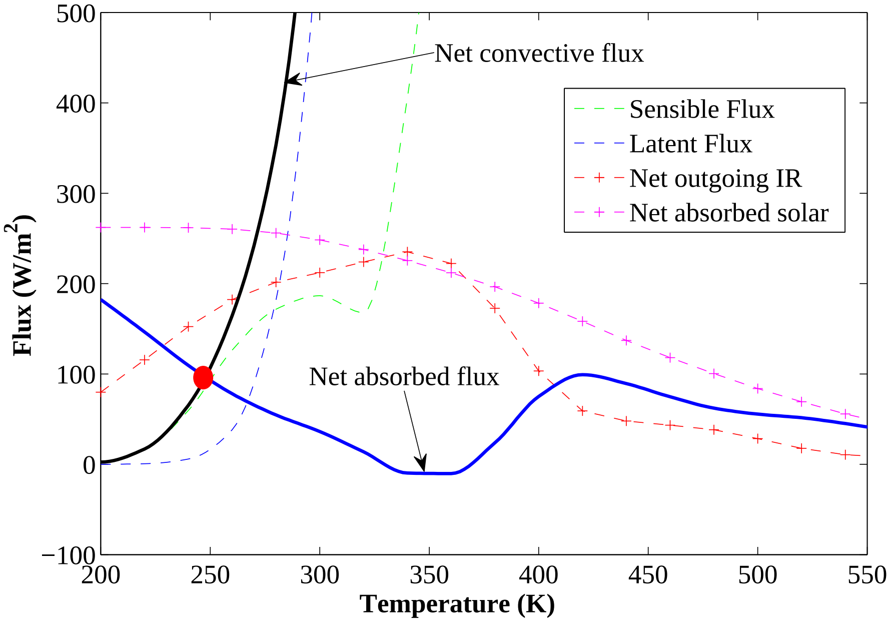

Fig. 2.— Assuming a fixed surface relative humidity, R_{surf}, of 1%, this figure shows the large increase in the net convective heat flux (black solid curve), which is dominated by the latent heat flux (dashed gray curve). Surface energy balance is maintained between the net convective heat flux and the net absorbed radiative flux (solid blue curve) at only one temperature (red filled circle) that is much colder than the typical temperatures encountered at the inner edge of the HZ. Beyond this point, any increase in temperature leads to an unphysical situation in which the net convective flux is greater than the net absorbed flux.

Fig. 2 illustrates this problem by showing various fluxes as a function of surface temperature, assuming R_{surf} is fixed at 1%. There is only one temperature (red filled circle) at which the net absorbed flux at the surface (blue solid curve) equals the net convective flux (black solid curve). Beyond this point (i.e, at higher surface temperatures) the net convective flux increases exponentially, chiefly due to the increase in the latent heat flux. Surface energy balance cannot be achieved at these temperatures if liquid water is present. Hence, the only physically possible solution is for the surface to be dry. But this, then, is not a

[1] B decreases with an increase in temperature, while ρ first decreases marginally, then increases beyond 350 K, which exacerbates the increase in F_L.

habitable planet. Surface liquid water might be stable in cooler regions, as in the "Dune" planets discussed in the main text, but such planets cannot be simulated self-consistently in a 1-D climate model.

Another way to look at this problem is to calculate the lifetime, t_{life}, of liquid water against evaporation assuming some water content on the surface of the planet:

$$t_{life} = \frac{L\rho\Delta z}{F_L} \tag{6}$$

where L and F_L are defined in Eq. (3), ρ is the density of water and Δz is the thickness of the layer of water spread evenly on the planet's surface. For the case of $R_{surf} = 1\%$ (Fig. 2), at a surface temperature of 300 K, $F_L = 600$ Wm^{-2} $= 6 \times 10^5$ erg cm^{-2} s^{-1}. Assuming the same amount of water content as Earth's oceans (1.4×10^{21} kg, which is unphysical for $R_{surf} = 1\%$, but we are aiming here for the longest possible time scale), $\Delta z \sim 3 \times 10^5$ cm. Substituting these values in Eq.(6), $t_{life} = 1.25 \times 10^{10}$ seconds ~ 400 years. The authors of [10] estimate that hot desert worlds have 100 times smaller liquid water content than Earth. If that is true, then $t_{life} \sim 4$ years. Therefore, even with the most optimistic estimates of water content on the surface, the lifetime of the liquid water against evaporation is very small on a planet with low relative humidity.

REFERENCES

[1] Ramanathan, V., *The Role of Ocean-Atmosphere Interactions in the* CO_2 *Climate Problem*, Journal of the Atmospheric Sciences, 38(1981), 918-930

[2] Boer, G. J., *Climate Change and the Regulation of the Surface Moisture and Energy Budgets*, Climate Dynamics, 8(1993), 225-239

[3] Trenberth, K. E., *Atmospheric Moisture Residence Times and CyclingL Implications for Rainfall Rates and Climate Change*, Climate Change, 39(1998), 667-694

[4] Allen, M. R., & Ingram, W. J., *Constraints on Future Changes in Climate and the Hydrologic Cycle*, Nature, 419(2002), 224-232

[4] Held, I. M., & Soden, B. J., *Robust Responses of the Hydrological Cycle to Global Warming*, American Meteorological Society, 19(2006), 5686-5699

[5] Soden, B. J., & Held, I. M., *An Assessment of Climate Feedbacks in Coupled Ocean-Atmosphere Models*, Journal of Climate, 19(2006), 3354-3360

[6] Davies-Jones, R., *An Efficient and Accurate Method for Computing the Wet-Bulb Temperature along Pseudoadiabats*, Monthly Weather Review, 136(2008), 2764-2785

[7] Garratt., *The atmospheric boundary layer*. Cambridge University Press., 1992, 316 pp.

[8] Hartmann, D. L., *Global Physical Climatology*. Academic Press, 1994, 411 pp.

[9] Arya, P. 2001. *Introduction to micrometeorology*, 79, Academic Press, second edition

[10] Zsom, A., Seager, S., De Wit, J., & StamenKovic, V., *Towards the Minimum Inner Edge Distance of the Habitable Zone*, Accepted to Astrophysical Journal, 2013.
