# Stochastic Security for Operations Planning With Significant Wind Power Generation

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*Abstract—***In their attempt to cut down on greenhouse gas emissions from electricity generation, several countries are committed to install wind power generation up to and beyond the 10%–20% penetration mark. However, the large-scale integration of wind power represents a challenge for power system operations planning because wind power 1) cannot be dispatched in the classical sense; and 2) its output varies as weather conditions change. This warrants the investigation of alternative short-term power system operations planning methods capable of better coping with the nature of wind generation while maintaining or even improving the current reliability and economic performance of power systems. To this end, this paper formulates a short-term forward electricity market-clearing problem with stochastic security capable of accounting for nondispatchable and variable wind power generation sources. The principal benefit of this stochastic operation planning approach is that, when compared to a deterministic worst-case scenario planning philosophy, it allows greater wind power penetration without sacrificing security.**

*Index Terms—***Electricity markets, expected load not served, level of penetration, reserve, stochastic security, uncertainty, unit commitment, wind power generation.**

## I. INTRODUCTION

**TURBING** emissions of greenhouse gases causing global warming is currently one of the most pressing issues facing the electricity generation sector in industrialized nations. To that end, several continental European countries, most notably Denmark, Germany, and Spain, are increasing the level of penetration of renewable and low carbon electricity generation resources, wind power generation (WPG) being the prime resource. The United Kingdom, although lagging its continental counterparts, is committed to cover 10% of its electricity demand from renewable resources by 2010 and to reach the 20% mark by 2020 [1]. In North America, although federal authorities in both the United States and Canada have been less proactive in the reduction of greenhouse gas emissions [2], [3], several state and provincial jurisdictions have taken steps to increase the penetration of WPG and other renewable generation technologies. Good examples include Texas and

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Alberta with their respective 1995 MW and 284.5 MW of installed wind capacity in early 2006 [4], [5].

It is well known that WPG cannot be scheduled and dispatched in the classical sense because of its intrinsic dependence on constantly-varying weather conditions. Large, megawatt-range modern wind turbines generally have mechanisms that attempt to regulate their output as the wind speed varies [6]. However, these local control schemes are designed to extract the maximum power from the wind rather than to respond to the dispatch instructions of a grid operator or to system frequency excursions. Specifically, the integration of WPG in power systems poses three challenges.

- 1) The WPG system input is uncertain and cannot be predicted accurately.
- 2) The WPG system input can vary immensely over time; in a given hour, it may be quite high and then drop to a very low value (and vice-versa) over to the next hour.
- 3) The correlation between the WPG system input and the load may be negative. This is problematic, especially in situations when load is low and WPG is high.

As a result, WPG needs sufficient and appropriate backup from hydrothermal generating units to perform the primary, secondary, and tertiary regulation actions necessary to maintain a secure grid operation [7]. Obviously, in systems where WPG represents a significant proportion of the installed classic hydrothermal generation (HTG) capacity, the regulation needs imposed on the system may be important and have significant costs [8]–[14]. In fact, it is well recognized that increasing the level of WPG penetration requires a full reassessment of operating methodologies and standards, especially in setting operating reserve requirements governing primary, secondary, and tertiary regulation [9], [11]–[19].

In addition to the security and economic issues, the integration of WPG in existing grids has to be made in accordance with current electricity market structures [9], [11], [14], [20]. Nowadays, WPG may be sold in hour-ahead or in real-time electricity markets [9]. Nonetheless, there is no clear agreement on how WPG should offer energy in these markets. It is evident, however, that the quality of WPG offerings is coupled to the accuracy of short-term (24 to 1 h ahead) wind forecasting techniques [21]. Accurate WPG prediction techniques are also crucial to grid operators in order to schedule appropriate levels and types of operating reserves needed to perform the different regulation tasks.

There is therefore a need to investigate alternatives and improvements to current short-term power system operations planning methods to be able to cope with and take advantage of the nature of this new generation mix. This paper outlines one such

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proposal based on the electricity market-clearing with stochastic security approach developed in [22] and [23]. In this paper, we adapt the concept of stochastic security to perform secure-economic short-term forward market-based scheduling of generation, load, and tertiary reserves for power systems with uncertain WPG and load.

The formulation we develop here goes beyond efforts in developing scheduling methods for small-scale isolated power systems containing wind and other nondispatchable resources [24]. When moving to larger systems, in a quest for computational tractability, some authors make use of *offline* Monte Carlo simulations [8], while others use analytical work based on empirical data [9], [15]–[19]. These techniques do not address all three of the complicating aspects of operations planning with WPG simultaneously as it is done in this paper. They only deal with the inherent uncertainty in WPG predictions through offline computation of the levels of operating reserves. The unifying characteristic of these contributions is their attempt to establish *a priori* system-wide levels of reserves to be provided by HTG to cope with the uncertainty in WPG predictions (Challenge 1). These approaches assume that reserve requirements are sufficient in setting an appropriate unit commitment and scheduling ample ramping capacity to follow the possibly erratic WPG swings (Challenge 2) and to manage possible negative correlation between WPG and load (Challenge 3).

The major contribution of this paper is the design of an electricity market-clearing algorithm addressing *simultaneously all three challenges* of operations planning with WPG. The scenario-based approach we adopt in modeling the uncertain WPG and load makes it possible to condition explicitly—through scenario-specific unit commitment and ramping constraints—the unit commitment, reserve levels, and HTG dispatches to the possibly erratic and negatively-correlated WPG and load. The other contribution of this paper is the scenario modeling and generation methods required to approximate the continuously-valued WPG and load over the scheduling horizon (as opposed to the discrete nature of the uncertain events modeled in [22] and [23]). This paper also formalizes of the use of WPG spillage for economic or technical reasons as part of a market-clearing algorithm. Finally, this paper contributes to demonstrate the value of added scheduling flexibility (through voluntary or involuntary load shedding and WPG spillage) in improving the technical and economic properties of electricity markets with important WPG penetration.

