Decline-Curve Analysis Using Type Curves—Analysis of Gas Well Production Data

by J.C. Palacio, ECOPETROL(Colombia)/Texas A&M U. and T.A. Biasingame, Texas A&M U.

This is a preprint -- subject to correction.

ABSTRACT
The analysis of gas production data is presently being done under many constraints that do not allow for rigorous modeling of these data. The traditional efforts not only assume highly idealized production conditions, such as constant bottomhole pressures, but these methods also treat the variation of the gas properties with pressure in an approximate manner at best. The abstractions neglect these variations at worst.

This paper illustrates theoretical developments and methods of application that can be used to analyze gas well performance data rigorously with decline curve analysis via type curves. These methods are based on the use of "modified time functions" and a new algorithm to compute gas in place which are capable of modeling the behavior of production data for variable rate and/or variable pressure drop conditions. Currently, the only methods to solve for gas in place are iterative schemes. The new algorithm is based directly on a steady state relation that avoids the necessity of iteration.

In addition, a modified "Fekovich-Carter" type curve is presented. This new graph gives the performance of constant rate and constant pressure gas flow solutions, the traditional transient flow solutions, and Arps decline curve stems.

Finally, detailed analysis procedures and interpretation strategies are provided. The new analysis techniques introduced are verified with simulation, and the application of the methods are also illustrated using field data.

INTRODUCTION
Decline curve analysis of production data is, in essence, a technique where actual production rate and time data are history matched to a theoretical model using either type curves or computer programs. The theoretical model chosen is then used to predict ultimate oil or gas in place volumes as well as formation properties.

References and illustrations at end of paper

Not only is rate-time analysis founded on proven flow principles but it is also capable of providing virtually the same information as a conventional transient test at no cost in lost production.

The present work introduces new theoretical developments along with their proper implementation techniques that have proven useful in analyzing and interpreting both liquid and gas production with decline curve methods. Emphasis is placed on the study of gas flow, and the problem is solved in a "totally liquid equivalent" fashion that allows the liquid solutions to be used to model gas flow.

In any case, the previous limitations imposed by either constant bottomhole pressure or constant rate are overridden, making it feasible to analyze production data obtained under actual field conditions.

The new solution for the gas problem is one based on a material balance-like time function and a new algorithm which allows: (1) the use of decline curves developed for liquids, (2) modeling of actual variable rate/variable pressure drop production conditions accurately, and (3) an explicit computation of gas in place.

Another important result of this work is a modified decline type curve that is based on the Fekovich type curve for liquid, and the Carter type curve for gas. This modified curve considers several possibilities of analysis for both single phase gas or liquid and when combined with the rest of the theoretical developments introduced in this work, it can be used to interpret production data regardless of the production scenario.

The tools introduced in this paper are important because (1) they provide a simple, direct and fast way to compute gas in place, (2) they allow inexpensive production tests to replace expensive transient tests, (3) in many cases they avoid the necessity of a long pressure buildup especially when dealing with tight, low permeability reservoirs, (4) they are not constrained by highly idealized assumptions regarding the production conditions of either liquid or gas and finally, (5) they rigorously account for the change in gas properties with pressure.
HARMONIC DECLINE FOR LIQUID

In Appendix A of this paper the combination of a material balance relation and the pseudosteady-state flow equation for liquid is shown to yield a very useful theoretical expression for decline curve analysis. This expression is given by Eq. A-10 and is repeated here as Eq. 1.

\[
\frac{q_L}{(p_L - p_{wf}} \ b_{Lm} = \frac{1}{1 + \left[ \frac{m_s}{b_{Lm}} \right]^2} \tag{1}
\]

where

\[
m = \frac{1}{N_{cr}} \tag{2}
\]

\[
b_{Lm} = \frac{141.2 \beta \mu \phi}{k_A} \left[ \frac{1}{2} \ln \left( \frac{A}{A_o} \right) \right] \tag{3}
\]

and

\[
\bar{t} = \frac{N_{cr}}{b_{Lm}} \tag{4}
\]

The dimensionless form of Eq. 1 is

\[
q_{Lm} = \frac{1}{1 + \bar{t}} \tag{5}
\]

where

\[
q_{Lm} = \frac{q_L}{(p_L - p_{wf}} \ b_{Lm} \tag{6}
\]

and

\[
\bar{t} = \frac{m_s}{b_{Lm}} \bar{t} \tag{7}
\]

Appendix A proves that Eq. 1 and Eq. 5 should retread exactly a harmonic decline on a Fetkovich\textsuperscript{2,3} type curve. This is done by comparing Eq. 5 to Arps\textsuperscript{5} general hyperbolic decline equation also in dimensionless form, given by

\[
q_{Lm} = \frac{1}{1 + (b_{Lm})^{\bar{t}}} \tag{8}
\]

Thus, whenever a decline curve exponent, \(b\), of unity is substituted into Eq. 8, the resulting equation has exactly the same form as Eq. 5. This is called a harmonic decline.

