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Mechanics of Hydraulic Fracturing

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ABSTRACT

A theoretical examination of the fracturing of rocks by means of pressure applied in boreholes leads to the conclusion that, regardless of whether the fracturing fluid be of the penetrating or non-penetrating type, the fractures produced should be approximately perpendicular to the axis of least stress. The general state of stress underground is that in which the three principal stresses are unequal. For tectonically relaxed areas characterized by normal faulting, the least stress should be horizontal; the fractures produced should be vertical with the injection pressure less than that of the overburden. In areas of active tectonic compression, the least stress should be vertical and equal to the pressure of the overburden; the fractures should be horizontal with injection pressures equal to or greater than the pressure of the overburden.

Horizontal fractures cannot be produced by hydraulic pressures less than the total pressure of the overburden.

These conclusions are compatible with field experience in fracturing and with the results of laboratory experimentation.

INTRODUCTION

The hydraulic-fracturing technique of well stimulation is one of the major developments in petroleum engineering of the last decade. The technique was introduced to the petroleum industry in a paper by J. B. Clark,[1] of the Stanolind Oil and Gas Co. in 1948, and since then its use has progressively expanded so that by the end of 1955 more than 100,000 individual treatments had been performed.

The technique itself is mechanically related to three other phenomena concerning which an extensive literature had previously developed. These are: (1) pressure parting in water injection wells in secondary-recovery operations, (2) lost circulation during drilling, and (3) the breakdown of formations during squeeze-cementing operations, all of which appear to involve the formation of open fractures by pressure applied in a wellbore. The most popular interpretation of this mechanism has been that the pressure had parted the formation along a bedding plane and lifted the overburden, notwithstanding the fact that in the great majority of cases where pressures were known they were significantly less than those due to the total weight of the overburden as determined from its density.

Prior to 1948, this prevalent opinion had already been queried by Dickey and Andresen,[2] in a study of pressure parting; and by Walker,[3,4] who, in studies of squeeze cementing, pointed out that the pressures required were mostly less than those of the overburden, and inferred that the fractures should be vertical. J. B. Clark,[1] in his paper introducing hydraulic fracturing, and later Howard and Fast,[5] and Scott, Bearden, and Howard,[6] also of Stanolind, postulated that the entire weight of the overburden need not be lifted in producing horizontal fractures, but that it was only necessary to lift an "effective overburden," requiring a correspondingly lower pressure. Hubbert,[7] in discussing the paper by Scott and associates, pointed out that the normal state of stress underground is one of unequal principal stresses; and in tectonically relaxed areas, characterized by normal faults, the least stress should be horizontal. Therefore, in most cases, fracturing should be possible with pressure less than that of the overburden and, moreover, such fractures should be vertical. Harrison, Kieschnick, and McGuire,[8] also on the expectation that the least principal stress should be horizontal, argued strongly in favor of vertical fracturing.

Scott, Bearden, and Howard[6] observed that, when

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[1]References given at end of paper.

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using penetrating fluids, hollow, cylindrical cores could be ruptured at less than half the pressures required using non-penetrating fluids. They also observed that with penetrating fluids the fractures occurred parallel to the bedding, irrespectively of the orièntation of the bedding with respect to the axis of the core; whereas with non-penetrating fluids the fractures tended to be parallel to the axis of the core. Finally, Reynolds, Bocquet, and Clark[9] described additional experiments confirming the earlier work of Scott, Bearden, and Howard and on the basis of this concluded that it should be possible to produce horizontal fractures with penetrating fluids and vertical fractures with non-penetrating fluids.

During the last two years the present authors have been engaged in a critical re-examination of this problem, and since the results obtained have sustained the conjecture offered earlier by Hubbert,[7] the principal content of this paper is an elaboration of that view.

THE STATE OF STRESS UNDERGROUND

The approach frequently made to the problem of underground stresses is to assume that the stress field is hydrostatic or nearly hydrostatic with the three principal stresses approximately equal to one another and to the pressure of the overburden. That this cannot generally be true is apparent from the fact that over long periods of geologic time the earth has exhibited a high degree of mobility wherein the rocks have been repeatedly deformed to the limit of failure by faulting and folding. In order for this to occur, substantial differences between the principal stresses are required.

The general stress condition underground is therefore one in which the three mutually perpendicular principal stresses are unequal. If fluid pressure were applied locally within rocks in this condition, and the pressure increased until rupture or parting of the rocks results, that plane along which fracture or parting is first possible is the one perpendicular to the least principal stress. It is here postulated that this plane is also the one along which parting is most likely to occur (Fig. 1).

In order, therefore, to have a mechanical basis for anticipating the fracture behavior of the rocks in various localities, it is necessary that something be known concerning the stress states that may be expected. The best available evidence bearing upon these stress conditions is the failure of the rocks themselves, either by faulting or by folding.

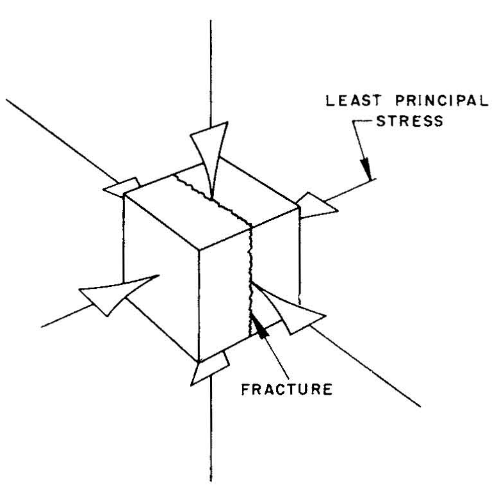

Fig. 1—Stress Element and Preferred Plane of Fracture.

The manner in which the approximate state of stress accompanying various types of geologic deformation may be deduced was shown in a paper by Hubbert,[10] published in 1951, of which the remainder of this section is a paraphrase. In addition, Figs. 2 to 9 have been taken from that earlier paper and are here reproduced by permission of the Geological Society of America.

Figs. 2 and 3 show a box having a glass front, and containing ordinary sand. In the middle there is a partition which may be moved from left to right by turning a hand screw. The white lines are markers of powdered plaster of paris which have no mechanical significance. As the partition is moved to the right, a normal fault with a dip of about 60° develops in the left-hand compartment, as shown in Fig. 2. With further movement a series of thrust faults with dips of about 30° develop in the right-hand compartment, as shown in Fig. 3.

The general nature of the stresses which accompany the failure of the sand may be seen in Fig. 4. Adopting the usual convention of designating the greatest, intermediate, and least principal stresses by σ_1, σ_2, and σ_3, respectively (here taken as compressive), in the left-hand compartment σ_3 will be the horizontal stress, which

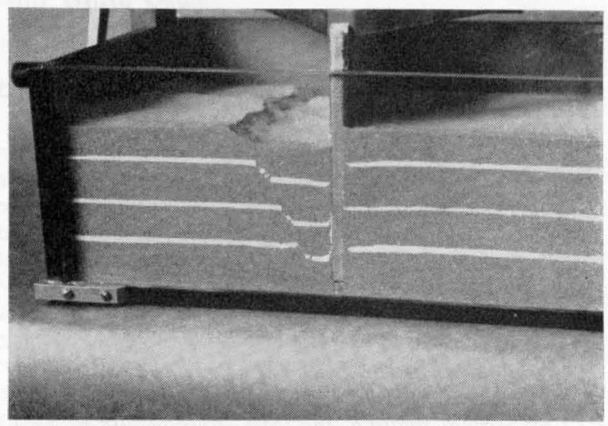

Fig. 2—Sand-box Experiment Showing Normal Fault.

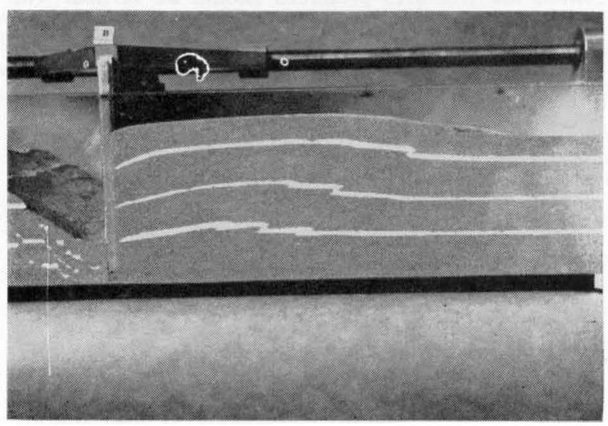

Fig. 3—Sand-box Experiment Showing Thrust Fault.

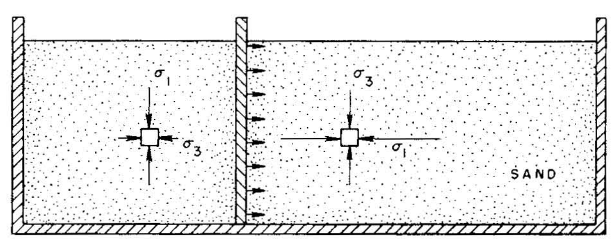

Fig. 4—Section Showing Approximate Stress Conditions in Sand Box.

is reduced as the partition is moved to the right, and σ_1 will be the vertical stress, which is equal to the pressure of the overlying material. In the right-hand compartment, however, σ_1 will be horizontal, increasing as the partition is moved, and σ_3 will be vertical and equal to the pressure of the overlying material. A third type of failure, known as transcurrent faulting, is not demonstrated in the sand-box experiment. This occurs when the greatest and least principal stresses are both horizontal and failure occurs by horizontal motion along a vertical plane. In all three kinds of faults, failure occurs at some critical relationship between σ_1 and σ_3.

To determine this critical relationship it is first necessary to obtain an expression for the values of the normal stress σ and shear stress τ acting across a plane perpendicular to the σ_1,σ_3-plane and making an arbitrary angle α with the direction of least principal stress σ_3. As shown in Fig. 5, this may be done by balancing the equilibrium forces which act upon a small triangular prism of the sand. The resulting expressions for σ and τ are

$$\left.\begin{aligned}\sigma &= \frac{\sigma_1+\sigma_3}{2}+\frac{\sigma_1-\sigma_3}{2}\cos 2\alpha\,,\\ \tau &= \frac{\sigma_1-\sigma_3}{2}\sin 2\alpha\,.\end{aligned}\right\} \quad . \quad . \quad (1)$$

A very convenient method of graphically representing these expressions, known as the Mohr stress representation, consists in plotting values of normal and shear stress from Eq. 1 with respect to σ,τ-coordinate axes for all possible values of the angle α, as shown in Fig. 6. The locus of all σ,τ-values is a circle and it can be seen that as α approaches zero and the plane becomes normal to σ_1, the normal stress becomes equal to σ_1 and the shear stress disappears. On the other hand, as α approaches 90° and the plane becomes normal to the least principal stress σ_3, the normal stress becomes equal to σ_3 and the shear stress again disappears. This figure completely describes all possible combinations of normal and shear stress acting on planes perpendicular to the plane of σ_1 and σ_3.