#### II. STOCHASTIC SECURITY

Power system operation is subject to the occurrence of random events which include sudden line and generator failures as well as demand variations. So far, the industry practices used to plan for such contingencies have been based on rules-of-thumb. These preventive actions include most notably the scheduling of levels of operating reserves (spinning and supplemental) to cover for the potential loss of generating units. Reserve scheduling is however based on deterministic models and usually ignores the likelihood and the potential consequences of the random contingencies. This is a weakness that has been recognized for some time now, and several

authors have investigated ways to better represent the impact of contingency probabilities on operating reserve levels [25], [26]. Recently, in Bouffard *et al.* [22], [23] the concept of *stochastic security* was proposed as a way to further improve the systematic scheduling of reserves. The key advantage of stochastic security is that it accounts for the expected costs of 1) preventive security actions through unit commitment, generation and load dispatch, as well as reserve scheduling; and 2) post-disturbance corrective security actions which include the deployment of reserves via generation and *voluntary* load re-dispatch in addition to the possibility of using *involuntary* load shedding. The important distinctions with respect to prior proposals are the explicit optimization of post-disturbance corrective actions and the consideration of involuntary load shedding as an emergency action to respond to contingencies. The approach recognizes, however, that involuntary load shedding should only be used when the likelihood of disturbances is very small, and the costs to customers, in terms of the cost of lost load, are also small. The salient feature of preliminary case studies for a day-ahead electricity market-clearing formulation shown in [23] was that stochastic security can potentially generate non-negligible economic savings while still ensuring high levels of customer service reliability.

The unexpected failures of transmission lines and generating units were the uncertainty factors considered in [22] and [23]. Nevertheless, as shown briefly in Appendix A, the basic marketclearing problem with stochastic security is general enough to accommodate any type of power system uncertainty. In this paper, we exploit this generality by considering the short-term power system operation planning problem for which the sources of uncertainty are WPG and demand forecast errors. We note that the type of uncertainty dealt with in this paper is in the hourly *average* levels of wind generation and demand. That is, we consider only the scheduling of energy and tertiary regulation resources in the form of reserves, assuming that the reserves required for the primary and secondary regulation tasks are scheduled through a mechanism separate from the actual market-clearing problem. Ideally, as argued in [7], energy as well as all resources necessary to perform the different regulation actions should be scheduled simultaneously. However, with the currently available computing power, this is hardly possible.

## III. WIND POWER GENERATION AND DEMAND FORECAST UNCERTAINTY

### *A. Demand*

Short-term electricity demand prediction tools are numerous and have been the subject of extensive research and development [27]. Here, we assume that a prediction technique is available and provides an hour-by-hour sequence of load forecasts (or any time step, as required by the scheduling horizon),  $\ddot{d}_{mt}$ <sup>1</sup> megawatts for the successive periods of the scheduling horizon  $t = 1, \ldots, T$  and demand entities  $m = 1, \ldots, M$ . We note that, for simplicity of exposition in this paper, we ignore transmission network effects.

<sup>&</sup>lt;sup>1</sup>As a convention for the remaining of this paper, variables and parameters is sion network effects.<br><sup>1</sup>As a convention for the remaining of this paper, variables and parameters<br>written with a "hat,"  $\hat{x}$ , denote, respectively, the expected values of those variables and the forecasted values of those parameters.

Since the demand forecasts are generally inaccurate, we model the forecast errors as zero-mean normally-distributed random variables (RVs)  $\theta_{mt}$  with (predicted) standard deviation  $\sigma_{mt}$  megawatts for  $t = 1, ..., T$  and  $m = 1, ..., M$ . The normality assumption of the demand forecast error is standard in the literature [27]. It is justified through the wide diversity of the electricity demand across geographical areas and consumer classes combined with an invocation of the central limit theorem [28].

In an electricity market setting, a number of caveats about demand forecasts and forecast errors must be addressed. First, demand forecasts are performed by the load-serving entities  $m =$  $1, \ldots, M$ , which then bid accordingly in the market on behalf of their consumers. As a result, the system-wide demand forecast can be seen as a by-product of demand-side bidding in forward (e.g., day-ahead and hour-ahead) electricity markets. That is, the forecast sequences  $d_{mt}$  for  $t = 1, ..., T$  and  $m = 1, ..., M$ are determined through the benefit functions of demands, which reflect consumers' price elasticities. Yet, we assume here that the error in the demand forecast is independent of the consumer benefit functions. This assumption is justified because forecast errors are caused generally by uncontrollable factors exogenous to the consumers, with weather being the prime factor [27]. Likewise, a normal distribution model implies that the underlying RV modeling the uncertain demand prediction may be negative-valued, something which is not physically possible. One way to avoid this modeling inconsistency would be to use a log normal distribution to represent the prediction error [28]. Nevertheless, the assumption here is that the demand model essentially can ignore occurrences of negative-valued loads since these events, from a practical mathematical point of view, have low enough probabilities [29].

### *B. Wind Power Generation*

WPG prediction techniques are currently the subject of extensive ongoing research and development [21]. Nevertheless, here we assume—like in the case of the demand—that a prediction tool provides an hour-by-hour (or any time step, as required by the scheduling problem) system aggregate WPG forecast sequence,  $\hat{w}_t$  megawatts for  $t = 1, \dots, T$ . As with the demand, we model the forecast errors with zero-mean normally-distributed RVs  $\theta_{wt}$  with (predicted) standard deviation  $\sigma_{wt}$  megawatts for  $t=1,\ldots,T$ .

Statistical models for wind speeds at specific locations do not fit normal distributions but rather Rayleigh distributions [6]. In addition, this reality combined with the wind turbines' nonlinear wind speed-to-power output relationships result in that the probability distributions of the power output of *individual* wind generators are not normal. However, like in the case of the demand, the large number and the geographical dispersion of wind turbines permit the invocation of the central limit theorem to justify the normality assumption of the prediction error. This assumption is also motivated by empirical evidence; see [30] for a real life example. Of course, there may be cases where the poor geographical diversity of the wind-based generation capacity cannot justify making this assumption. In such cases, forecast error model modifications have to be made accordingly;

this is, however, out of the scope of this paper. Nevertheless, the approach we detail here based on the normally-distributed forecast error (for both the load and WPG) is general enough to be applicable with any forecast error probability distribution. We also note that the problem encountered with negative-valued load levels applies as well with WPG. Yet, we may assume here that the WPG model also forbids negative-valued power output levels because these have probabilities which are basically insignificant.