This fact implies that as long as the "material balance time", \(\bar{t}\), is used rather than ordinary time in the analysis, it is possible to model variable rate and/or variable pressure production scenarios for the single phase liquid case using the Fetkovich\textsuperscript{2,3} harmonic (\(b = 1\)) decline type curve stem.

Eq. 1 is obtained from a relation (Eq. A-5) that is strictly valid for pseudosteady-state. Nevertheless, it has been shown\textsuperscript{3} that Eq. A-5 works fairly well for transient flow. In other words, when either rate or pressure is changed, a transient is introduced. However, as long as the transient effect remains negligible, compared to the influence of the outer boundary, so that the well is not kept from reaching or maintaining pseudosteady-state like flow, Eq. 1 should yield accurate answers.

The decline curve analysis method proposed here is based on Eqs. 1, 5 and 8. These relations suggest that if \(\bar{t}\) is correctly calculated, then a scaled log-log plot of \(q_L\) vs. \(\bar{t}\) overlays the \(q_{Lm}\) versus \(q_{Lm}\) trend exactly for a harmonic decline of \(b = 1\) on the Fetkovich\textsuperscript{2,3} type curve. The relations that should be used to compute oil in place and reservoir properties are given by Eqs. A-14, A-15 and A-16.

HARMONIC DECLINE FOR GAS

Following a similar procedure presented for liquid, the appropriate gas flow analysis equations are derived in Appendix B. In Appendix B it is shown that for gas the most useful form of the flow equation that should be used to perform decline analysis under variable rate/variable pressure drop conditions is

\[
\frac{q_g}{P_{sc} - P_{wf}} \ b_{gas} = \frac{1}{1 + \left[ \frac{m_s}{b_{gas}} \right]^2} \tag{9}
\]

where

\[
m_s = \frac{1}{G_{st}} \tag{10}
\]

\[
b_{gas} = \frac{141.2 \beta \mu \phi}{k_A} \left[ \frac{1}{2} \ln \left( \frac{A}{A_o} \right) \right] \tag{11}
\]

\[
P_g = \frac{P_{sc} - P_{wf}}{P_{sc}} \int_0^p \frac{p}{\mu(p) \phi(p)} dp \tag{12a}
\]

and

\[
h_0 = \frac{P_{sc} - P_{wf}}{q_g} \int_0^t \frac{q_g}{\mu(p) \phi(p)} dt \tag{12b}
\]

The dimensionless form of Eq. 9 is

\[
q_{g'} = \frac{1}{1 + \bar{t}_{gas} q_{g'}} \tag{13}
\]

with

\[
q_{g'} = \frac{q_g}{P_{sc} - P_{wf}} \ b_{gas} \tag{14}
\]

It must be noted that the \(q_{g'}\) definition is now in terms of normalized pseudopressures and the modified dimensionless time function \(\bar{t}_{gas}\) is not in terms of real time nor material balance time, but in terms of the material balance pseudotime (Eq. 12b). Still, Eq. 13 should retread the path of a harmonic decline on the Fetkovich\textsuperscript{2,3} type curve because its form is identical to the Arps\textsuperscript{5} hyperbolic decline relation (Eq. 8) when a \(b = 1\) value is assumed.

This implies that the same techniques of analysis may be employed for both gas and liquids. However, although the computation of the pseudotime function in the gas case involves a much larger effort than the simple material balance time for the liquid case.

Specifically, an estimate of gas in place should be available in order to obtain the correct pseudotime values because that volume would permit the computation of the average reservoir pressure profile and this in turn would allow the estimation of the gas properties involved in the pseudotime integral. However, it is clear that \(G\) is an objective rather than an input value for decline curve analysis. Fram and Wattenbarger\textsuperscript{8} propose an iterative scheme on \(G\) to solve this problem in their constant bottomhole pressure study. Blasingame and Lee\textsuperscript{9} also suggest a similar iterative process in their reservoir limits testing work.

This work introduces a new algorithm that, for the most part, avoids the necessity to iterate on \(G\) for the most part. This algorithm is presented in the next section of this paper. Suffice it to say now that as long as the "material balance pseudotime", \(\bar{t}\), is used rather than ordinary time in the dimensionless decline
function, it is possible to model variable rate and/or variable pressure production scenarios for the single phase gas case, using the Fetkovich\textsuperscript{,3} liquid type curve for a harmonic decline.

Eq. 9 is also derived from a boundary dominated flow relation (Eq. B-4). However, the same comments made for Eq. 1 regarding the validity for both transient and boundary dominated flow are also applicable here.

The decline curve analysis procedure that is proposed for the gas case is based on Eqs. 9, 13 and B. These relations suggest that if \( q_s \) is calculated correctly, a scaled log-log plot of \( \frac{q_s}{P_{mf} \cdot P_{m}^{0.5}} \) vs. \( t_d \) overlays the original \( q_s \) versus \( t_d \) trend for a harmonic decline of \( b=1 \) on the Fetkovich type curve\textsuperscript{,3}. The relations which are used to compute gas in place and reservoir properties are given by Eqs. B-21, B-22 and B-23.