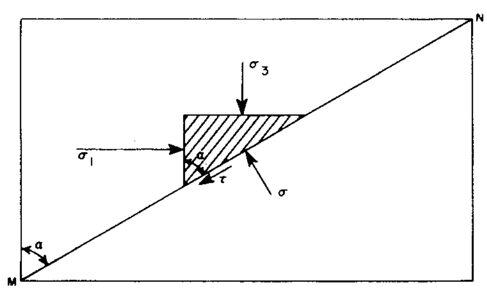

Fig. 5—Stresses σ and τ on Plane of Arbitrary Angle α.

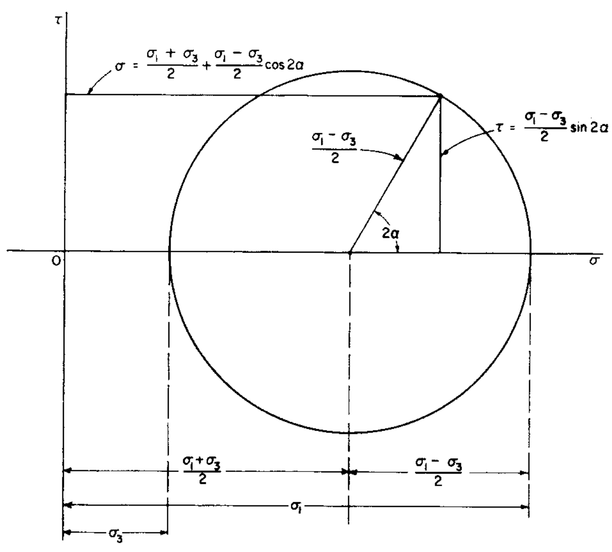

Fig. 6—Mohr Diagram Showing Normal Stress σ and Shear Stress τ on Plane of Orientation α in Terms of σ_1, σ_2, and α.

It is next necessary to determine the combination of shear and normal stresses which will induce failure. This information may be obtained from a standard soil-mechanics test which is illustrated in Fig. 7. A horizontally divided box is filled with sand which is then placed under a vertical load. The shearing force which is necessary to displace the upper box is then measured for various values of vertical stress. In this way it is found that the shearing stress for failure is directly proportional to the normal stress, or that

$$\frac{\tau}{\sigma} = \tan\phi\,, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where ϕ is known as the angle of internal friction and is a characteristic of the material. For loose sand ϕ is approximately 30°. These critical stress values may be plotted on a Mohr diagram, as shown in Fig. 8. The two diagonal lines comprise the Mohr envelopes of the material and the area between them represents stable combinations of shear stress and normal stress, whereas the area exterior to the envelopes represents unstable conditions. Fig. 8 thus indicates the stability region within which the permissible values of σ and τ are clearly defined. The stress circles may then be plotted in conjunction with the Mohr envelopes to determine the conditions of faulting. This is illustrated in Fig. 9 for both normal and thrust faulting. In both cases one of the principal stresses will be equal to the overburden pressure, or σ_z. In the case of normal faulting the horizontal principal stress is progressively reduced thereby increas-

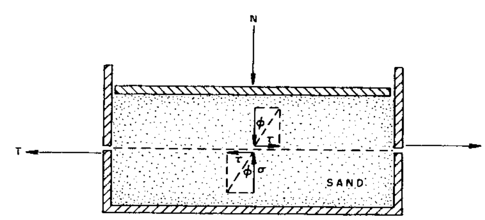

Fig. 7—Shear Box for Measuring Ratio τ/σ at Which Slippage Occurs.

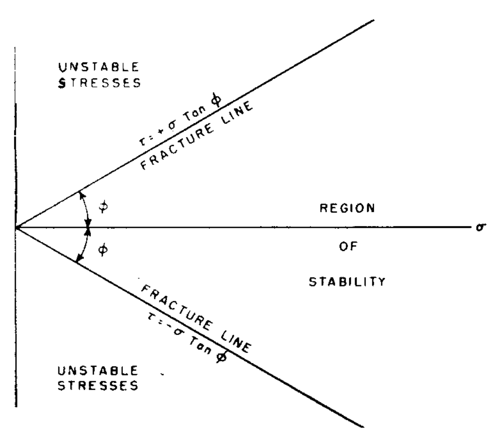

Fig. 8—Mohr Envelopes for Sand Showing Curves of Values of σ and τ at Which Slippage Occurs.

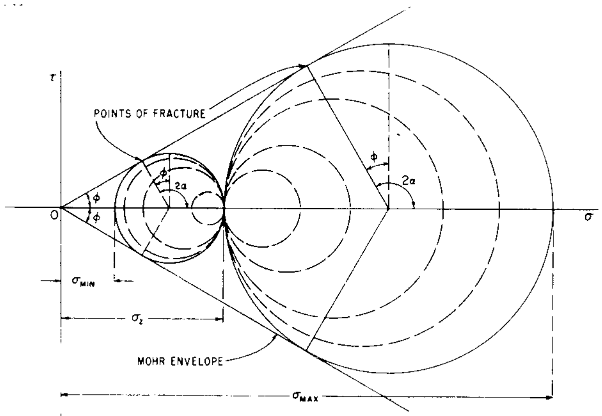

FIG. 9—MOHR DIAGRAM SHOWING THE POSSIBLE RANGE OF THE HORIZONTAL STRESS FOR A GIVEN VERTICAL STRESS σ_Z. THE HORIZONTAL STRESS CAN HAVE ANY VALUE RANGING FROM APPROXIMATELY ONE-THIRD THE VERTICAL STRESS, CORRESPONDING TO NORMAL FAULTING, TO APPROXIMATELY THREE TIMES THE VERTICAL STRESS, CORRESPONDING TO REVERSE FAULTING.

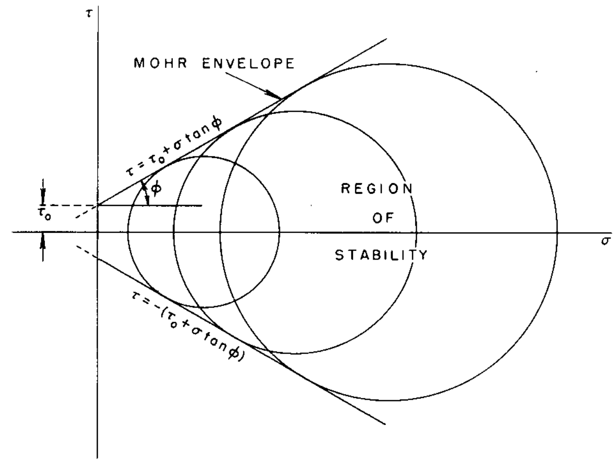

FIG. 10—TYPICAL MOHR ENVELOPES FOR ROCK.

ing the radius of the stress circle until it becomes tangent to the Mohr envelopes. At this point unstable conditions of shear and normal stress are reached and faulting occurs on a plane making an angle of $45° + \phi/2$ with the least stress. For sand having an angle of internal friction of 30°, the normal fault would have a dip of 60°, which agrees with the previous experiments. For the case of thrust faulting, the least principal stress would be vertical and would remain equal to the overburden pressure while the horizontal stress is progressively increased until unstable conditions occur and faulting takes place on a plane making an angle of $45° + \phi/2$ with the least principal stress, or $45° - \phi/2$ with the horizontal. For sand this would be a dip of about 30°, which again agrees with the experiment.

It can be seen that, for sand having an angle of internal friction of 30°, failure will occur in either case when the greatest principal stress reaches a value which is about three times the least principal stress, and that the failure will occur along a plane making an angle of about 60° to the least principal stress. Also, for a fixed vertical stress σ_z, the horizontal stress may have any value between the extreme limits of one-third and three times σ_z.

MOHR DIAGRAM FOR ROCKS

The foregoing theoretical analysis is directly applicable to solid rocks provided the Mohr envelopes have been experimentally determined. In order to do this it is necessary to subject rock specimens to a series of triaxial compression tests under wide ranges of values of greatest and least principal stresses σ_1 and σ_3. It has been found that at sufficiently high pressures nearly all rocks deform plastically and the Mohr envelopes become approximately parallel to the σ-axis. However, at lower pressures most rocks fail by brittle fracture and within this domain the envelopes are approximated by the equation

$$\tau = \pm (\tau_o + \sigma \tan \phi) \quad , \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where the angle of internal friction ϕ has values usually between 20° and 50° and most commonly not far from 30°, and τ_o is the shearing strength of the material for zero normal stress (Fig. 10).

Fortunately, Eq. 3 is applicable to most rocks within drillable depths. Exceptions would occur in the cases of the plastic behavior of rock salt and unconsolidated clays.

One additional modification in the theoretical analysis is needed before it is directly applicable to geologic phenomena. Sedimentary rocks are both porous and permeable and their pore spaces are almost invariably occupied by fluids, usually water, at some pressure p. It is necessary to know the effect which is produced by the fluid pressure upon the mechanical properties of the rock.

This question was specifically investigated by Douglas McHenry,[11] of the U. S. Bureau of Reclamation, who ran a large series of tests on duplicate specimens with and without enclosure by impermeable jackets, using nitrogen gas to produce the pressure p. He found for the unjacketed cases, when the axial compressive stress S was corrected for the opposing fluid pressure p, that the value of the residual effective stress, $\sigma = S - p$, at which failure occurred was to a close approximation constant and independent of the pressure p of the permeating fluid.

This result is directly applicable to the behavior of rocks underground. Porous sedimentary rocks are normally saturated with fluid under pressure and constitute a mixed solid-fluid stress system. The stress field existing in this system may be divided into two partial stresses: (1) the hydrostatic pressure p which pervades both the fluid and solid constituents of the system, and (2) an additional stress in the solid constituent only. The total stress is the sum of these two.

If, across a plane of arbitrary orientation, S and T are the normal and tangential components, respectively, of the total stress, and σ and τ the corresponding components of the solid stress, then, by superposition,

$$\left. \begin{aligned} S &= \sigma + p \ , \\ T &= \tau \ , \end{aligned} \right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

are the equations relating the stress fields.

The pressure p produces no shearing stress and hence has no tendency to cause deformation. Moreover, as demonstrated by the work of McHenry, it has no significant effect on the properties of the rock. Therefore, with respect to the stress of components σ and τ, the rock has the same properties underground as those exhibited in the triaxial testing machine using jacketed specimens.

This fact has long been recognized in soil mechanics where the partial stress of components σ and τ is known as the *effective stress,* and the pressure p as the *neutral stress* (Terzaghi,[12]). The effective stress defined in this manner is not to be confused with the postulated "effective overburden pressure" invoked by the Stanolind group and others to justify their assumption of horizontal fracturing in response to pressure less than that of the overburden.

Therefore, the entire Mohr stress analysis is directly applicable to porous rocks containing fluid under pressure provided that effective stresses only are used.

It is interesting to consider the behavior of the effective vertical stress under various fluid-pressure conditions. Under any condition the total vertical stress S_z is very nearly equal to the weight of the overlying material per unit area. The effective vertical stress σ_z, however, is given by

$$\sigma_z = S_z - p \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Under normal hydrostatic conditions p is somewhat less than half the total pressure of the overburden and the effective vertical stress is therefore slightly more than one-half the overburden pressure. However, with abnormally high fluid pressures, such as occur in some parts of the Gulf Coast, the effective vertical stress is correspondingly reduced, and in the extreme case of fluid pressure equal to the total overburden pressure, the effective vertical stress becomes zero.