#### *C. Net Load*

From the demand and wind power generation forecasts, it is possible to define what is generally termed the *net load* forecast  $\hat{n}_t$  and its associated forecast error RV  $\theta_{nt}$  [18], [12]. Given that both the discrete time demand and WPG random processes have similar frequency spectra, during some time period  $t$ , we define the net load forecast  $\hat{n}_t$  as the difference between the demand and the WPG forecasts

$$
\hat{n}_t = \sum_{m=1}^{M} \hat{d}_{mt} - \hat{w}_t.
$$
\n(1)

Since it is generally assumed that the WPG and load forecast errors are uncorrelated normal RVs, then the standard deviation of the forecast error associated with the net load  $\sigma_{nt}$  is given by

$$
\sigma_{nt} = \sqrt{\sum_{m=1}^{M} \sigma_{mt}^2 + \sigma_{wt}^2}
$$
 (2)

for  $t = 1, \ldots, T$ . In the remaining parts of this paper, to simplify the notation, we will use the net load concept in formulating the electricity market-clearing problem with WPG and demand uncertainty. In so doing, the net load forecast  $\hat{n}_t$  will be used along its zero-mean normally-distributed forecast error RV  $\theta_{nt}$ with standard deviation  $\sigma_{nt}$ .

#### *D. Net Load Scenario Construction*

The continuously-valued net load forecast error model just developed is not computationally practical to formulate a market-clearing problem. This is because this model would require the formulation of a mathematical program needing to meet all constraints over the spectrum of net load values. It would also require the objective function to be computed as an integral over the continuum of the possible net loads. It is more reasonable to consider an approximation whereby the continuous probability distribution of the net load error RV is discretized (sampled) through a number of representative "slices" [31].

Fig. 1 shows an example of such a discretization of the continuous distribution function of the net load forecast error. Here, seven intervals are centered on the zero mean and each of the intervals are one net load forecast error standard deviation  $(\sigma_{nt})$ -wide. Obviously, other slicing designs can be adopted with more intervals to improve the quality of the approximation at the expense of a larger problem size. Likewise, uneven  $\overline{4}$ 

 $3.5$  $\overline{\mathbf{3}}$  $2<sup>1</sup>$ 

Probability density  $\mathfrak{I}$ 

 $\mathbf{1}$ 

 $\overline{1}$ 

 $0.5$ 

 $_{-0.4}^{0}$   $\overline{ }$ 

 $-0.3$ 

 $-0.2$ 

Fig. 1. Typical discretization of the probability distribution of the net load forecast error.

 $-0.1$ 

slicing patterns can be used whereby more intervals are clustered closer to the mean and fewer are used to model the tails of the distribution.

The probability distribution slicing process produces discrete-valued random variables  $\theta_{nt}(j)$  for  $j = 1, \ldots, J$  and  $t = 1, \dots, T$ . For each slice  $j, \theta_{nt}(j)$  is defined at the center of its respective interval with its corresponding probability evaluated using standard techniques [28]. For example, in Fig. 1, the discrete realizations of the net load error RV are  $\theta_{nt}(1) = -0.3$ per unit,  $\theta_{nt}(2) = -0.2$  per unit, and so on.

Under the assumption that the net load forecast error can only adopt a finite number of values for each period, the marketclearing problem will be solved only over a finite number of net load forecast error trajectories. The forecast error trajectories are made up of sequences of *nodes*  $j \in \{1, \ldots, J\}$  representing one of the possible discrete realizations of the net load error RV. A specific instance of a trajectory, a *net load forecast error scenario*  $k \in \{0, 1, \ldots, K\}$  and denoted as  $S_k$ , is an ordered sequence of nodes  $\{j_{k1}, j_{k2}, \ldots, j_{kT}\}\.$  We also define the *error-free scenario*  $S_0 = \{j_{01}, j_{02}, \ldots, j_{0T}\}$  for which the realization of the net load error is nil for all  $t = 1, \ldots, T$ . The collection of all net load forecast error scenarios is called the *scenario tree*  $\mathcal{T} = \{S_0, S_1, \ldots, S_K\}$ . We assume that the scenario tree always contains at least the error-free scenario. Note that to avoid excessive notational zeal, we let  $\theta_{nt}(k) \equiv \theta_{nt}(j_{kt})$  as it is understood that under scenario  $S_k$ , it is the realization indexed by  $j_{kt}$  which occurs during period  $t$ . Furthermore, for each scenario  $S_k$ , there is a corresponding time-indexed sequence of probabilities  $p_t(k)$  calculated from first principles.

Fig. 2 shows an example for a case with  $J = 3$  defining "Low," "As predicted," and "High" net load forecast error slices evolving over  $T = 2$  time periods. In Fig. 2, all transitions are allowed, leading to a scenario tree containing nine scenarios. The error-free scenario is easily identified as the sequence {As predicted, As predicted}.

Statistical studies of inter-hour wind generation variations in Scandinavia [12], [32] indicate that most of the time recorded inter-hour variations in the wind power generation remain within plus or minus  $5\%$ -10% of the installed wind capacity. Likewise, the observed inter-hour demand deviations from

Fig. 2. Net load forecast error scenario tree example.

the load forecast are usually well bounded. Consequently, in formulating a market-clearing problem with wind/demand uncertainty, it may be possible—and, in fact, necessary—to optimize over a scenario tree that is made up of those scenarios that do not contain very "unlikely" inter-period transitions. These further simplifications should make the problem more computationally tractable in light of the computational explosion arising when all probable inter-period transitions are allowed. For example, the scenario tree corresponding to the seven-slice net load forecast error probability distribution in Fig. 1 running over an horizon of 24 h would have over  $1.9 \times 10^{20}$  scenarios if all possible inter-hour transitions were allowed. In streamlining the scenario tree, *ad hoc* scenario rejection techniques based on empirical evidence could be used. Other systematic techniques applicable to generic stochastic optimization problems can be investigated as well [33].

## IV. MARKET-CLEARING UNDER NET LOAD FORECAST UNCERTAINTY

In this section, we characterize the social cost objective as well as the feasible set of the electricity market-clearing problem defined for a net load forecast uncertainty scenario tree  $T$ . We examine more specifically aspects of scenario-per-scenario power balancing through wind energy spillage, involuntary load shedding, as well as HTG and demand-side reserve deployment.