**EXPLICIT GAS IN PLACE COMPUTATION**

In order to avoid the use of cumbersome iterative techniques to solve the problem at hand, a new procedure to compute gas in place has been devised. The development of the proposed new procedure is presented in Appendix C. The two basic relations that allow that a one-time calculation be made to obtain \( G \) with only a few back substitutions are given below. First, we have the material balance equation

\[
\frac{dG}{d(\frac{t_d}{t_d^0})} = \frac{1}{P_{mf}} \left[ 1 - \frac{P_{m}^{0.5}}{P_{mf}^{0.5}} \right] \quad (16)
\]

and second, the gas flow equation for pseudosteady-state flow

\[
\frac{q_s}{P_{mf} \cdot P_{m}^{0.5}} = \frac{G}{t_d^0} \quad (17)
\]

The new algorithm makes use of the fact that both \( G \) and \( t_d^0 \) are fixed values in time. Thus, the combination of these two equations permits that the correct \( \frac{t_d}{t_d^0} \) profile in time be obtained. This profile is then used in Eq. 16 and a plot of \( G \) vs. \( t_d \) is made. The correct value for \( G \) is read from this plot where the computed data trend depicts a horizontal straight line as expected since the original volume in place is not a time dependent value. All this is done in an independent form, making the computation of the average pressure profile much more straightforward yielding a significant simplification for type curve analysis using the new pseudosteady function.

After having estimated the correct average reservoir pressure profile using the estimated gas in place based on the extrapolation method given above, the pseudosteady values needed for the decline curve analysis can now be computed. This method requires the use of the average pressures to estimate the fluid properties in order to allow integration. The proper estimation of suitable values of these fluid properties is straightforward since we only need to interpolate from a table of PVT data for the particular gas being studied.

**MODIFIED FETKOVICH-CARTER TYPE CURVE**

This section introduces a new type curve that has been created based on two previous type curves used for decline curve analysis. The first of the basic type curve is the one presented by Fetkovich\textsuperscript{,3} which illustrates liquid rigorous solutions and is used to obtain approximate solutions for gas. The second set of curves includes the solution presented by Carter\textsuperscript{,4} which contains rigorous liquid solutions and semi-rigorous gas stems.

The new type curve is referred to simply as the "modified curve" throughout the remainder of this paper.

**Type Curve Description**

The modified curve is shown here as Fig. 1. The first aspect that needs to be mentioned is that the plot combines the original solutions presented by both Fetkovich\textsuperscript{,3} and Carter\textsuperscript{,4} in order to collect those solutions into a single graph. It was necessary to plot Carter's type curve in terms of Fetkovich equivalent variables. What this means is that since Carter\textsuperscript{,4} presented his plot in terms of simple dimensionless variables \( q_{D0} \) vs. \( t_d \), these variables can be transformed to their equivalent Fetkovich\textsuperscript{,3} decline dimensionless variables \( q_{D0}^{*} \) vs. \( t_d^{*} \).

The variables used to generate the Fetkovich part of the modified curve are liquid types of variables. On the other hand, Carter's type curve involves pseudopressure in the dimensionless decline rate axis, but still uses actual time for the dimensionless decline time computation. Carter's\textsuperscript{,4} type curve represents a gas solution. However, it must be clear that Carter's\textsuperscript{,4} type curve is not a strictly rigorous technique, because it uses actual time rather than pseudopressure, and it assumes a constant bottomhole flowing pressure scenario. This last consideration allows for the variations in gas properties to be accounted for by simply identifying the value of the drawdown parameter, \( \lambda \). This parameter is the ratio between the viscosity-compressibility product at initial conditions and the value at the average reservoir pressure.

The variables for the type curve generation are given by the following relations which use nomenclature that avoids specifying the type of fluid

\[
q_{D0} = 141.2 B_{d} \frac{p_{f}}{k h} \frac{q}{(P_{mf} \cdot P_{m}^{0.5})} \ln \left( \frac{t_{d}}{t_{d0}} \right) - \frac{1}{2}
\]

\[
t_d^{*} = 0.00633 \frac{k}{p_{d0} \phi \mu \sigma_{f}} \left( \frac{1}{2} \frac{1}{r_{d}^{2}} \right) \ln \left( \frac{t_{d} - \frac{1}{2}}{t_{d0} - 0.5} \right)
\]

It must be noted that these equations represent a circular reservoir geometry. The \( P_{mf} \) variable represents the appropriate pressure function in each case, and \( t_{d0} \) is the appropriate time function for the Fetkovich\textsuperscript{,3} part of the modified curve. \( q_{D0} \) is taken as actual pressure and \( t_d^{*} \) is actual time. The original stems given by Fetkovich\textsuperscript{,3} (liquid case) are the solid lines in the modified curve. On the other hand, for the Carter stems on the type curve, \( P_{mf} \) is taken as pseudopressure rather than actual pressure, but \( t_d^{*} \) is still actual time.