Returning now to the mechanical properties of rocks, for loosely consolidated sediments such as those of the Gulf Coast area, the limiting envelopes on the Mohr diagram will approximate those for loose sand shown in Fig. 8. In older and stronger rocks, the Mohr envelopes are given approximately by Eq. 3:

$$\tau = \pm (\tau_o + \sigma \tan \phi) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

These also are similar to the envelopes for loose sand except, as shown in Fig. 10, they project to an intersection at some distance to the left of the origin, indicating that the rocks have some degree of tensile strength and also have a shear strength of finite magnitude τ_c when the normal stress is equal to zero. The Mohr envelopes for tests on a sandstone and an anhydrite made by J. W. Handin of the Shell Development Exploration and Production Research Laboratory are shown in Figs. 11 and 12.

In either case, however, it will be observed that, at other than shallow depths, the value of σ_3, the least stress, at the time of faulting, will be of the order of one-third of the value of σ_1, the greatest stress.

Since these are the extreme states of stress at which failure occurs, it follows that when actual faulting is not taking place the stress differences which may prevail are somewhat less than these limits. However, in most regions a given type of deformation is usually repetitive over long geologic periods of time, indicating that the stresses of a given type are persistent and not far from the breaking point most of the time.

The orientation of the trajectories of the principal stresses in space is largely determined by the condition which they must satisfy at the surface of the earth. This is a surface along which no shear stresses can exist. Since for unequal stresses the only planes on which the shear stresses are zero are those perpendicular to the principal stresses, it follows that one of the three trajectories of principal stress must terminate perpendicular to the surface of the ground, and the other two must be parallel to this surface. Thus, in regions of gentle topography and simple geologic structures, the principal stresses should be respectively nearly horizontal and vertical, with the vertical stress equal to the pressure of the overlying material.

Therefore, in geologic regions where normal faulting is taking place, the greatest stress σ_1 should be approximately vertical and equal to the effective pressure of the overburden, while the least stress σ_3 should be horizontal and most probably between one-half and one-third the effective pressure of the overburden.

On the other hand, in regions which are being shortened, either by folding or thrust faulting, the least stress should be vertical and equal to the effective pressure of the overburden, while the greatest stress should be horizontal and probably between two and three times the effective overburden pressure.

In the case of transcurrent faulting, both the greatest and least stresses should be horizontal, with the intermediate stress σ_2 equal to the effective vertical stress.

As an example, the Tertiary sediments of the Texas and Louisiana Gulf Coast have undergone recurrent normal faulting throughout Tertiary time and up to the present. This indicates that a normal-fault stress system must have been continuously present which intermittently reached the breaking points for the rocks, causing the stresses temporarily to relax and then gradually to build up again. Hence, during most of this time, including the present, a stress state must have existed in this region for which the least stress has been horizontal and probably between one-half and one-third of the effective pressure of the overburden. Since the

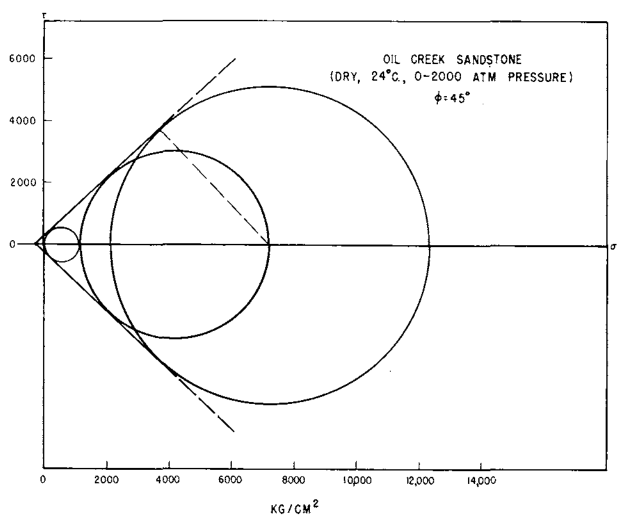

FIG. 11—MOHR ENVELOPES FOR OIL CREEK SANDSTONE (MEASUREMENTS BY JOHN HANDIN).

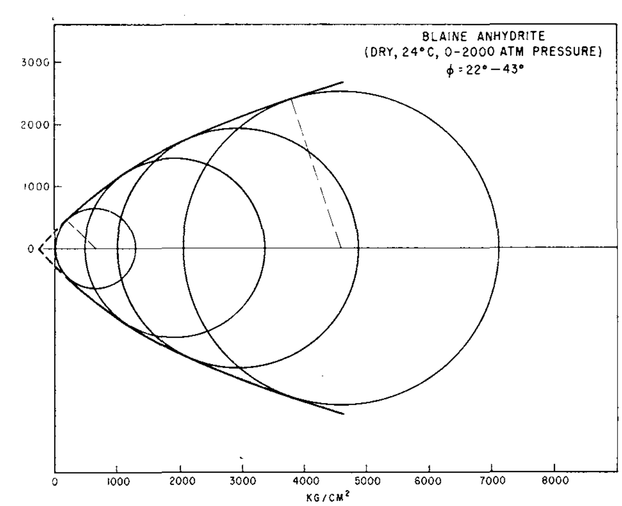

FIG. 12—MOHR ENVELOPES FOR BLAINE ANHYDRITE (MEASUREMENTS BY JOHN HANDIN).

faults in this area, except around salt domes, are mostly parallel to the strike of the rocks, the axis of least stress must be parallel to the dip.

A large part of the region of West Texas and the Mid-Continent is also a region of tectonic relaxation characterized by older normal faults. The situation here is somewhat more ambiguous than that of the Gulf Coast since faulting in these regions is not now active. However, since evidences of horizontal compression are lacking, it is still reasonable to assume that a relaxed stress state in these areas is the more probable one at present.

California, on the other hand, is in a region where active tectonic deformation is occurring at the present, as indicated by the recurrence of earthquakes, by extensive folding and faulting of the rocks during the Recent geological period, and by slippages along faults and measurable movements of elevation bench marks during the last few decades. All three of the types of stress pattern described earlier probably occur in different parts of this region; but, in areas still undergoing active compression, the greatest stress must be essentially horizontal, whereas the least stress would be the effective weight of the overburden.

It should be understood that the foregoing analysis of faulting is employed only as a means of estimating the state of stress underground and that the shearing mechanism of faulting is quite distinct from the mechanism of producing hydraulic fractures, which are essentially tension phenomena. However, with an understanding of the regional subsurface stresses, it is now possible to analyze the stress conditions around the borehole and to determine the actual conditions under which hydraulic tension fractures will be formed.

STRESS DISTORTIONS CAUSED BY THE BOREHOLE

The presence of a wellbore distorts the pre-existing stress field in the rock. An approximate calculation of this distortion may be made by assuming that the rock is elastic, the borehole smooth and cylindrical, and the borehole axis vertical and parallel to one of the pre-existing regional principal stresses. In general, none of these assumptions is precisely correct, but they will provide a close approximation to the actual stresses. The stresses to be calculated should all be viewed as the effective stresses, carried by the rock in addition to a hydrostatic fluid pressure, p, which exists within the wellbore as well as in the rock. The calculation is made from the solution in elastic theory for the stresses in an infinite plate containing a circular hole, with its axis perpendicular to the plate, which was first obtained by Kirsch,[13] and is also given by Timoshenko,[14] and by Miles and Topping.[15]

Expressed in polar coordinates with the center of the hole as the origin, the plane-stress components at a point θ, r, exterior to the hole in a plate with an otherwise uniform uniaxial stress, σ_A, are given by

$$\left.\begin{aligned}
\sigma_r &= \frac{\sigma_A}{2}\left[1 - \frac{a^2}{r^2}\right] + \frac{\sigma_A}{2}\left[1 + 3\frac{a^4}{r^4} - 4\frac{a^2}{r^2}\right]\cos 2\theta \ , \\
\sigma_\theta &= \frac{\sigma_A}{2}\left[1 + \frac{a^2}{r^2}\right] - \frac{\sigma_A}{2}\left[1 + 3\frac{a^4}{r^4}\right]\cos 2\theta \ , \\
\tau_{r\theta} &= \frac{\sigma_A}{2}\left[1 - 3\frac{a^4}{r^4} + 2\frac{a^2}{r^2}\right]\sin 2\theta \ ,
\end{aligned}\right\} \quad \ldots \ldots \ldots \ldots \ldots \quad (6)$$

where a is the radius of the hole and the θ-axis is taken parallel to the axis of the compressive stress σ_A. The same solution for a regional principal stress σ_B at right angles to σ_A, in which $(\theta + 90°)$ is used for the angular coordinate, may be superposed (added) onto Eq. 6 to give the complete horizontal components of stress in the vicinity of the borehole. The values of the horizontal stresses across the principal planes in the vicinity of the borehole have been calculated in this way for various relative values of the σ_B/σ_A-ratio and are presented in Figs. 13 and 14.

It can be seen that in every case the stress concentrations are local and that the stresses rapidly approach the undisturbed regional stresses within a few hole diameters. The principle of superposition of the two parts of the stress field is illustrated in Fig. 13 for the case in which σ_B/σ_A is 1.0. For σ_A alone, the circumferential stress at the walls of the hole varies from a minimum value of $-\sigma_A$ (tensile) across the plane parallel to the σ_A-axis to a maximum of $+3\sigma_A$, across the plane normal to the σ_A-axis. When the two stresses are superposed, the stress field has radial symmetry and the circumferential stress at the walls of the hole is $+2\sigma_A$. The resultant stress fields for other ratios of σ_B/σ_A are shown in Fig. 14. In the extreme case when $\sigma_B/\sigma_A = 3.0$, the circumferential stress at the walls of the hole varies from a minimum of 0 to a maximum of $+8\sigma_A$.

The vertical component of the stress is also distorted in the vicinity of the borehole. The initial vertical

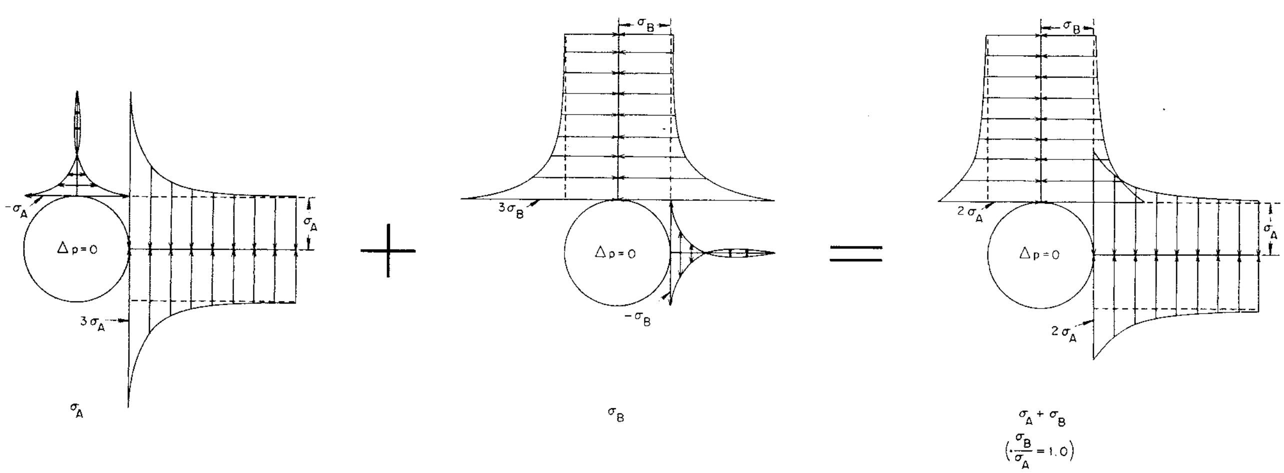

FIG. 13—SUPERPOSITION OF STRESS STATES ABOUT A WELLBORE DUE TO TWO HORIZONTAL PRINCIPAL STRESSES OF EQUAL MAGNITUDE.

stress is equal to the effective pressure of the overburden. The distortion in the vertical stress is a function of the values of the regional horizontal stresses σ_A and σ_B. However, the magnitude of this distortion is small in comparison with the concentrations of the horizontal stresses and it rapidly disappears with distance away from the wellbore.