## *A. Objective*

The goal of the electricity market-clearing problem is to minimize a measure of the expected social cost

$$
\min \sum_{t=1}^{T} \left[ C_g(\mathbf{u}_t) + C_r \left( \mathbf{r}_t^{\text{up}}, \mathbf{r}_t^{\text{dn}} \right) \right] - \sum_{\substack{(k,t) \in T}} p_t(k) \left[ B_d(\mathbf{d}_t(k)) - C_g(\mathbf{g}_t(k)) - \mathbf{v}_t^T \ell_t(k) \right].
$$
\n(3)

We distinguish between two components of the expected social cost function:



 $\overline{\text{High}}$ 

As predicted

As predicted

Low ่า

Low

0 High

 $t=2$ 

∩

- those which materialize with probability one and can only be acted upon in the initial decision stage (at the time of market-clearing or day-ahead unit commitment), which are:
	- the scheduling costs of tertiary reserve services during period  $t$  (generation and voluntary demand-side up and down spinning reserves),  $C_r(\mathbf{r}_t^{\text{up}}, \mathbf{r}_t^{\text{dn}});$
	- the fixed running and startup costs of HTG during period  $t$ , which depend on their associated 0–1 unit commitment variables,  $C_g(\mathbf{u}_t)$ ;
- those second-stage components that materialize with a probability  $p_t(k)$  during period t, under scenario  $S_k$ , which are:
- the demand benefits,  $B_d(\mathbf{d}_t(k))$ ;
- the generation running costs,  $C_g(\mathbf{g}_t(k))$ ;
- the costs of involuntary load shedding,  $v_t^T \ell_t(k)$ , where v represents the vector of values of lost load.

We point out that boldfaced symbols denote vectors of a given variable representing either all generators  $(i = 1, \ldots, I)$  or all demands  $(m = 1, \ldots, M)$ . For instance,  $\mathbf{g}_t(k) = [g_{1t}(k) g_{2t}(k) \cdots g_{It}(k)]^T$  is the vector of HTG outputs during period  $t$  under the realization of scenario  $\boldsymbol{k}$ .

The reason why components of the expected social cost function are assigned a probability of one is that their associated sets of decisions variables (i.e., unit commitment and reserve scheduling decisions) have to be taken prior to the revelation of the uncertainty—*first-stage decisions*. On the other hand, the components of the expected social cost function which are assigned probabilities  $p_t(k)$  are those which only materialize once the uncertainty is revealed—*second-stage decisions*. These components measure the expected social cost associated with the reserve deployment and load shedding patterns needed to keep the power system balanced during the full length of the scheduling horizon and for all the considered scenarios.

In the expected social cost function, we do not assign an operating cost component to the WPG. It is probably not realistic at the present moment for WPG to submit nonzero running cost offers given their relative incapacity to regulate effectively their output. We assume that WPG cannot be dispatched in the classical sense; the only possible control action for WPG is wind power curtailment, which we also call "wind power spillage." Clearly, the development of techniques aimed at the formulation of effective market offer strategies for WPG warrants further investigation.

## *B. Power Balance*

For all pairs  $(k, t)$  belonging to the scenario tree T, the power balance must be satisfied

$$
\sum_{i=1}^{I} g_{it}(k) + \sum_{m=1}^{M} \ell_{mt}(k) - s_t(k) = \sum_{m=1}^{M} d_{mt}(k) - \hat{w}_t + \theta_{nt}(k). \tag{4}
$$

We recall that the variables  $g_{it}(k)$  represent the power output of generator i during period t under scenario  $S_k$  of the net load error RV  $\theta_{nt}$ . In (4), the variables  $\ell_{mt}(k)$  correspond to involuntary load shedding that could be applied, where of necessity

we impose

$$
0 \le \ell_{mt}(k) \le \sum_{m=1}^{M} [d_{mt}(k) + \theta_{mt}(k)], \quad k \ne 0.
$$
 (5)

In (5),  $\theta_{mt}(k)$  is the forecast error associated with demand m during period t under scenario  $S_k$ . Again, (5) is defined for all time intervals and realizations of the net load RV forming the scenario tree  $\mathcal T$ , except for the error-free scenario  $\mathcal S_0$  for which we require that  $\ell_{mt}(0) = 0.2$ 

In (4) and (5), the demand-side variables  $d_{mt}$  are also indexed by the scenario index  $k$  to model the demand-side adjustments that may be requested by the grid operator as part of voluntary responses to errors in the combined wind power and demand forecasts. In the case of the error-free scenario  $k = 0$ , the demand variables are given by  $d_{mt}(0) = \hat{d}_{mt}$ , where  $\hat{d}_{mt}$  is the forecasted demand which is a by-product of the market bidding by demand entity  $m$ . Here, the forecasted wind power generation  $\hat{w}_t$  is not indexed by k since, unlike the demand, it may not be controlled directly through voluntary adjustments.

Further in (4), the variable  $s_t(k)$  models the rate at which wind power generation is curtailed ("spilled") during period  $t$ under scenario  $S_k$ . Much like the involuntary load shedding term, WPG spillage is bounded from below by zero and from above by the actual wind generation

$$
0 \le s_t(k) \le \hat{w}_t + \theta_{wt}(k) \tag{6}
$$

for all  $(k, t) \in \mathcal{T}$ . Here,  $\theta_{wt}(k)$  is the forecast error associated with the WPG during period t under scenario  $S_k$ . We also note that unlike the demand, we allow for WPG to be curtailed under the error-free scenario.

The use of spillage may seem counterintuitive because WPG is free energy. However, under low probability situations for which the amount of wind generation is very high and the demand is very low, the expected social cost may be lower if the wind energy is simply curtailed. In such cases, the expected costs of the required down-spinning reserve services that would have to be supplied by hydrothermal units and demands can easily outweigh the benefits of the free, but excessive, wind. Wind energy spillage is generally obtained through active and passive mechanical controls of the wind turbines' blade pitch angle and nacelle yaw angle [6].

#### *C. Other Market-Clearing Constraints*

Referring to the generic market-clearing with stochastic security problem detailed in Appendix A, we have addressed so far the modeling of the objective function and of the power balance under the various forecast error scenarios. What remains to be modeled are the operational constraints applying to individual HTG units and demand entities. Clearly here, because of the intrinsic and complex time dynamics of the market-clearing problem, the ramping limitations of the HTG are prime factors

<sup>2</sup>The reason why we do not allow nonzero load shedding decisions in the error-free scenario is essentially philosophical. We believe that under perfectly forecasted conditions, the demand should be able to fully benefit from available generation capacity. However, the opposite can also be acceptable as permitting load shedding under the error-free scenario may allow lower levels of expected social costs and higher wind power penetration.

to be modeled. The interested reader is referred to Appendix B for a more detailed treatment of these constraints. The other important feature of the constraints detailed in Appendix B is the explicit coupling they impose between first-stage (unit commitment and reserve levels) and some of the second-stage decision variables (HTG generation outputs and responsive demand levels). Lastly, we impose constraints on variables sharing common scenario paths to model forecast error nonanticipation. We do so by enforcing that a variable  $x_t(k) = x_t(k')$  for  $1 \leq t \leq \tau$  if the error realizations corresponding to scenarios  $S_k$  and  $S_{k'}$  are identical over the interval  $1 \le t \le \tau$ .