The second important aspect is represented by the additional curves included. There are two additional cases closely related to Carter\textsuperscript{,4} original type curve. The first of these additions is the inclusion of new \( \lambda \) stems (0.875, 0.625, 0.30) that have values different from the original values (1, 0.75, 0.55). All these stems, original and new, appear as dotted lines on the modified curve.

The next addition that is that of the \( q_{D0}^{*} \) vs. \( t_d^{*} \) stems for different constant rate cases using Carter's strategy. These new solutions are the dashed lines on the modified curve. Carter\textsuperscript{,4} did not include these cases because he was only concerned with the constant bottomhole flowing pressure case. Since Carter's solution is intended to solve gas problems, these new additional solutions should prove useful in getting at least an approximate answer when dealing with a gas production scenario under constant flow rate. The generation of these stems was done by means of a \( t_{d0} \) based on actual time and a special \( q_{D0}^{*} \) termed \( q_{D0}^{*} \) which is given by

\[
q_{D0}^{*} = \frac{141.2 B_{d} \frac{p_{f}}{k h}}{P_{mf} \cdot P_{m}^{0.5} \cdot (p_{p,ab} - P_{mf})} \ln \left( \frac{t_{d0}}{t_{d0} - 0.5} \right)
\]

where \( p_{p,ab} \) is the abandonment pressure of the reservoir. This is done to ensure that these curves indicate production to the point where the reservoir would be totally depleted.

Theory has it that the \( b=1 \) solution is the constant rate liquid
solution. Therefore it should be expected that these additional curves lie close to the Fetkovich harmonic stems. It can be seen from the plot that this latter condition is not totally true. The reason why the additional curves do not match the h=1 stem is that gas rather than liquid is considered. Thus, the fluid property variations are not properly being accounted for since time rather than pseudotime is being used.

The analysis of liquid or gas production data using the modified curve requires suitable definitions of both \( p_{\text{hms}} \) and \( t_{\text{hms}} \) according to the needs and desired accuracy. The table given below summarizes the variables that are used for the generation of all the stems on the modified curve.

Table 1 - Generalized Variables for the Modified Type Curve

<table>
<thead>
<tr>
<th>Type Curve</th>
<th>( p_{\text{hms}} )</th>
<th>( t_{\text{hms}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fetkovich (original)</td>
<td>( p )</td>
<td>( t )</td>
</tr>
<tr>
<td>Carter (constant rate)</td>
<td>( p )</td>
<td>( t )</td>
</tr>
</tbody>
</table>

Type Curve Analysis Methods

We present the two tables below to clarify the usefulness of the different decline curve analysis tools in order to analyze and interpret single phase production data. The tables include the description of the data plotting functions that must be used in each case according to the specified requirements (liquid or gas flow, specific rate and pressure behavior). The additional definitions involved in Table 2 are the normalized pseudopressure and conventional pseudotime which are respectively

\[
p_{\text{f}} = \left( \frac{p_{\text{f}}(t)}{p_{\text{f}}(0)} \right) \int_0^t \frac{p(t')}{p_{\text{f}}(t')} dt'
\]

and

\[
t_{\text{f}} = \frac{1}{h_{\text{f}}(t)} \int_0^t \frac{1}{h_{\text{f}}(t')} dt'
\]

Table 2 - Current Decline Curve Analysis Methods

<table>
<thead>
<tr>
<th>Case</th>
<th>Pressure Function</th>
<th>Time Function</th>
<th>Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>General Liquid</td>
<td>( p )</td>
<td>( t )</td>
<td>Fetkovich ( h ) stems as necessary</td>
</tr>
<tr>
<td>Liquid constant</td>
<td>( p )</td>
<td>( t )</td>
<td>Fetkovich ( h=0 )</td>
</tr>
<tr>
<td>( P_{\text{ef}} ) (rigorous)</td>
<td>( P_{\text{ef}} )</td>
<td>( t )</td>
<td>Carter ( h ) stems as necessary</td>
</tr>
<tr>
<td>General Gas</td>
<td>( P_{\text{ef}} )</td>
<td>( t )</td>
<td>Fetkovich ( h=0 )</td>
</tr>
<tr>
<td>(semi-rigorous)</td>
<td>( P_{\text{ef}} )</td>
<td>( t )</td>
<td>Carter ( h ) stems as necessary</td>
</tr>
<tr>
<td>Gas constant ( P_{\text{ef}} ) (rigorous)</td>
<td>( P_{\text{ef}} )</td>
<td>( t )</td>
<td>Fetkovich ( h=0 )</td>
</tr>
</tbody>
</table>

In Table 3 the only additional definition required is that of the material balance pseudotime given by Eq. 12. This is the only variable that not only rigorously accounts for the gas properties variation, but also permits any pressure or rate profile for gas flow to be studied and matched to a harmonic decline stem.