THE EFFECT OF PRESSURE APPLIED IN THE BOREHOLE

The application within the borehole of a fluid pressure in excess of the original fluid pressure produces additional stresses. In the case of a non-penetrating fluid, these stresses may be derived from the Lamé solution for the stresses in a thick-walled elastic cylinder, which is given by Timoshenko. If the outer radius of the cylinder is allowed to become very large and the external pressure is set equal to zero, the solution becomes applicable to the wellbore problem and the radial, circumferential, and vertical stresses become

$$\left.\begin{aligned} \sigma_r &= +\,\Delta p \frac{a^2}{r^2}\,, \\ \sigma_\theta &= -\,\Delta p \frac{a^2}{r^2}\,, \\ \sigma_z &= 0\,, \end{aligned}\right\} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (7)$$

in which Δp is the increase in fluid pressure in the wellbore over the original pressure, a is the hole radius, and r is the distance from the center of the hole.

The circumferential stresses due to a pressure Δp in the wellbore are shown in Fig. 15. The stresses given are those caused by Δp alone, and to obtain the complete stress field it is necessary to superpose these stresses upon those caused by the pre-existing regional stresses which were calculated previously. This is illustrated in Fig. 16 in which a pressure equal to $1.6\sigma_A$ is applied to the wellbore for the case in which $\sigma_B/\sigma_A = 1.4$, and is just sufficient to reduce the circumferential stress to zero across one vertical plane at the walls of the hole. In all cases when the σ_B/σ_A-ratio is greater than 1, the vertical plane across which σ_θ first becomes zero as the wellbore pressure is increased is that perpendicular to σ_A, the least horizontal stress.

RUPTURE PRESSURES

In order to determine the rupture or breakdown pressures required to initiate fractures under various conditions, it is necessary to consider the properties of the rocks being fractured. The tensile strength of rock is a notoriously undependable quantity. For flawless specimens it ranges from zero for unconsolidated materials to several hundred pounds per square inch for the strongest rocks. However, as observation of any outcrop will demonstrate, flawless specimens of linear dimensions greater than a few feet rarely occur. In addition to the bedding laminations across which the tensile strength ordinarily is a minimum, the rocks usually are interesected by one or more systems of joints comprising partings with only slight normal dis-

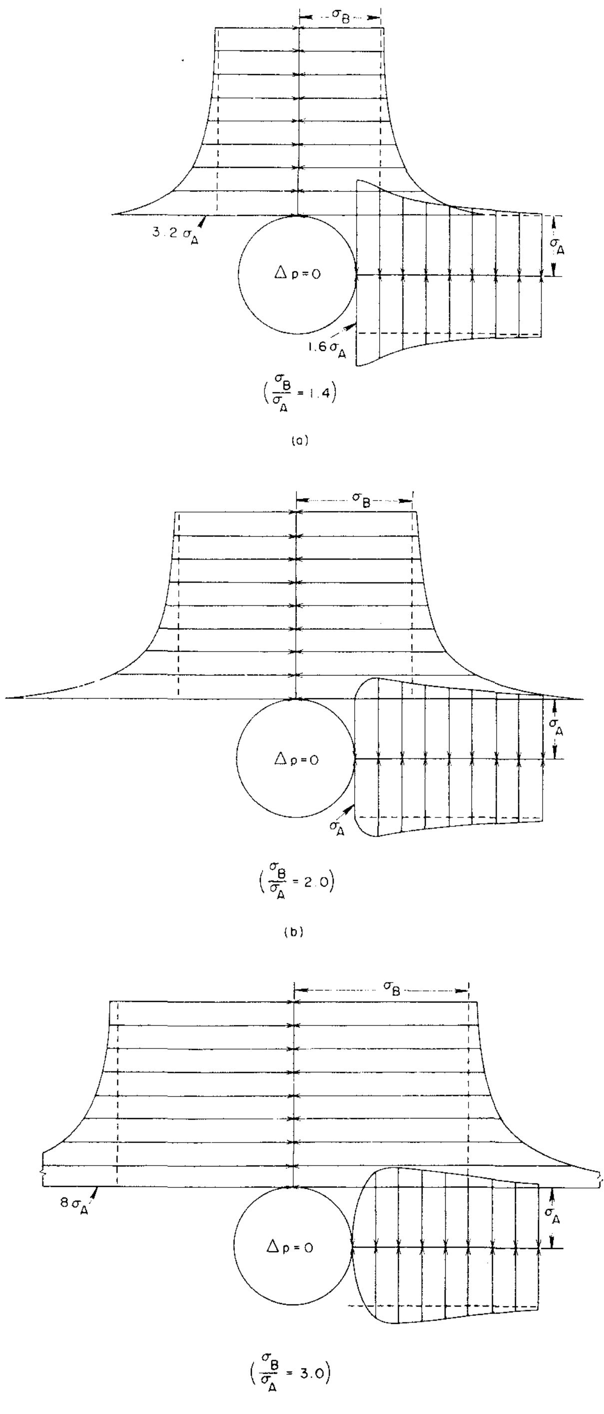

FIG. 14—STRESS STATES ABOUT A BOREHOLE FOR REGIONAL-STRESS RATIOS σ_B/σ_A OF 1.4, 2.0, AND 3.0.

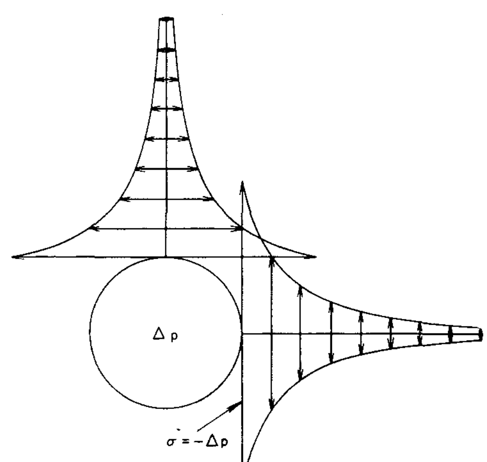

FIG. 15—STRESSES CAUSED BY A PRESSURE Δp WITHIN THE WELLBORE.

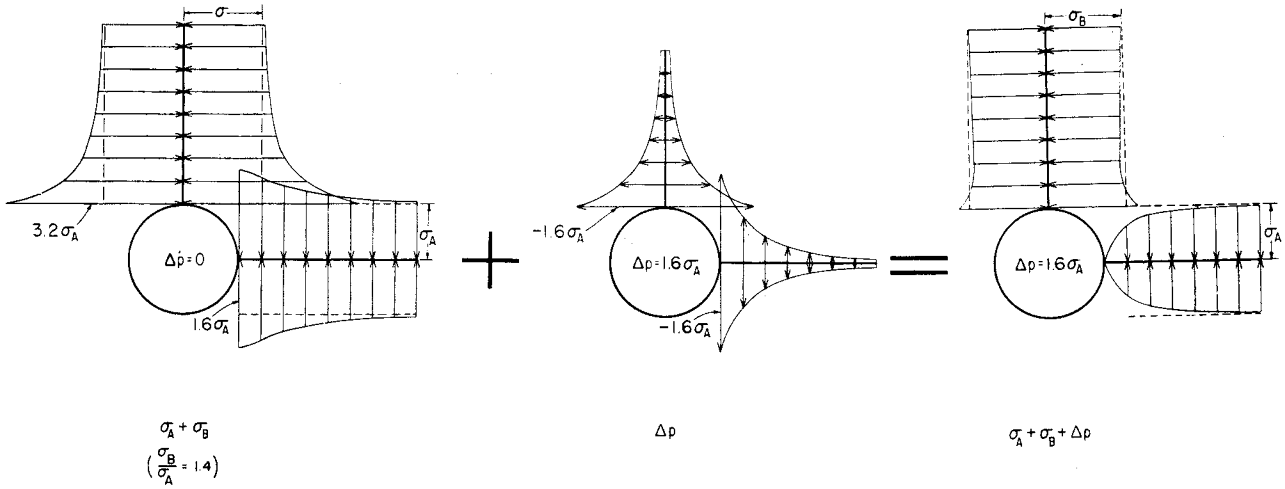

FIG. 16—SUPERPOSITION OF THE STRESSES DUE TO A PRESSURE Δp OF 1.6 σ_A UPON THE STRESSES AROUND A WELLBORE WHEN σ_B/σ_A IS 1.4.

placements. Across these joint surfaces the tensile strength is reduced essentially to zero.

In any section of a wellbore a few tens of feet in length, it is probable that many such joints have been intersected. It appears likely, therefore, that the tensile strengths of most rocks that are to be subjected to hydraulic fracturing by pressure applied in wellbores is effectively zero, and that the pressure required to produce a parting in the rocks is only that required to reduce the compressive stresses across some plane in the walls of the hole to zero.

As the pressure is increased, the plane along which a fracture will commence will be that across which the compressive stress is first reduced to zero. In the case of a smooth cylindrical wellbore, this plane must be vertical and perpendicular to the least principal regional stress. For the cases illustrated in Fig. 14, the least compressive stress across a vertical plane at the walls of the hole varies from twice σ_A to zero. Therefore, the down-the-hole pressure required to start a vertical fracture with a non-penetrating fluid may vary from a value of twice the least horizontal regional stress to zero, depending upon the σ_B/σ_A-ratio.

It can be seen from Eq. 7 that pressure inside a cylindrical hole in an infinite solid can produce no axial tension, suggesting that it is impossible to initiate horizontal fractures. However, under actual conditions in wellbores, end effects should occur at well bottoms or in packed-off intervals in which axial forces equal to the pressure times the area of the cross section of the hole would be exerted upon the ends of the interval. Furthermore, irregularities exist in the walls of the borehole which should permit internal pressures to produce tension.

In particular, as has been suggested by Bugbee,[16] the initial fractures may often be joints which have separated sufficiently to allow the entrance of the fluid, in which case it is only necessary to apply sufficient pressure to hold open and extend the fracture.