## *D. Incorporating HTG Contingencies*

The proposed electricity market-clearing formulation would be incomplete without the inclusion of a discussion of the impact of HTG equipment outages. From the point of view of the problem formulation, adding HTG outages is straightforward because it boils down to the mere generation of extra scenarios whose probabilities are found by calculating the convolution of the probability distributions of the net load forecast error with those of the contingencies.

From a computational point of view, however, the addition of hydrothermal generator contingencies would render the solution process of the market-clearing problem much more challenging. The addition of a single generator contingency, also considering its possible times of failure, would multiply the number of scenarios by the number of periods of the scheduling horizon. We must recall that with each extra scenario, there are corresponding extra variables and constraints. As a result, realistically-sized problems may be very hard to handle with current computing tools in the reasonable amount of time required for day-ahead market-clearing purposes.

Some modeling simplifications could be considered. One possibility is to formulate a hybrid problem in which a deterministic reserve criterion (e.g.,  $N-1$  criterion) covers the HTG contingencies, while the probabilistic method developed here takes care of scheduling reserves for the WPG and demand uncertainty. A second solution could make use of the scenario reduction techniques mentioned earlier [33]. Lastly, decomposition techniques [31] are promising because they exploit the intrinsic decomposable structure of the problem—whereby each scenario is optimized individually under the command of a master coordinating problem. These aspects, however, lie outside of the scope of this paper.

## V. ILLUSTRATIVE STUDY

This section outlines the results of an illustrative example of how the electricity market-clearing with stochastic security under demand and WPG uncertainty works. Primarily, we aim to demonstrate the following.

- When planning operations under uncertain load and wind forecasts, voluntary and involuntary load shedding are valuable scheduling options for the system operator.
- Under stochastic market-clearing, the acceptable level of wind power penetration can be more important than under operating schemes based on deterministic "worst-case scenario" security criteria.

TABLE I HOURLY DEMAND FORECAST

		Time $t(h)$			
$\hat{d}_t$	(MW)	- 30	80	-110	40

TABLE II PER-UNITIZED HOURLY WPG FORECAST



As the WPG penetration level is increased—WPG penetration is defined as the ratio of the WPG installed capacity to the total HTG capacity—for some arbitrary WPG and demand forecasts, the stochastic market-clearing is solved and its outcomes are analyzed against the outcomes of a corresponding deterministic security-constrained market-clearing formulation. Here the deterministic market-clearing formulation schedules the power system while ensuring that the power balance is met under all realizations of the net load forecast error scenarios without resorting to involuntary load shedding and without considering the relative likelihood of the scenarios. It minimizes the cost of unit commitment, reserves, and of the generation dispatch corresponding to the predicted (error-free) net load scenario. This approach is similar to the security-constrained scheduling problem formulated in [34].

To facilitate conceptual understanding and exposition, we consider a small system consisting of three HTG units with a aggregate capacity of 250 MW whose technical and cost data are detailed in Appendix C. The system is scheduled over four consecutive hours assuming the inelastic demand forecast given in Table I. The standard deviation of the demand forecast error is assumed to equal 2% of the hourly demand forecast. The demand forecast error probability distribution is approximated by a discrete distribution made up of seven one-standard-deviation-wide slices as previously shown in Fig. 1. This slicing arrangement gives rise to a scenario tree with 2401 paths spreading over the four hour-long scheduling horizon.

Here we investigate the performance of market-clearing with stochastic security with respect to the level of penetration of WPG. For this, we first assume we have the per-unitized wind power forecast of Table II, which is scaled to the increasing penetration level. Second, we take the WPG forecast error model from Fabbri *et al.* [13]. Assuming that the WPG prediction was made 24 h prior to the first hour of the schedule for an ensemble of wind farms contained in a region with a diameter of 140 kilometers, the standard deviation of the WPG forecast error is approximated by

$$
\sigma_{wt} = 0.02 + 0.2\hat{w}_t \tag{7}
$$

where  $\sigma_{wt}$  and  $\hat{w}_t$  are both in per unit of the installed wind capacity for  $t = 1, \ldots, 4$ .



Fig. 3. Total expected social costs of scheduling as a function of the WPG penetration.

## *A. Results and Analysis*

Fig. 3 shows the evolution of the expected social cost of the schedule with increasing levels of WPG penetration for both stochastic and deterministic market-clearing approaches. The deterministic and stochastic optimal social costs cannot be readily compared because they are obtained through completely different objective functions. Nonetheless, an expected social cost comparison can be made between the two if we compute the expected cost of operation (i.e., the expected social cost of the generation dispatches required to meet the power balance under all net load scenarios without having to resort to any load shedding) under the proviso that the market was initially cleared in a worst-case scenario deterministic fashion. We obtain the expected social cost of the deterministic schedule by adding this expected cost of operation to the deterministic unit commitment and reserves costs. This expected social cost is comparable to that of the expected cost of the stochastic approach because it considers simultaneously the costs of unit commitment, reserve scheduling, and generation dispatches under all scenarios as the objective function of the stochastic approach (3) does.

The expected cost under stochastic market-clearing undergoes a steady decrease as the amount of WPG increases. This contrasts with the deterministic schedule cost which decreases until the 8% penetration mark where it starts increasing until no feasible schedules exist when the penetration level reaches 10%. The observed behavior of the deterministic-based expected scheduling cost indicates that at one point, the need for reserves to cover all forecast error scenarios overwhelms the expected savings brought about by the increasing level of free, but uncertain, WPG.

Noticeably in Fig. 3, the size of the gap between the two curves widens with the increase in the WPG penetration. This size of the gap represents the value of the stochastic solution (VSS) [31], which is the expected value of the savings brought about by using a stochastic approach. We observe a slow increase in the VSS until the 8% penetration level. The faster increase in between 8% and 9% is explained by the need for the deterministic schedule to use expensive voluntary demand-side reserve (at \$50/MWh) to balance power under all scenarios—even those with very low probabilities.