Table 3 - Decline Curve Analysis Methods Introduced in This Work

<table>
<thead>
<tr>
<th>Case</th>
<th>Pressure Function</th>
<th>Time function</th>
<th>Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liquid, variable pressure model</td>
<td>( p )</td>
<td>( t )</td>
<td>Fetkovich ( h=0 )</td>
</tr>
<tr>
<td>Gas, variable pressure model</td>
<td>( P_{\text{ef}} )</td>
<td>( t )</td>
<td>Fetkovich ( h=0 )</td>
</tr>
</tbody>
</table>

ANALYSIS OF GAS PRODUCTION DATA FOR VARIABLE RATE/VARIABLE PRESSURE DROP CONDITIONS

The general analysis procedure provided below includes two options which represent two different forms of estimating the value of gas in place. The first of these options is based on the use of the simple material balance time function \( t \), that this work has proven to be theoretically rigorous, yielding a harmonic decline for the case of liquid flow. This approach is proposed because it has been noticed that even though the use of the \( t \) function is not a rigorous approach for gas, it can still provide an approximate value of gas in place.

The second option makes direct use of the new algorithm described above to estimate the value of gas in place. All the examples in this paper will make use of the new gas in place algorithm so that a full understanding of its capability and functioning can be gained.

For variable pressure drop/variable flow rate conditions the proposed method to analyze gas production data involves the following basic steps:

1. Computation of the Basic Plotting Function for the Vertical Axis

Conversion of pressures \( p \) to normalized pseudopressures \( P_{\text{ef}} \) and computation of the \( P_{\text{ef}} \) group. This group constitutes the simplest vertical axis plotting function to perform type curve matching.

2. Computation of Gas in Place

Option A:

Calculate the \( t \) values for each time recorded in the input data set. This function is

\[
t = \frac{1}{q_{\text{f}}(t)} \int_0^t q_{\text{f}}(t') dt' = \frac{G_{\text{f}}}{q_{\text{f}}}
\]

Compute two complementary plotting functions for the vertical axis based on \( t \). That is, the integral of \( P_{\text{ef}} - P_{\text{ct}} \) with respect to \( t \) and the derivative of that integral, again with respect to \( t \). These two additional plotting functions have mathematical forms that follow the well testing practice which intends to keep the units for both the integral and the integral derivative the same as those of the original function. Thus, those plotting functions are given...
\[
\begin{align*}
\left( \frac{q_t}{P_{wi} - P_{rwi}} \right) &= \frac{1}{t_a} \int_0^t \frac{q_t}{P_{wi} - P_{rwi}} \, dt \\
\left( \frac{\partial q_t}{\partial (P_{wi} - P_{rwi})} \right) &= \frac{1}{\partial (P_{wi} - P_{rwi})} \left[ \frac{\partial q_t}{\partial (P_{wi} - P_{rwi})} \right]
\end{align*}
\]

The reader is referred to the original references\textsuperscript{10} where these definitions and the appropriate type curves were first presented for a thorough discussion on this matter. These variables can be used to perform type curve matching to different decline curves that have been presented previously in the literature.\textsuperscript{1,2,10} All the different type curves used here are based on the same principles as the Fetkovich type curve\textsuperscript{2}.

Thus, in all cases, a \(b=1\) stem indicates a harmonic type of decline. Since this case is the fundamental principle on which the new techniques rely, no further comments regarding the type curves will be made and their use will be explained in the example section.

After computing these plotting functions, we match the three different vertical plotting functions versus \(t\), to liquid decline type curves, such as those given by Fetkovich\textsuperscript{1,2} and McCray\textsuperscript{10} in order to estimate the best possible value of gas in place, \(G\).

\[
G = \frac{1}{c_u} \int_{P_{wi}}^{P_{rwi}} \left( \frac{q_t}{(P_{wi} - P_{rwi})} \right) \, dP_{wi}
\]

The subscript "\(m\)" in the latter relation can be either deleted or substituted with "\(r\)" or "\(i\)." This means that the group with the "\(m\)" subscript may be the basic plotting function for the vertical axis, its integral or its integral derivative.

Option B:

Use the new algorithm based on the combination of the material balance equation and the boundary dominated equation for gas flow to obtain the \(P\) profile. Make a plot of \(G\) versus time and determine the best possible value for gas in place, reading it where the line becomes straight and flat. To compute the values of \(G\) versus time, a rearrangement of the material balance equation given by the following relation must be used

\[
G = \frac{G_{wi}}{1 - (1 - G_{wi}) t}
\]

3. Computation of the Rigorous Plotting Function for the Horizontal Axis

Using the \(G\) value computed in the previous step, by either method, reevaluate the average reservoir pressure profile in time using the material balance equation, and calculate the material balance pseudotime functions given by Eq. 12. These relations are

\[
\begin{align*}
\tilde{t}_a &= \frac{P_{rwi}}{q_t} \int_0^{t_a} \frac{q_t}{P_{rwi} \Omega(t)} \, dt \\
k &= \frac{B_{pwi} \mu_i}{h} \left[ \frac{f_{wi}}{r_{wi}} \right]^{1/2} \left( \frac{P_{wi} - P_{rwi}}{P_{rwi}} \right) \mu_i \Omega(t)
\end{align*}
\]