INJECTION PRESSURES

Once a fracture has been started the fluid penetrates the parting of the rocks and pressure is applied to the walls of the fracture. This reduces the stress concentration that previously existed in the vicinity of the wellbore and the pressure Δp required to hold the fracture open in the case of a non-penetrating fluid is then equal to the component of the undistorted stress field normal to the plane of the fracture. A pressure only slightly greater than this will extend the fracture indefinitely provided it can be transmitted to the leading edge. This can be seen from an analysis of an ideally elastic solid, as shown in Fig. 17. The normal stresses across the plane of a fracture near its leading edge are shown for the case in which the applied pressure Δp is slightly greater than the original undistorted stress field σ_A. This solution is derived directly from the solution for the stresses in a semi-infinite solid produced by a distributed load, which is presented by Timoshenko.[14]

The tensile stress near the edge of the fracture approaches an infinite magnitude for a perfectly elastic material. For actual materials this stress will still be so large that a pressure Δp only slightly greater than σ_A will extend the fracture indefinitely. The minimum down-the-hole injection pressure required to hold open and extend a fracture is therefore slightly in excess of the original undistorted regional stress normal to the plane of the fracture. The actual injection pressure will

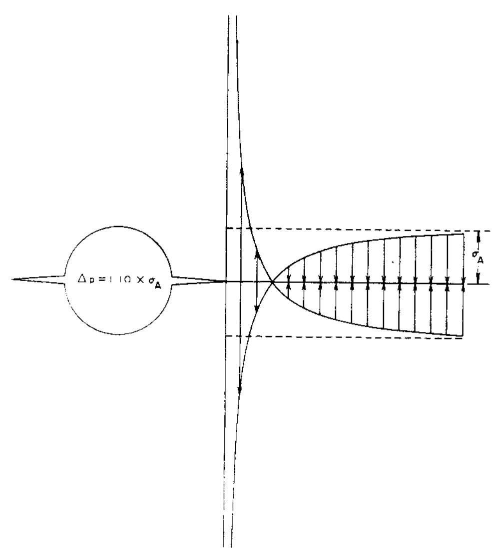

FIG. 17—STRESSES IN THE VICINITY OF A CRACK IN A STRESSED ELASTIC MATERIAL WHEN THE PRESSURE ACTING ON THE WALLS OF THE CRACK IS SLIGHTLY GREATER THAN THE STRESS WITHIN THE MATERIAL.

in general be higher than this minimum because of friction losses along the fracture.

PRESSURE BEHAVIOR DURING TREATMENT

A comparison of the breakdown and injection pressures required using non-penetrating fluids and for various values of the σ_B/σ_A-ratio shows that there are in general two types of possible down-the-hole pressure behavior during a fracturing treatment. These are illustrated in Fig. 18. The pressures $\triangle p$ are increases measured with respect to the original fluid pressure in the rocks. In one case the breakdown pressure may be substantially higher than the injection pressure. This would probably correspond to a horizontal fracture from a relatively smooth wellbore or to a vertical fracture under conditions in which the two horizontal principal stresses σ_A and σ_B were nearly equal. In the second case there is no distinct pressure breakdown during the treatment indicating that the pressure required to start the fracture is less than or equal to the injection pressure. This would correspond to a horizontal or vertial fracture starting from a pre-existing opening or to a vertical fracture in a situation where the ratio σ_B/σ_A of the horizontal principal stresses was greater than 2.0.

THE EFFECT OF PENETRATING FLUIDS

When a penetrating fluid is used in a fracturing operation, a more complicated mechanical situation exists. As noted previously the total normal stress S across any plane may be resolved into the sum of a residual solid stress σ and the fluid pressure p, or

$$S = \sigma + p \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Furthermore, with a non-penetrating fluid an increase in pressure $\triangle p$ equal to σ, or a total pressure $P = p + \triangle p$ equal to S, is required to hold open and extend a fracture along this plane.

For the case of a penetrating fluid an increment of pressure in the fracture, which now may be designated by $\triangle p_o$, will produce an outward flow of fluid into the rock with a resulting variable increment $\triangle p$ to the pressure within the formation. The gradient of this incremental pressure will exert an outward directed force upon both the rock and the contained fluid in each of the walls of the fracture. Let the normal component

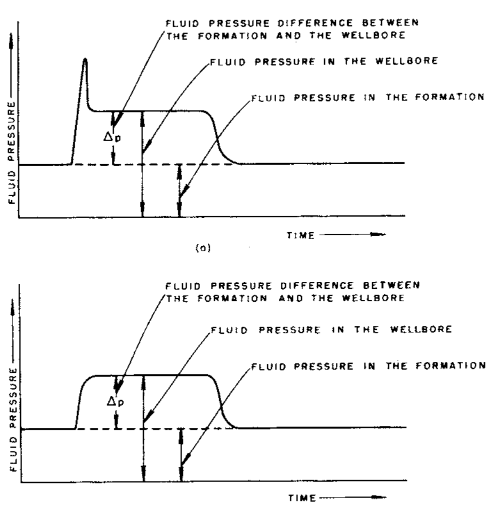

FIG. 18—IDEALIZED DIAGRAM OF TWO POSSIBLE TYPES OF PRESSURE BEHAVIOR DURING FRACTURE TREATMENT DEPENDING UPON VARIOUS UNDERGROUND CONDITIONS.

of this force acting upon the rock content of unit bulk volume be H_{n1} and that upon the fluid content H_{n2}. Since the volume of the fluid per unit bulk volume is the porosity f, and that of the rock $(1 - f)$, it follows that

$$H_{n1} = -(1-f)\frac{\partial(\triangle p)}{\partial n}, \quad . \quad . \quad . \quad . \quad . \quad (8)$$

$$H_{n2} = -f\frac{\partial(\triangle p)}{\partial n} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

However, due to viscous coupling the force H_{n2} acting upon the fluid is entirely transmitted to the rock so that the total outward force exerted upon the rock per unit of bulk volume will be

$$H_n = H_{n1} + H_{n2} = -\frac{\partial(\triangle p)}{\partial n} \quad . \quad . \quad . \quad . \quad (10)$$

Similarly, the total outward force per unit area of the fracture wall will be the integral of all the forces exerted upon the rock contained within a column of unit area of cross section normal to the fracture, or

$$\frac{F}{A} = -\int_0^\infty \frac{\partial(\triangle p)}{\partial n}dn = -\int_{\triangle p_o}^{0} d(\triangle p) = \triangle p_o \quad . \quad . \quad . \quad . \quad (11)$$

In order for the fracture to be held open and extended, this outward directed force per unit area must be equal to σ. Therefore, for this case

$$P = p + \triangle p = p + \sigma = S \quad , \quad . \quad . \quad . \quad (12)$$

which is exactly the same as the pressure required to hold the fracture open when a non-penetrating fluid is used.

In the case of radial flow away from a wellbore, the situation differs somewhat from that of flow away from a plane fracture. In the radial-flow case, a force acts outward whose magnitude per unit bulk volume is

$$\mathbf{H} = -\text{grad}(\triangle p) \quad , \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

and the effect of this distributed field of force is to diminish the stress concentration at the face of the hole. This in turn reduces the excess pressure that otherwise would be required to produce breakdown. Once the fracture is started, however, the flow field and the stress field become those associated with a plane fracture given in Eq. 12.

Therefore, the only effect of using a penetrating fluid is in the reduction of the breakdown pressure. The minimum injection pressure, for both penetrating and non-penetrating fluids, must be greater than the pre-existing normal stress across the plane of the fracture.

ORIENTATION OF THE FRACTURES PRODUCED

Returning to the earlier postulate that the fractures should occur along planes normal to the least principal stress, the minimum injection pressure should then be equal to the least principal stress. Considering the injection pressures and fracture orientations for various tectonic conditions, it follows that, in regions characterized by active normal faulting, vertical fractures should be formed with injection pressures less than the overburden pressure; whereas, in regions characterized by active thrust faulting, horizontal fractures should be formed with injection pressures equal to or greater than the overburden pressure.

In the particular case of horizontal fracturing, the total normal stress across the plane of the fracture is equal to the pressure due to the total weight of the overburden, and therefore the minimum injection pressure, regardless of whether the fluid is penetrating or non-penetrating, is also equal to the overburden pressure. It thus appears to be mechanically impossible for horizontal fractures to be produced with total fluid pressures less than the total overburden pressure.

Since the great majority of fracturing operations in the Gulf Coast, Mid-Continent, and West Texas-New Mexico regions require injection pressures less than the overburden pressure, it is difficult to escape the conclusion that most of these fractures are vertical. Furthermore, since the minimum pressures should be independent of the fluids used, there appears to be no valid basis for the claims that vertical vs horizontal fracturing can be controlled by variations in the penetrability of the fracturing fluids. In either case, it appears, the orientation of the fractures should be controlled by the pre-existing stress field of the rocks into which the fluid is injected.

PREDICTED INJECTION PRESSURES

It is interesting to estimate the actual values of the minimum injection pressures under conditions of incipient normal faulting such as may exist in many parts of the Gulf Coast area.

As has been pointed out (Eq. 5), the undisturbed effective vertical stress σ_z is equal to the total pressure of the overburden S_z less the original fluid pressure p existing within the rocks prior to disturbances such as fluid withdrawals. In algebraic form

$$\sigma_z = S_z - p \quad \ldots \ldots \ldots \ldots \quad (5)$$

Under conditions of incipient normal faulting, the least principal stress σ_A will be horizontal and will have a value of approximately one-third the effective overburden pressure σ_z. Therefore,

$$\sigma_A \cong (S_z - p)/3 \quad \ldots \ldots \ldots \quad (13)$$

Since the additional fluid pressure $\triangle p$ required to hold open and extend a fracture should be equal to the least principal stress, then

$$\triangle p \cong (S_z - p)/3. \quad \ldots \ldots \ldots \quad (14)$$

However, the total injection pressure P is given by

$$P = \triangle p + p. \quad \ldots \ldots \ldots \ldots \quad (15)$$

Therefore,

$$P \cong (S_z + 2p)/3. \quad \ldots \ldots \ldots \quad (16)$$

Dividing by depth z then gives

$$P/z \cong (S_z/z + 2p/z)/3 \quad \ldots \ldots \quad (17)$$

which is the approximate expression for the minimum injection pressure required per unit of depth in an area of incipient normal faulting.

The value of S_z/z is approximately equal to 1.00 psi/ft of depth for normal sedimentary rocks in most areas. Under normal hydrostatic fluid-pressure conditions, p/z is about 0.46 psi/ft of depth. Substituting these values into Eq. 17 gives

$$P/z \cong 0.64 \text{ psi/ft},$$

as the approximate minimum value that should be expected in the Gulf Coast.

Let us consider the values of P which would occur under conditions in which the original fluid pressure was other than hydrostatic. In those cases of an original fluid pressure less than hydrostatic, it can be seen from Eq. 16 that P would be correspondingly reduced. On the other hand, where abnormally high original fluid pressures prevail, P would become higher until in the limit, when the original pressure p approaches the total overburden pressure S_z, P also approaches the total overburden pressure, and fracturing will occur at pressures only slightly greater than the original fluid pressure.

Walker[4] has described an interesting example of lost circulation which might be explained on the basis of the foregoing analysis. In a Gulf Coast well drilling below 10,000 ft, the specific weight of the drilling mud, which was a little over 18 lb/gal, had to be kept constant to within 0.3 lb/gal, or about 2 per cent, to prevent either lost circulation when the density was too high, or else "kicking" by the formation fluids when the density was too low.