Fig. 4. Cost of reserves as a function of the WPG penetration.

The behavior of the expected costs and of the VSS can be further investigated by inspecting Fig. 4, which shows the progression of the reserve costs as the WPG penetration increases. There is nothing more to say about the case of the deterministic-based market-clearing; it is clear that the reserve costs increase sharply with the level of WPG penetration. On the other hand, the cost of reserves under stochastic market-clearing follows a different pattern: 1) between 0% and 2% penetration, the cost of reserves decreases because, on average, it is cheaper to spill wind and to shed load involuntarily than to schedule more reserves because of the small size of the WPG capacity; 2) between 2% and 15%, more reserves are required since the costs of load shedding and the opportunity costs of spilling wind increase faster than those of reserves; and 3) for 15% of WPG onward, the cost of reserves remains relatively constant as the provision of more reserves cannot further decrease the global expected scheduling costs. This phenomenon is reflected in the amounts of involuntary load shedding and wind energy spillage shown in Fig. 5. The faster increase in the wind energy spillage occurring for WPG penetration levels above 15% is reflected in the flat part of the curve for the cost of reserves since under these higher penetration levels, wind energy spillage, which does not cost anything, is used in lieu of expensive demand-side downspinning reserve.

One of the peculiarities of the demand and wind forecasts in this case study is the occurrence of a low demand period during  $t = 1$ , which is at the same time a high wind period. This has serious implications as can be seen in Table III for a 15% WPG penetration level. During this period, generator 3 is constrained by its minimum output level (10 MW), impeding any kind of down-regulating actions. Thus, a significant amount of wind energy has to be curtailed—an average value of 2306.3 kWh. In fact, here one could argue that the maximal WPG penetration level should not exceed 15% unless demand increases because at this point, HTG generators have no more freedom to perform down-going regulation without having to go offline. Obviously, this is an extreme example on a very small and idealistic system. However, with higher and higher levels of wind penetration, such phenomena may become prevalent and of practical importance. Appropriate procedures and technologies to manage these



Fig. 5. Expected load not served and expected wind energy spillage as a function of the WPG penetration.

TABLE III ERROR-FREE GENERATION, RESERVES, EXPECTED LOAD NOT SERVED, AND EXPECTED WIND ENERGY SPILLAGE FOR 15% WPG PENETRATION

		Time $t(h)$			
		1	$\overline{2}$	3	4
$g_{1t}(0)$	(MW)	0.0	16.9	56.3	0.0
$r^{up}_{1t}$	(MW)	0.0	11.2	8.0	0.0
$r_{1t}^{\widetilde{d}n}$	(MW)	0.0	3.7	2.7	0.0
$g_{2t}(0)$	(MW)	0.0	0.0	0.0	0.0
$r_{2t}^{up}$	(MW)	$_{0.0}$	$_{0.0}$	0.0	$_{0.0}$
$r_{2t}^{\overline{d}n}$	(MW)	$_{0.0}$	0.0	0.0	$_{0.0}$
$g_{3t}(0)$	(MW)	10.0	50.0	50.0	30.6
	(MW)	9.2	0.0	0.0	5.5
$\frac{r_{3t}^{up}}{r_{3t}^{dn}}$	(MW)	0.0	0.0	0.0	0.0
$r^{up}$	(MW)	0.0	0.0	0.0	0.0
$r_{dt}^{\frac{dt}{dn}}$	(MW)	0.0	0.0	0.0	0.0
$\hat{\ell}_t$	(kWh)	30.5	0.0	0.0	17.0
$\hat{s}_t$	(kWh)	2306.3	272.7	194.4	1047.1

events have to be developed. Large-scale energy storage infrastructure may offer part of the answer to this problem once its capital cost has come down [35].

Table IV shows the expected marginal social costs of energy  $\lambda_t^E$  and security  $\lambda_t^S$  for the stochastic market-clearing calculated as in [23]. We notice again the effect of the constraint on down-going regulation active during period  $t=1$  through the low values for the expected marginal costs of energy and security (below the \$20/MWh offered marginal running cost of generator 3). These lower marginal expected costs indicate that, by increasing their load during that period, consumers could benefit from the plentiful supply of free wind.

#### *B. Computational Complexity*

The dimensions of the simple market-clearing problem here are not trivial; they are reported in the second column of Table V. The large dimensions are a consequence of the fact that no scenario reduction techniques were applied. Moreover, the number of discrete net load forecast error samples (seven per hour) is a factor directly influencing the size of the problems. Obviously,

TABLE IV EXPECTED MARGINAL SOCIAL COSTS OF ENERGY AND SECURITY FOR 15% WPG PENETRATION

		Time $t(h)$			
$\lambda_{\pm}^E$	$(S/MWh)$ 14.17		30.00	30.00	20.00
$\lambda_t^S$	$(S/MWh)$ 14.17		25.60	28.32	20.00

TABLE V PROBLEM DIMENSIONS UNDER DEMAND AND WPG UNCERTAINTY WITH AND WITHOUT SOLVER PREPROCESSING



using fewer slices would reduce the problem size at the expense of accuracy. In addition, with the help of the mixed-integer linear programming solver preprocessing engine, one can generally reduce significantly the size of the problem, as shown in the third column of Table V.

The market-clearing problem was solved on a Pentium 4, 1.8 MHz with 512 MB of RAM using version 9.0.2 of CPLEX under GAMS [36]. The solution time was of 39.0 s for the 10% WPG penetration case. Similar computation times were obtained for other penetration levels. Reiterating what has already been mentioned regarding the computational complexity of the proposed formulation, any practical implementation would require further investigation into the application of scenario reduction techniques. Moreover, the application of decomposition methods should equally be the subject of future research efforts.

## VI. CONCLUSION

Current electricity market-clearing schemes cannot fully integrate the most essential features of nondispatchable generation technologies like wind power. This limitation is becoming an issue for grid operators as there is more and more public and political pressure to increase the penetration of renewable generation technologies, which depend on randomly-varying weather conditions. Using the stochastic security framework developed in previous work, this paper proposed an electricity market-clearing formulation that can account explicitly for this type of uncertainty and variability.