4. Computation of the Additional Plotting Functions for the Vertical Axis

Compute the complementary plotting functions for the vertical axis based on \(G\). That is, the integral of \(1 - G\) with respect to \(t\) and the derivative of that integral, again with respect to \(t\). These two additional plotting functions are mathematically\textsuperscript{10} expressed in a manner to keep the units for both the integral and the integral derivative the same as those of the original function. In this case, these plotting functions are given\textsuperscript{10} as:

\[
\left( \frac{q_t}{P_{wi} - P_{rwi}} \right) = \int_{t_a}^t \left( \frac{q_t}{P_{wi} - P_{rwi}} \right) \, dt
\]

and

\[
\left( \frac{\partial q_t}{\partial (P_{wi} - P_{rwi})} \right) = \int_{t_a}^t \left( \frac{\partial q_t}{\partial (P_{wi} - P_{rwi})} \right) \, dt
\]

5. Matching of the Different Plotting Functions to Liquid Type Curves

Use the Fetkovich\textsuperscript{1,2} and McCray\textsuperscript{10} type curves to match the plotting functions to the harmonic stems of each type curve in all cases, then establish the match point and the appropriate \(\mu_i\) value. Using these latter estimates, compute the formation permeability and skin as well as the final gas in place value. If a circular reservoir geometry is assumed, the following relations should be used:

\[
G = \frac{1}{c_u} \int_{P_{wi}}^{P_{rwi}} \left( \frac{q_t}{(P_{wi} - P_{rwi})} \right) \, dP_{wi}
\]

\[
k = \frac{B_{pwi} \mu_i}{h} \left[ \frac{f_{wi}}{r_{wi}} \right]^{1/2} \left( \frac{P_{wi} - P_{rwi}}{P_{rwi}} \right) \mu_i \Omega(t)
\]

\[
r_{wi} = r_{wi}
\]

\[
s = \ln \left( \frac{r_{wi}}{r_{wi}} \right)
\]

APPLICATION EXAMPLES

Prior to presenting the examples, we want to emphasize that the design of the type curves allows us to match one curve or set of curves then translate that match to other type curves. Through this arrangement, we can match on the Fetkovich and McCray type curves separately or in unison, then share the results.

Constant Bottom Hole Flowing Pressure Case

The first example is a simulated data set. The pertinent reservoir and fluid data that are input into the simulator are provided in Table 4. A numerical simulator\textsuperscript{11} was used to obtain the corresponding rate profile. Then, the step procedure described in the previous section was followed, using option B for the computation of gas in place. The resulting plot of \(G\) vs. time is shown here as Fig. 4.
Table 4 - Data Input for First Simulated Example

<table>
<thead>
<tr>
<th>Reservoir Properties</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Reservoir Area, A</td>
<td>80 acres (24.745 ft)</td>
</tr>
<tr>
<td>Wellbore radius, r_w</td>
<td>0.2 ft</td>
</tr>
<tr>
<td>Net pay thickness, h</td>
<td>30 ft</td>
</tr>
<tr>
<td>Permeability, k</td>
<td>0.3 md</td>
</tr>
<tr>
<td>Initial Reserve Pressure, p_i</td>
<td>4000 psia</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fluid Properties</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Gas gravity, γ_g</td>
<td>0.65 (air=1)</td>
</tr>
<tr>
<td>Gas volume factor at p_i, B_g</td>
<td>0.6929 RB/Mscf</td>
</tr>
<tr>
<td>Gas viscosity at p_i, μ</td>
<td>2.3485x10^{-5} cp</td>
</tr>
<tr>
<td>Gas compressibility at p_i, c_g</td>
<td>1.4783x10^{4} psia^{-1}</td>
</tr>
<tr>
<td>Temperature, T</td>
<td>200 °F</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Production Characteristics</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Bottomhole pressure, p_r</td>
<td>500 psia</td>
</tr>
</tbody>
</table>

It is clear from this plot that the value of 4.15x10^6 Mscf obtained for G with the new method is very close to the actual gas in place (4.0x10^6 Mscf). This verifies, at least partially, the validity of the new algorithm. Using the gas in place value from Fig. 4, the “material balance” pseudotime functions and all the plotting functions described in the previous section are calculated. Figs. 5 and 6 summarize the results in a graphical fashion.