FIELD EVIDENCE

Present field data derived from experience with hydraulic fracturing, squeeze cementing, and lost circulation are fully consistent with the foregoing conclusions. In the Gulf Coast area Recent normal faulting indicates that vertical fractures should be formed with injection pressures less than the total overburden pressure. In the Mid-Continent and West Texas regions, older normal faulting, although comprising more ambiguous evidence, also favors vertical fracturing.

Howard and Fast[5] have summarized the pressure data from 161 squeeze-cementing and acidizing jobs performed in the Gulf Coast area and the West Texas-New Mexico area. Also, published data by Harrison, Kieschnick, and McGuire,[8] and by Scott, Bearden, and Howard,[6] describe injection pressures for large samples by hydraulic-fracturing operations in the Gulf Coast, Mid-Continent, and West Texas regions. With but few exceptions the injection pressures have been substantially less than the total overburden pressure and thus imply that vertical fractures are actually being formed.

In addition to the above data, the occurrence of lost circulation throughout the Gulf Coast area at pressure substantially less than that due to the weight of the overburden also supports the conclusion that the least stress should be horizontal in this area.

On the other hand, in much of California tectonic compression is taking place, and in these areas horizontal fractures should occur with injection pressures greater than the total overburden pressure. Although comparatively few fracturing operations have been performed in California, extremely high pressures are required with injection pressures commonly greater than the overburden pressure (Hassebroek[17]).

A phenomenon very similar to artificial formation fracturing but on a large scale is that of dike emplacement. It has been pointed out by Anderson[18] that igneous dikes should be injected along planes perpendicular to the axis of least principal stress. This situation is entirely analogous to that for artificial formation fracturing. A striking field example of the effect of a regional stress pattern upon the orientation of igneous dikes is afforded by the Spanish Peaks igneous complex in Colorado.

A map of this area is shown in Fig. 19, and a photograph of West Spanish Peak from the northwest, showing dikes cutting flat-lying Eocene strata, is presented in Fig. 20. H. Odé[19] has made a mathematical solution of the regional stress field which would most likely result from the presence of the structural features in the area. A comparison of the radial-dike system with the mathematical solution shows the dikes to be almost

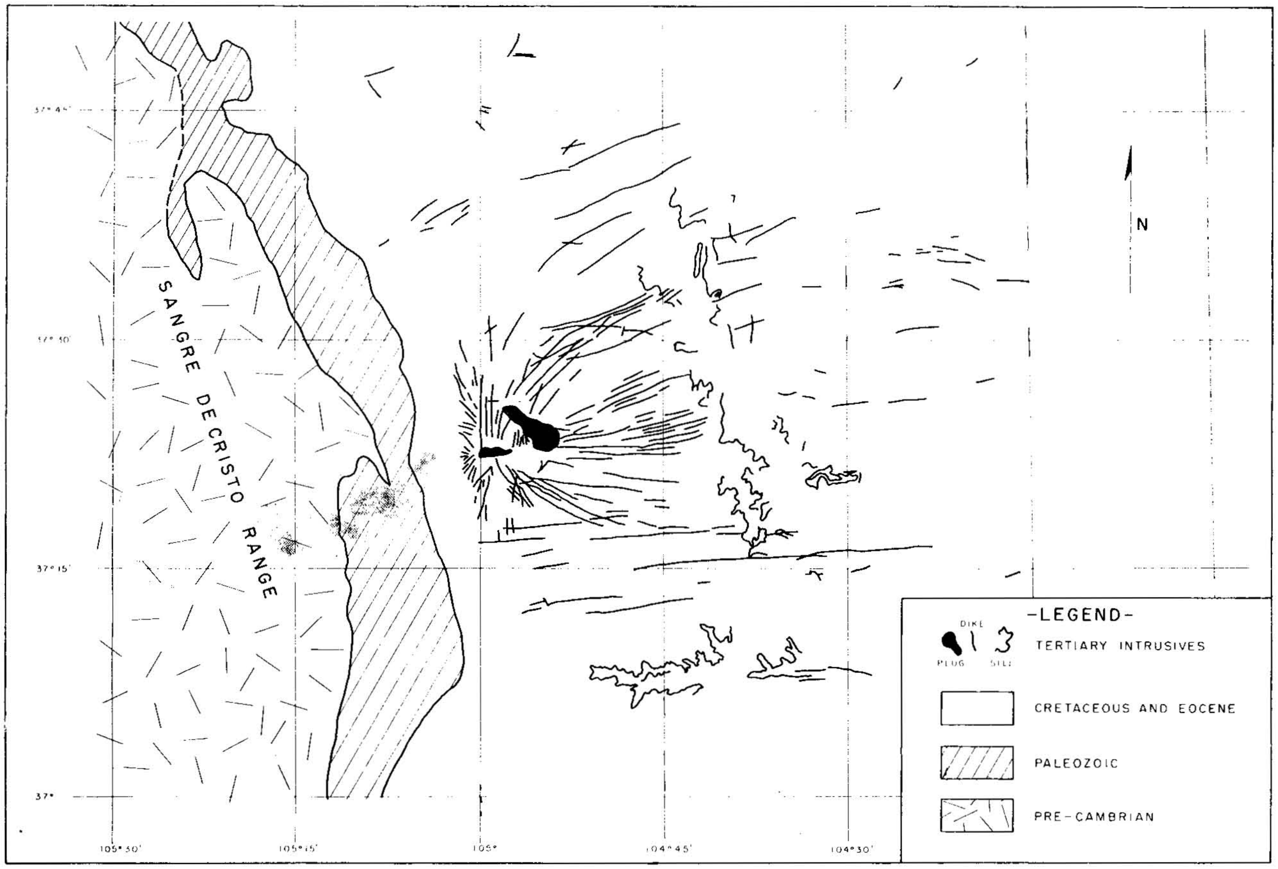

FIG. 19—DIKE PATTERN OF THE SPANISH PEAKS AREA, COLO.

exactly perpendicular to the trajectories of the least principal stress.

EXPERIMENTAL FRACTURING DEMONSTRATION

In order to verify the inferences obtained theoretically, a series of simple laboratory experiments has been performed. The general procedure was to produce fractures on a small scale by injecting a "fracturing fluid" into a weak elastic solid which had previously been stressed. Ordinary gelatin (12 per cent solution) was used for the solid, as it was sufficiently weak to fracture easily, was readily molded with a simulated wellbore, and was almost perfectly elastic under short-time application of stresses. A plaster-of-paris slurry was used as a fracturing fluid since this could be made thin enough to flow easily and could also be allowed to set and thus provide a permanent record of the fractures produced.

It is interesting to note that, in a model experiment conducted in this way, the stress distributions are entirely independent of scale. Provided the material is elastic, similitude will exist no matter on what length scale the experiment is conducted.

The experimental arrangement consisted of a 2-gal polyethylene bottle, with its top cut off, used as a container in which was placed a glass tubing assembly consisting of an inner mold and concentric outer casings. The container was sufficiently flexible to transmit externally applied stresses to the gelatin. The procedure was to place the glass tubing assembly in the liquid gelatin, and after solidification to withdraw the inner mold leaving a "wellbore" cased above and below an open-hole section. Stresses were then applied to the gelatin in two ways. The first, Fig. 21, was to squeeze the polyethylene container laterally, thereby forcing it into an elliptical cross section, and producing a compression in one horizontal direction and an extension at right angles in the other. The least principal stress was therefore horizontal, and vertical fractures should be expected in a vertical plane, as shown in Fig. 21.

In other experiments the container was wrapped with rubber tubing stretched in tension, Fig. 22, thus producing radial compression and a vertical extension. In this case, the least principal stress was vertical, and horizontal fractures could be expected, as shown in Fig. 22.

The plaster slurry was injected from an aspirator bottle to which air pressure was applied by means of a squeeze bulb.

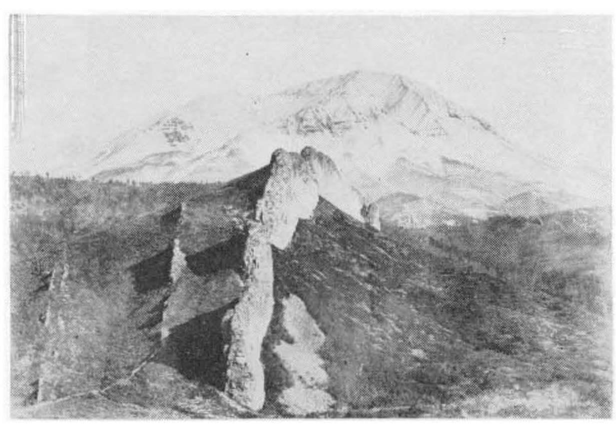

FIG. 20—PHOTOGRAPH OF WEST SPANISH PEAK FROM THE NORTHWEST, SHOWING DIKES CUTTING FLAT-LYING EOCENE STRATA (G. W. STOSE, U. S. GEOLOGICAL SURVEY).

Four experiments were performed under each of the two stress conditions, and in every case the fractures were formed perpendicular to the least principal stress. A vertical fracture is shown in Fig. 23 and a horizontal fracture in Fig. 24.

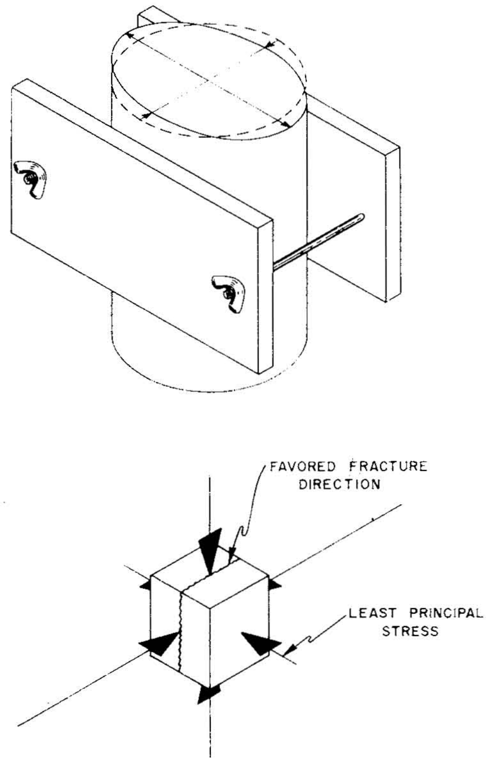

FIG. 21—EXPERIMENTAL ARRANGEMENT FOR PRODUCING THE LEAST STRESS IN A HORIZONTAL DIRECTION.

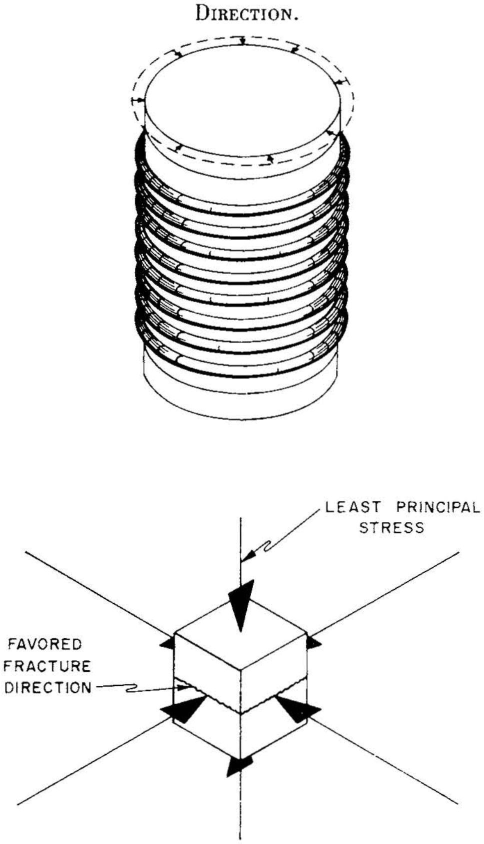

FIG. 22—EXPERIMENTAL ARRANGEMENT FOR PRODUCING THE LEAST STRESS IN A VERTICAL DIRECTION.