There are several advantages to the use of a stochastic approach over the more conservative deterministic approaches. The latter assumes worst-case scenario wind and demand conditions to be as likely as those close to forecasted conditions. On the other hand, the former, since it assigns probabilities of occurrence to each wind-demand scenario, is biased more toward the most likely forecasted conditions. This has the consequence of permitting the improvement of the economic performance of the market by taking advantage of the freely-available wind power and by reducing reserve scheduling and HTG unit commitment costs. In addition, the stochastic approach, because it includes the extra flexibility provided by coordinated

involuntary load shedding and wind spillage actions, permits the expansion of the feasible space of the security-constrained market-clearing problem. A collateral benefit of this is an increment in allowable WPG penetration levels. A simple case study showed how the proposed formulation does that in comparison to a worst-case-based deterministic technique.

Notwithstanding the higher efficiency and flexibility of the proposed stochastic market-clearing approach, work on fitting better wind forecast error models and scenario construction should be conducted. This applies in cases where the normality assumption of the error may not be valid—in regions with poor geographic distribution of wind farms, for instance—and where there is inter-hour correlation in wind and load levels. The other important research needed is the improvement of its computational tractability through the application of scenario reduction and decomposition methods.

## APPENDIX A MARKET-CLEARING WITH STOCHASTIC SECURITY

Consider the following security-constrained market-clearing problem formulated as a two-stage stochastic optimization problem with recourse [31]. It allows for unit commitment decisions to be made only in the first decision-making stage:

$$
\min \sum_{t=1}^{T} \left\{ V(\mathbf{u}_t, \mathbf{y}_t) + \sum_{k=0}^{K} p(k) \left[ W(\mathbf{x}_t(k)) + \mathbf{v}_t^T \ell_t(k) \right] \right\} \tag{8}
$$

subject to, for all time periods of the market-clearing scheduling horizon  $t = 1, \ldots, T$ 

$$
\mathbf{H}(\mathbf{x}_t(0),0) = 0\tag{9}
$$

$$
\mathbf{H}(\mathbf{x}_t(k), k) + \ell_t(k) = 0; \quad k = 1, ..., K \tag{10}
$$

$$
\mathbf{G}(\mathbf{y}_t, \mathbf{u}_t, \mathbf{x}_t, \ell_t) \geq 0. \tag{11}
$$

The vectors  $\mathbf{u}_t$  and  $\mathbf{y}_t$  represent the first-stage decision variables taken prior to the revelation of uncertainty:  $\mathbf{u}_t$  is the vector of discrete unit commitment decision variables and  $y_t$  is the vector of reserve scheduling decisions applying to period  $t$ . The second-stage variables, which are realized once uncertainty has been revealed, include the controllable generation and load dispatch variables  $x_t$  as well as the involuntary load shedding variables  $\ell_t$  for period t. The problem formulation is based on the possible occurrence of  $k = 0, 1, \dots, K$  scenarios for which we associate individual probabilities of occurrence  $p(k)$ . We denote the scenario indexed  $k = 0$  as the pre-disturbance (or, as in the context of this paper, the *error-free*) scenario, whereas the scenarios indexed by  $k = 1, ..., K$  are the post-disturbance (or, as in the context of this paper, the *error*) scenarios.

The objective function (8) minimizes the expected social cost of operating the power system under pre- and post-disturbance operation, including any possible involuntary load shedding. On the one hand, the function  $V(\mathbf{u}_t, \mathbf{y}_t)$  is a measure of the net social cost—unit commitment and reserve scheduling—of preparing to operate under the full collection of scenarios  $k =$  $0, 1, \ldots, K$  and is thus incurred with a probability equal to one.

On the other hand, the net social cost under the uncertain scenarios  $k = 0, 1, ..., K$ , denoted by  $W(\mathbf{x}_t(k))$ , includes the costs of deploying generation-side and demand-side reserves through dispatch decisions once the uncertainty is revealed. In addition, under the disturbed states, the cost of any involuntary load shedding used is included— $\mathbf{v}_t^T \ell_t(k)$  for  $k = 1, ..., K$ only, where the vector  $v_t$  represents the value of lost load [37]. Here, we impose that  $\ell_t(0) = 0$  as we do not allow for load shedding under the pre-disturbance (*error-free*) scenario,  $k = 0$ .

The vector constraint (9) represents the power balance under the pre-disturbance scenario for which no load shedding is allowed, while (10) represents the power balance conditions for each of the  $k = 1, \ldots, K$  disturbance scenarios wherein involuntary load shedding is permitted. The vector inequality (11) gathers all the remaining network and technological constraints, and it contains the constraints coupling the pre- and post-disturbance conditions.

One of the most important aspects of electricity marketclearing with stochastic security is that reserves in the first stage are scheduled in an implicit manner as dictated by the power balance (9) and (10). The result of this is that there are no needs for the *a priori* specification of minimal reserve levels for the entire system. Instead, the levels of reserves are specified for each generating unit and time period as a by-product of the pre- and post-disturbance power balances. For example, the provision of up-going spinning reserve provided by generator  $i$ during some period  $t, r_{it}^{\text{up}}$ , satisfies the inequalities

 $\overline{r}$ 

$$
\begin{aligned} \n\stackrel{\text{up}}{=} \ge 0 \n\end{aligned} \n\tag{12}
$$

$$
r_{it}^{\text{up}} \ge g_{it}(k) - g_{it}(0); \quad k = 1, ..., K \tag{13}
$$

$$
r_{it}^{\text{up}} \le r_{it}^{\text{up max}} u_{it} \tag{14}
$$

where the variables  $g_{it}(0)$  represent the HTG generation levels under the pre-disturbance state  $k = 0$ , the variables  $g_{it}(k)$  represent the post-disturbance generation outputs under the spectrum of scenarios,  $k = 1, ..., K$ , and the variables  $u_{it}$  denote the unit commitment status for generator i during period  $t$ — $u_{it} = 1$  indicates that unit i is online during period t; otherwise,  $u_{it} = 0$ . Finally,  $r_i^{\text{up max}}$  is the maximum amount of up-going reserve offered by generator  $i$  during period  $t$ .

## APPENDIX B HTG AND DEMAND-SIDE CONSTRAINTS

Classical multiperiod market-clearing problems with unit commitment include provisions for modeling restrictions on the operation of HTG units. These restrictions include most notably minimum/maximum power output limits, ramping limitations, and minimum up- and down-time constraints [38].