As can be seen from Figs. 5 and 6, a match to a harmonic decline with a value of unity was always obtained. This example verifies the use of t_bar for variable rate processes as well as accounting for the variation in gas properties with pressure. The value of the match point value appears as a legend in each plot. Using this match point it is possible to find the final gas in place value, and the formation permeability by means of the equations presented in the preceding section. Thus,

\[
G = \left( \frac{1}{1.4783 \times 10^4 \text{ psia}^{-1}} \right) \left( \frac{165 \text{ Mscf/d}}{1} \right) \left( \frac{1}{1} \right) = 4.018 \times 10^6 \text{ Mscf}
\]

Now, we use a zero value for the skin factor as was considered in the data input into the simulator. This gives

\[
k = 141.2 \left( \frac{0.6929 \text{ RB/Mscf}(2.3485 \times 10^{-5} \text{ cp})}{30 \text{ ft}} \right) \left( \frac{1}{165 \text{ Mscf/d}} \right) \left( \frac{1}{0.2 \text{ ft}} \right) \left( \frac{1}{2} \right) = 97.6 \text{ md}
\]

It can be seen that the results obtained after performing the type curve match agree with the input data for both G and k, proving that the analysis is correct.

### Variable Flow Rate Case

The second example considers a scenario of step wise changes in gas flow rate and uses the response obtained from inputting such a rate profile into the simulator. The basic input data are the same as those used in the previous example. However, the permeability value is changed from 100 md to 1 md. The flow rate is made to vary in step-wise changes and this profile is given by a series of decreasing and increasing rates shown below.

<table>
<thead>
<tr>
<th>t (days)</th>
<th>q_i (Mscf/D)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-365</td>
<td>700</td>
</tr>
<tr>
<td>365-1095</td>
<td>600</td>
</tr>
<tr>
<td>1095-2191</td>
<td>450</td>
</tr>
<tr>
<td>2191-2356</td>
<td>500</td>
</tr>
<tr>
<td>2357-2922</td>
<td>650</td>
</tr>
<tr>
<td>2923-3650</td>
<td>780</td>
</tr>
</tbody>
</table>

The gas in place plot is presented again as both Cartesian and semilog graphs. These graphs are shown here as Figs. 7 and 8. It is clear that the sudden changes in gas flow rate introduce transients in the reservoir. These transients cause some spikes in the expected behavior of G with time, but once a pseudo-steady-state flow is reestablished, the data points tend to lie back on the expected horizontal straight line.

It is also seen in Figs. 7 and 8 that the data points corresponding to the last two sudden changes in flow rate do not quite reach a point where they level off to depict a horizontal straight line, as was the case for the first four changes. This behavior can be understood as the result of a transient introduced whose effects did not disappear, preventing the reservoir from returning to full pseudo-steady-state flow. Given that the flow relation used in the new algorithm to compute gas in place is certainly a boundary-dominated flow type of equation, the shape of the lines in the gas in place plots is not unexpected.

In spite of this behavior, the adequate horizontal straight line is still readily defined with the first four rate changes, so that the value of gas in place can be estimated with a good degree of accuracy. In fact, in this case the approximate value of gas in place was computed as 4.082x10^6 Mscf which is very close to the actual G value of 4.04x10^6 Mscf.

Now, just as in the previous example, the computed gas in place value is used to calculate the pseudotime functions and then a match to the same type curves used before was attempted. Figs. 9 and 10 show the results obtained for the type curve matching procedure in this case.

Although the variable rate profile produced spikes in the bottomhole flowing pressure values, the magnitude of the pressure changes does not distract from a good estimate of gas in place. The magnified "departure effect" that is caused by an integration of the normalized rate data and later by further differentiation of the results of that integration, is not very significant.

As before it is perfectly reasonable to perform the gas in place and permeability computations using the match point read for any of the three type curves. Using exactly the same procedure and equations employed in the previous example and also assuming that the formation damage parameter, s, has a zero value, the values found for G and k are

\[
G = \left( \frac{1}{1.4783 \times 10^4 \text{ psia}^{-1}} \right) \left( \frac{325 \text{ D}}{1} \right) \left( \frac{1.85 \text{ Mscf/d}}{1} \right) = 4.067 \times 10^6 \text{ Mscf}
\]

and

\[
k = 141.2 \left( \frac{0.6929 \text{ RB/Mscf}(2.3485 \times 10^{-5} \text{ cp})}{30 \text{ ft}} \right) \left( \frac{1}{1.85 \text{ Mscf/d}} \right) \left( \frac{1}{0.2 \text{ ft}} \right) \left( \frac{1}{2} \right) = 1.094 \text{ md}
\]

The G value from the new algorithm with the actual one of 4.04x10^6 Mscf. The computed k value also agreed with the input value of 1.0 md.
**Variable Bottomhole Pressure Case**

The third example to be illustrated here is also a simulated case. This case, however, involves a production scenario of variable bottomhole flowing pressure. The basic input data are the same as those used in the variable rate example except now the bottomhole pressure is made to vary in step wise changes. The $p_{w}$ profile is shown below.