The saucer shape of the horizontal fracture is a result of the method of applying the stresses and requires some explanation. As the gelatin is compressed on all sides, it tends to be displaced vertically but is restrained by the walls of the container. This produces a shear stress causing the least principal stress to intersect the container at an angle from above. Therefore, when the fractures are formed normal to the least principal stress, they turn upward near the walls of the container producing the saucer shape shown in Fig. 24.

A further variation in the experiment consisted in stratifying the gelatin by pouring and solidifying alternate strong and weak solutions. One experiment was performed in this way under each stress condition. The

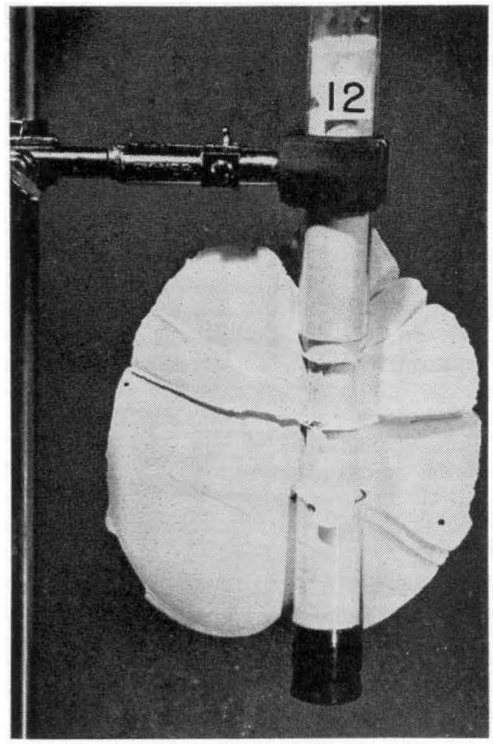

FIG. 23—VERTICAL FRACTURE PRODUCED UNDER STRESS CONDITIONS ILLUSTRATED IN FIG. 21.

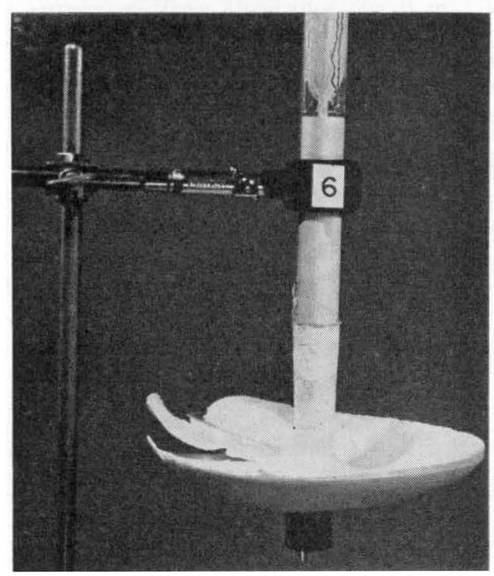

FIG. 24—HORIZONTAL FRACTURE PRODUCED UNDER STRESS CONDITIONS ILLUSTRATED IN FIG. 22.

vertical fracture is illustrated in Fig. 25, in which the weak gelatin appeared to fracture slightly more readily than the strong gelatin. Fig. 26 shows a horizontal fracture in stratified gelatin. In this case the fracture is not saucer shaped but appears to have followed a plane of weakness created by bubbles between two gelatin layers.

SIGNIFICANCE OF VERTICAL FRACTURING IN RESERVOIR ENGINEERING

In view of the foregoing evidence, it now appears fairly definite that most of the fracturing produced hydraulically is vertical rather than horizontal, so it remains to mention briefly what significance this may have in reservoir engineering. In geologically simple and tectonically relaxed areas, the regional stresses should be fairly uniform over extensive areas so that the horizontal stress trajectories in local areas should be nearly rectilinear. Consequently, when numerous wells in a single oil field are fractured, the fractures should be collimated by the stress field to almost the same strike.

This has serious implications, as Crawford and Collins[20] have pointed out, with respect to the direction of drive and the sweep efficiency in secondary-recovery operations. If the direction of drive should be parallel to the strike of the fractures, then the flow would be effectively short-circuited and the sweep efficiency would be very low. On the other hand, if the drive were normal to the strike of the fractures, the flow pattern would approximate that between parallel line sources and sinks and the sweep efficiency would approach unity.

This circumstance emphasizes the need, which is becoming increasingly more urgent, for the development of reliable down-hole instruments by means of which not only the vertical extent but also the azimuth of the fractures can be determined.

Since the foregoing paragraphs were written, these theoretical inferences have been strikingly confirmed by the fracturing experience during waterflood operations of North Burbank field, Okla. According to Z. Z. Hunter,[21] the initial pilot flood was based on the conventional five-spot pattern of injection and producing wells, but the results were anomalous. The injection wells were broken down at very low pressures (as low as one-fourth of the overburden pressure), and producing wells east and west of injection wells were frequently by-passed by the flood. Finally a sudden influx of water occurred in the isolated Stanley Stringer sand a mile to the east of the flood area.

Cumulative experiences of this kind, supplemented by fracture observations in oriented cores, led to the conclusion that the fractures were essentially vertical and oriented east and west. This realization led to a change of procedure wherein line drives were instituted from east-west rows of fractured injection wells to alternating rows of fractured producing wells. Greatly increased oil production without a corresponding increase in the water-oil ratio resulted.

The second question to be considered concerns the vertical migration of fluids. It needs hardly to be mentioned that vertical fractures will facilitate the vertical migration of fluids when the fractures intersect permeability barriers. They may in this manner interconnect a number of separate reservoirs in lenticular sands imbedded in shales, and may in fact tap some such reservoirs not otherwise in communication with the fractured well. There is a danger, however, in case a reservoir is overlain by a thin permeability barrier and a water-bearing sand, that a vertical fracture may also permit the escape of the oil and gas into the barren sands above.

A related question is that of the effect on water production of a vertical fracture which extends across the oil-water interface. In order to obtain an approximate idea of what this effect may be, consider a reservoir composed of a thick sand which is homogeneous and isotropic with respect to permeability. If production prior to fracturing is from an interval well above the

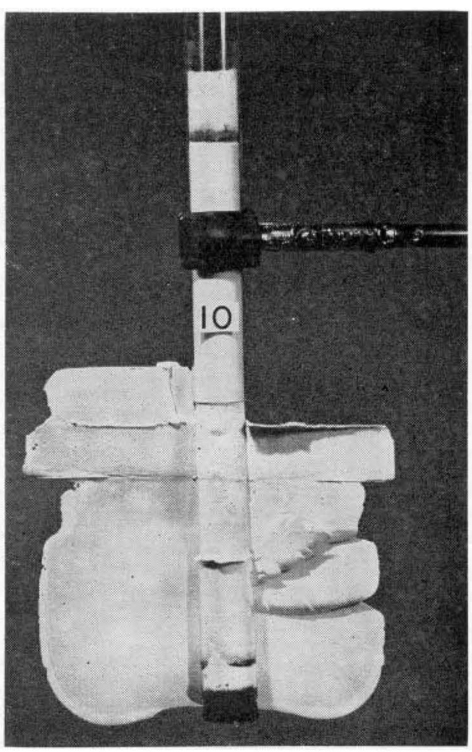

Fig. 25—Vertical Fracture in Stratified Gelatin.

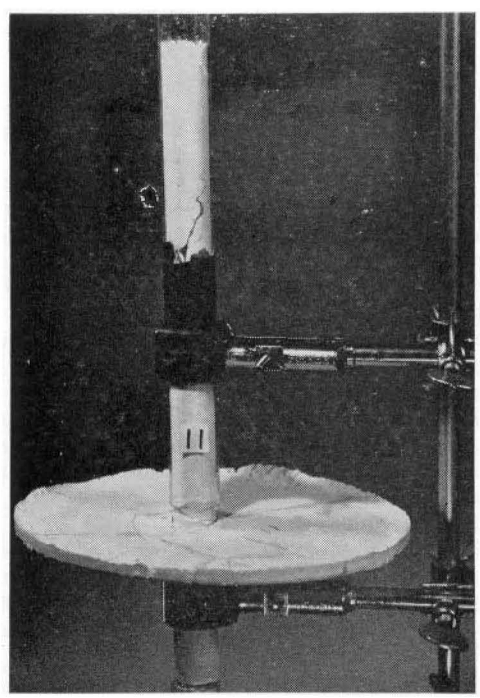

Fig. 26—Horizontal Fracture in Stratified Gelatin.

water table, the water will form a radially symmetrical cone, with a slope whose sine at any point is given by

$$\sin \theta = - \frac{\rho_o}{\rho_w - \rho_o} \cdot \frac{1}{g} \cdot \left| \text{grad } \Phi_o \right| , \quad . \quad (18)$$

where ρ_o and ρ_w are the densities of oil and water, respectively, g is the acceleration of gravity, and Φ_o the potential of the oil (Hubbert[22,23]).

The oil potential Φ_o at a given point is defined by

$$\Phi_o = gz + \frac{p}{\rho_o}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

where z is the elevation of the point with respect to sea level and p is the gauge pressure. Then by Darcy's law the volume of fluid crossing unit area in unit time will be

$$\mathbf{q} = -\frac{k\rho_o}{\mu} \text{grad } \Phi_o \;, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

where k is the permeability of the sand and μ the fluid viscosity. When this is substituted into Eq. 18, it gives for the tilt of the oil-water interface

$$\sin \theta = \frac{\rho_o}{\rho_w - \rho_o} \cdot \frac{\mu}{gk\rho_o} \cdot q. \quad . \quad . \quad . \quad (21)$$

Hence the sine of the angle of tilt is proportional to the rate of flow q of the oil along the interface.

We have now only to consider the flow patterns about the well without and with vertical fracturing. Without fracturing the flow converges radially toward the well with a rapidly increasing flow rate and a corresponding steepening of the cone. With fracturing, for the same rate of oil production from the well, the flow pattern approximates that of linear flow toward a vertical-plane sink. The maximum values of the flow velocity q for this case will be very much less than for the radial-flow case. Hence, for a given rate of oil production, a vertical fracture across the oil-water interface in a uniform sand, instead of causing an increase of water production, actually should serve to reduce markedly the water coning and consequently to decrease the production of water, a result in accord with reports of field experience wherein fracturing near the water table has not resulted in increased water production.

CONCLUSIONS

In the light of the foregoing analysis of the problem of hydraulic fracturing of wells, the following general conclusions appear to be warranted:

1. The state of stress underground is not, in general, hydrostatic but depends upon tectonic conditions. In tectonically relaxed areas, characterized by normal faulting, the least stress will be approximately horizontal; whereas, in areas of tectonic compression, characterized by folding and thrust faulting, the least stress will be approximately vertical and, provided the deformation is not too great, equal to approximately the overburden pressure.