As mentioned in the main text, HTG ramping limitations represent the most important limiting factor in the process of scheduling of tertiary reserves and its later deployment through generation re-dispatch. Ramp constraints set upper and lower bounds on the amount of power a unit can deliver at a later time in response to changing system conditions. For all scenarios  $S_k$ , we define upper bounds on the generation coming from unit  $i$  during period  $t$ 

$$
g_{it}(k) \le g_i^{\max} u_{it}; \quad t = 1, \dots, T \tag{15}
$$

$$
g_{it}(k) \le g_{i(t-1)}(k) + R_i^{\text{up}} u_{i(t-1)} + R_i^{\text{su}} (u_{it} - u_{i(t-1)}) + g_i^{\text{max}} (1 - u_{it}); \quad t = 2, ..., T
$$
 (16)

$$
g_{it}(k) \le g_{i0} + R_i^{\text{up}} u_{i0} + R_i^{\text{su}} (u_{it} - u_{i0}) + g_i^{\text{max}} (1 - u_{it}); \quad t = 1.
$$
 (17)

These are general enough and can apply for all HTG  $i = 1, \ldots, I$  and pairs  $(k, t) \in \mathcal{T}$ . In (15)–(17), the parameters applying to generator  $i, g_i^{\text{max}}, R_i^{\text{up}}, R_i^{\text{su}}, g_{i0}$ , and  $u_{i0}$ , are, respectively, the HTG unit maximum loading capacity, the runtime up-going ramp limit, the startup ramp limit, the initial power output, and unit commitment status at time  $t = 0$ . Likewise, lower bounds on  $g_{it}(k)$  for all HTG  $i = 1, \ldots, I$  and pairs  $(k, t) \in \mathcal{T}$  are expressed as

$$
g_{it}(k) \ge g_i^{\min} u_{it}; \quad t = 1, ..., T
$$
  
\n
$$
g_{it}(k) \ge g_{i(t-1)}(k) - R_i^{\rm dn} u_{it} - R_i^{\rm sd} (u_{i(t-1)} - u_{it})
$$
\n(18)

$$
g_{it(n)} \ge g_{i(t-1)}(n) - n_i u_{it} - n_i (u_{i(t-1)} - u_{it})
$$
  
-  $g_i^{\max} (1 - u_{i(t-1)})$ ;  $t = 2,...,T$  (19)

$$
g_{it}(k) \ge g_{i0} - R_i^{mu} u_{it} - R_i^{su}(u_{i0} - u_{it}) - g_i^{max}(1 - u_{i0}); \quad t = 1
$$
 (20)

where the parameters  $g_i^{\min}$ ,  $R_i^{\text{dn}}$ , and  $R_i^{\text{sd}}$  are the minimum generation output limit, the runtime down-going ramp limit, and the shutdown ramp limit, respectively. Minimum up- and down-time restrictions are not as critical here, and we shall omit them for the sake of brevity; the interested reader is referred to [38].

The demand-side limits have to account for the fact that only demand levels associated with the error-free scenario  $(d_{mt}(0) = d_{mt})$  are bounded by elasticity limits. That is, for  $m = 1, \ldots, M$  and  $t = 1, \ldots, T$ 

$$
d_{mt}^{\min} \le d_{mt}(0) \le d_{mt}^{\max}.\tag{21}
$$

Like in the case of market-clearing with equipment contingencies described in [22] and [23], the notion of reserve used here is different from what is the current industry definition. In the way illustrated in Appendix A, the levels of generation-side tertiary reserve (up-spinning,  $r_{it}^{\text{up}}$ , and down-spinning,  $r_{it}^{\text{dn}}$ ) assigned to particular HTG units satisfy

$$
0 \le r_{it}^{\text{up}} \le r_{it}^{\text{up max}} u_{it}; \quad i = 1, ..., I, t = 1, ..., T
$$
\n
$$
0 \le r_{it}^{\text{dn}} \le r_{it}^{\text{dn max}} u_{it}; \quad i = 1, ..., I, t = 1, ..., T
$$
\n(23)

and

$$
r_{it}^{\text{up}} \geq g_{it}(k) - g_{it}(0); \quad i = 1, \dots, I, \forall (k, t) \in \mathcal{T}
$$
\n
$$
r_{it}^{\text{dn}} \geq g_{it}(0) - g_{it}(k); \quad i = 1, \dots, I, \forall (k, t) \in \mathcal{T}.
$$
\n(25)

The levels of demand-side reserves are defined similarly.

TABLE VI HTG DATA

		Generator i		
			2	
min 9ï	(MW)	10.0	10.0	10.0
$g_i^{\mathrm{max}}$	(MW)	100.0	100.0	50.0
$a_{it}$	(S/MWh)	30.0	40.0	20.0
	(S/h)	100.0	100.0	100.0
$c_{i}^{\tiny su} \\ q_{it}^{\tiny up}$	(S/MWh)	5.0	7.0	8.0
$q_{it}^{\check{d}n}$	(S/MWh)	5.0	7.0	8.0

## APPENDIX C HTG DATA

The HTG unit data for this paper are found in Table VI. We assume here that the energy and reserve offers of the generators remain unchanged for all hours of the scheduling horizon. Moreover, the generators are assumed not to incur any fixed running costs, but they do incur constant-valued startup costs,  $c_i^{su}$ . The generation offering structure requires that each generator offers a single block of energy ranging between its technical minimum,  $g_i^{\text{min}}$ , and maximum,  $g_i^{\text{max}}$ , at the rate of  $a_{it}$ dollars per megawatt-hour. The bounds on the amounts of reserve services offered are set to be the largest possible; in other words, the upper bound on both up- and down-spinning reserve is  $g_i^{\text{max}} - g_i^{\text{min}}$ . The generation-side reserve services are offered at the rates, in dollars per megawatt-hour,  $q_{it}^{up}$  for up-going spinning reserve, and  $q_{it}^{\text{dn}}$  for down-going spinning reserve. The demand offers (voluntary) reserve (both up- and down-going) at the rate of \$50/MWh, while its value of lost load (VOLL) is \$1000/MWh. The HTG minimum up- and down-time constraints are assumed to be inactive, and their ramping capabilities are set to  $g_i^{\text{max}}$  megawatts per hour. Finally, we assume that all generators are in the off state at time  $t = 0$ .

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