2. Hydraulically induced fractures should be formed approximately perpendicular to the least principal stress. Therefore, in tectonically relaxed areas they should be vertical, while in tectonically compressed areas they should be horizontal.

3. Rupture or breakdown pressures are affected by the values of the pre-existing regional stresses, by the hole geometry including any pre-existing fissures, and by the penetrating quality of the fluid.

4. Minimum injection pressures depend solely upon the magnitude of the least principal regional stress and are not affected by the hole geometry or the penetrating quality of the fluid. In tectonically relaxed areas, the fractures should be vertical and should be formed with injection pressures less than the total overburden pressure. In tectonically compressed areas, provided the deformation is not too great, the fractures should be horizontal and should require injection pressures equal to or greater than the total overburden pressures.

5. It does not appear to be mechanically possible for horizontal fractures to be produced in relatively undeformed rocks by means of total injection pressures which are less than the total pressure of the overburden.

6. In geologically simple and tectonically relaxed areas, not only should the fractures in a single field be vertical but they also should have roughly the same direction of strike.

7. Vertical fractures intersecting horizontal permeability barriers will facilitate the vertical flow of fluids. However, in the absence of such barriers, vertical fractures across the oil-water, or gas-oil, interface will tend to reduce the coning of water, or gas, into the oil section for a given rate of oil production.

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DISCUSSION

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We are not in agreement with the authors' conclusions and feel that they have made a number of simplified theoretical assumptions which they have failed to support. Attempts to extrapolate their type of experimentation to reservoir conditions and to imply far-reaching conclusions is, to us, a serious mistake.

We are convinced that horizontal fractures can be created at any depth because we have seen the results of field experiments which indicated them. Spinner surveys and radioactive surveys run for the purpose of locating fractures are easier to believe than laboratory experiments in non-porous and non-permeable gelatin. Careful analysis of 18 surveys which gave positive results indicated 14 horizontal and four vertical fractures.

The authors have stated that low breakdown pressures in the bulk of a large number of fracturing operations imply the creation of vertical fractures. They have not supported this with experimental results. Our laboratory and field experimentations have led us to the opposite conclusion. In our opinion, their basic assumption of 1 lb of overburden pressure per foot of depth has not been supported experimentally.

We believe that fractures may be either horizontal or vertical. If incipient fractures (horizontal or vertical) exist, any type of fluid will extend them, rather than change their direction. If no fractures exist, we believe it has been proven that horizontal fractures can be started at any depth by penetrating breakdown liquids.

AUTHORS' REPLY to J. J. REYNOLDS and H. F. COFFER

Reynolds and Coffer query the authors' analysis and conclusions on the basis of the following propositions:

1. That horizontal fracturing at various depths has been demonstrated by spinner and radioactivity surveys.
2. That the existence of an overburden pressure gradient of approximately 1 lb/in.^{2}/ft is an unsupported assumption.
3. That any incipient fractures, whether vertical or horizontal, will be extended, and in the absence of such fractures penetrating fluids will produce horizontal fractures.

1. The critics' first point is based upon the assumption that spinner and radioactive surveys have reached a higher degree of reliability than the authors are willing to concede. The use of radioactive tracer techniques has recently been reviewed by G. L. Gore and L. L. Terry[1] of Dowell Inc., who, in discussing the use of these techniques in attempts to determine the nature of the fracturing, have made the following comments:

> "Logs made following fracturing treatments have indicated the presence of traces of radioactive material remaining in the wellbore. Such residual amounts of radioactive tracer usually result in high gamma-ray readings, resulting in erroneous interpretations. . . . The fact that a minor zone may take only a small amount of radioactive material, yet still give a strong reading, is one of the greater problems in the interpretation of the surveys. Actually, a 'cold spot' may have accepted most of the fracturing materials."

At best, spinner surveys, radioactive tracer observations limited to vertical freedom of motion, and electrical mud-loss instruments, can only determine with varying degrees of accuracy the vertical extent of the loss zone. A loss zone of substantial vertical extent is presumptive evidence of vertical fracturing; but a loss zone of limited vertical extent is not positive evidence that the fracture is horizontal. The reason for this is that a length of open hole is essentially "cased" by an annulus of stress concentration. Either in hydraulic fracturing or in the loss of circulation, the fluids may well break out through a small local hole through this stressed zone and then fan out into a large vertical fracture outside this zone of stress concentration, just as the vertical experimental fractures shown in Fig. 23 fan out behind the casing both above and below the open-hole section.

The most positive evidence of vertical vs horizontal fracturing would be that obtainable by some kind of a rotating scanning device, such as that shown by Gore and Terry in their Fig. 7. If the fracture is horizontal, the loss should have little correlation with azimuth; whereas, if it is vertical, a strong azimuthal correlation should be evident as the scanning tool is rotated in the hole.

2. The critics' second point is equivalent either to a contention that the bulk density of water-saturated sediments underground is unknown or else to a challenge of the validity of Newtonian mechanics.

If we consider a horizontal surface at some depth z, then the weight of all the material—sediments and their contained fluids—above that surface must be supported by the material below. This produces across such a surface a vertical normal stress of magnitude S_z, and at depths that are large as compared with the variations in topographic elevation, this normal stress will be nearly constant and will have the approximate value

$$S_z = \bar{\rho}_b g z = \bar{\gamma} z \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where $\bar{\rho}_b$ is the average bulk density of the overburden, g the acceration of gravity, and $\bar{\gamma}$ the average specific weight of the overburden. The quantity S_z is, by definition, the pressure due to the weight of the overburden.

The mean pressure gradient to the depth z is then obtained if we divide Eq. 1 by z:

$$\text{grad } S_z = S_z/z = \bar{\rho}_b g = \bar{\gamma} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Hence the vertical pressure gradient is known with the same accuracy to which $\bar{\rho}_b g$ or $\bar{\gamma}$ is known.

The value of g can be measured with great precision, and to a depth of 5 miles it varies by less than one part per 1,000. Hence, no significant error will re-

[1]References given at end of paper.

sult from regarding g as constant, and the pressure gradient will be known with approximately the accuracy to which the mean bulk density or mean specific weight of water-saturated sedimentary rocks can be determined.

Hundreds of published values of measured densities of sedimentary rocks are available. Table 2-6 of the *Handbook of Physical Constants*[2] contains 224 entries of measured densities of water-saturated sedimentary rocks. The values range between the extremes of 1.6 and 2.9 gm/cm^3, but 77 per cent fall within the range 2.2 to 2.6, and the highest frequency of occurrence is almost equally divided between 2.4 and 2.5.

L. F. Athy[3] has reported the density measurements on more than 2,200 samples, mostly of Pennsylvanian and Permian rocks from eastern Oklahoma and parts of Texas. His results were given in the form of dry bulk densities. When these are corrected for densities when water saturated, the mean values show the following variation with depth:

Depth (ft)	Water-Saturated Bulk Density (gm/cm^3)
0	2.26
1,000	2.41
2,000	2.52
3,000	2.59
4,000	2.62
5,000	2.64

Bulk densities can also be determined by the grain densities of the rock mineral and the porosity by means of the equation:

$$\rho_b = (1 - f)\ \rho_m + f\rho_w \ , \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where f is the porosity, ρ_m the mineral density, and ρ_w the density of water.

The porosities of 4,800 core samples taken from wells in nine states have been published by Rall, Hamontre, and Taliaferro[4] of the U. S. Bureau of Mines. The mean porosity is approximately 15 per cent. Then, with the density of water taken as 1.0 and the mean mineral density as 2.7 gm/cm^3, the mean rock density would be

$$\overline{\rho_b} = 0.85 \times 2.7 + 0.15 = 2.44 \text{ gm/cm}^3.$$

Densities of the Gulf Coast sediments have been determined from gravity studies of salt domes. The density of salt has the fixed value of 2.135 gm/cm^3, and salt domes are formed by the buoyant rise of less dense salt through overlying more dense sediments. According to Nettleton[5], gravity data give approximately the following variation of density with depth for the Gulf Coast sediments:

Depth (ft)	Density (gm/cm^3)
0	1.9
400	2.0
1,000	2.1
2,100	2.2
5,000	2.3
11,000	2.4
20,000	2.5

Another gravimetric determination of the densities of sediments in place has been made by Hammer[6], by measuring gravity at 200-ft intervals down a 2,247-ft vertical mine shaft near Barberton, Ohio. The mean density of the section, consisting of Devonian and Mississippian shales and sandstones, was 2.75 gm/cm^3.

These thousands of measurements are all consistent in indicating that the density of sedimentary rocks increases with age and depth of burial. For Tertiary rocks which have not been folded, the density increases from about 1.9 or 2.0 gm/cm^3 near the surface to about 2.2 at a depth of about 2,000 ft and 2.4 by about 10,000 ft. For older sedimentary rocks the mean density is close to 2.45.

Converting these results to pressure gadients in lb/in.^{2}/ft, water with a density of 1.0 gm/cm^3 has a specific weight of 6.24 lb/ft^3, and a vertical pressure gradient of 0.433 lb/in.^{2}/ft. A density of 2.0 gm/cm^3 will produce a gradient of 0.87 lb/in.^{2}/ft, and a density of 2.3 gm/cm^3 will produce a vertical pressure gradient of 1.0 lb/in.^{2}/ft. According to the density data just presented, it follows that the pressure gradient in Gulf Coast sediments should be about 0.87 lb/in.^{2}/ft at shallow depths, should increase to 1.0 at a depth of about 5,000 ft and should be greater than this amount at greater depths. For older rocks with a mean density of about 2.45 gm/cm^3, the mean pressure gradient should be about 1.06 lb/in.^{2}/ft.

Hence, unless the critics wish to repudiate Newtonian mechanics, the thousands of density determinations amply substantiate the generalization that the gradient of the overburden pressure in sedimentary rocks is to a close approximation equal to 1 lb/in.^{2}/ft of depth.

3. The validity of the critics' third point is largely dependent upon that of their second.

It was shown in the paper that the total fluid pressure P, which must be applied in any plane fracture to hold its walls apart, is equal to the total normal stress S tending to hold the walls together, independently of whether the fluid does or does not penetrate the rock; and that the minimum pressure required is equal to the least principal stress. If natural fractures occur essentially normal to the least stress, the fracture fluid may follow these, but it is unlikely that fractures transverse to this preferred direction will be opened.

In particular, in order to open up a horizontal fracture, a penetrating fluid would have to have a pressure great enough to lift the overburden, which, except at shallow depths in the Gulf Coast, would have to be a pressure in lb/in.^{2} equal approximately to the depth in feet.

That a penetrating fluid cannot lift the overburden with a pressure less than that required for a non-penetrating fluid can easily be demonstrated in the laboratory. If a vertical cylinder with a horizontal screen near the bottom is filled to the top with sand and water, a hydraulic pressure P can be applied at the bottom of the sand by means of water. When this is done, with and without an impermeable membrane at the bottom of the sand, it will be found that the water pressure required to lift the sand is the same in both cases, and is precisely equal to the total pressure exerted by the combined load of sand and water above the screen.

Consequently, when fracturing occurs at pressures well below that of the overburden, it appears to be mechanically impossible for those fractures to be horizontal.

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