<table>
<thead>
<tr>
<th>$t$ (days)</th>
<th>$p_{w}$ [psia]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-465</td>
<td>4000</td>
</tr>
<tr>
<td>562-730</td>
<td>2000</td>
</tr>
<tr>
<td>731-1095</td>
<td>800</td>
</tr>
<tr>
<td>1096-1719</td>
<td>500</td>
</tr>
<tr>
<td>2191-3287</td>
<td>300</td>
</tr>
</tbody>
</table>

The gas in place plots obtained in this case are presented as Figs. 11 and 12. The first of these plots is a semilog plot of $G$ vs. time that is presented along with the corresponding Cartesian plot (Fig. 12). These two forms of the $G$ plot are included to help visualize the real effects of the variable pressure profile and also facilitate the choice of the correct $G$ value. In these graphs it is seen that every sudden change in bottomhole flowing pressure introduces a transient in the reservoir that causes some spikes in the expected behavior of $G$ with time. Nevertheless, when pseudosteady-state flow is reestablished, the data points tend to lie back on a horizontal straight line, which is the result expected since $G$ is a fixed value.

The approximate value of $G$ found with the new algorithm is 4,152x10^6 Mcf, which is close to the actual one of 4,04x10^6 Mcf. The former value of $G$ was used to compute the material balance pseudotime functions and then the type curve matching attempt was attempted. The results of the type curve matching attempts are shown in Fig. 13. It is clear from this plot that the effect of the transients introduced have a significant effect on the integral curve.

It must be noted in Fig. 13 that the plot of plain normalized rates closely matches Petkovitch harmonic decline. However, the same is not true for the normalized rate integral.

This departure from the correct model is due to the fact that even though the sudden changes in bottomhole flowing pressures still allow us to use the raw rate data, when those data are integrated the effect is magnified making it very difficult to model the situation with boundary dominated flow equations. In other words, the simulator response to a sudden reduction in $p_{w}$ always includes flow rate values that are far from being feasible in any actual production field (the rate spikes usually diminish in less than two days). When these "spike" rates are used, as such, for type curve matching the boundary dominated flow equations are still capable of modeling the situation fairly accurately. However, the spike rates make it difficult to match these data to the appropriate integral type curve.

In spite of the previous considerations, Fig. 13 shows that although the data do not match McCray's type curve, a trend is still observed that seems to approach the harmonic decline stem. Using the normalized rate data does yield a good match to Petkovitch type curve, the required values of $G$ and $k$ were then computed in exactly the same form used in the previous example.

$$G = \frac{(320 D)(1.9 \text{ Mcf/Mdpsi})}{(1.4783 \times 10^{-4} \text{ psia}^{-1})} = 4.113 \times 10^6 \text{ Mcf}$$

### Field Case Example

As a field example we present Gas Well A which is in a low permeability gas reservoir. That is, the permeability, $k$, is generally less than 0.1 md. The bottomhole flowing pressures and flow rates are recorded on a daily basis. Finally, very significant liquid loading and seasonal effects have been detected.

The following table summarizes the values of the pertinent formation and fluid properties.

<table>
<thead>
<tr>
<th>Table 5 - Data Input for Field Example</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Reservoir Properties</strong></td>
</tr>
<tr>
<td>Reservoir Area, $A$</td>
</tr>
<tr>
<td>Wellbore radius, $r_w$</td>
</tr>
<tr>
<td>Net pay thickness, $h$</td>
</tr>
<tr>
<td>Porosity, $\phi$ (fraction)</td>
</tr>
<tr>
<td>Initial Reservoir Pressure, $p_i$</td>
</tr>
<tr>
<td><strong>Fluid Properties</strong></td>
</tr>
<tr>
<td>Gas gravity, $g$</td>
</tr>
<tr>
<td>Gas volume factor at $p_i$, $B_g$</td>
</tr>
<tr>
<td>Gas viscosity at $p_i$, $\mu_i$</td>
</tr>
<tr>
<td>Gas compressibility at $p_i$, $C_g$</td>
</tr>
<tr>
<td>Temperature, $T$</td>
</tr>
</tbody>
</table>

The type of variation in bottomhole flowing pressures and flow rates present in the well are shown in Figs. 14 and 15. It is clear from Fig. 14 that $p_{w}$ varies widely with no defined patterns.

Fig. 15 is a logarithmic plot of normalized instantaneous rates versus real time which shows that a match of this data set to any decline type curve is impossible at this point. In addition, Fig. 16 shows a poor performance for pressure drop normalized rate versus time.

The proposed procedure to analyze this kind of data was then used once more with option "B", and the resulting gas in place plots are shown as follows: Figs. 17 and 18. It can be seen that these plots do not exhibit the horizontal straight line that should be obtained for the fixed value of $G$. In fact, the poor performance observed was not unexpected given the production characteristics of this well. In particular, it must be remembered that all the theory presented in this work assumes single phase gas flow, and the significant liquid loading that exists in this well may invalidate this fundamental assumption.

Although the gas in place plots are far from ideal, a tendency toward a flat region and a high concentration of points is seen around $G$ of 0.2x10^6 Mcf. In order to determine if an interpretation of the data set from Gas Well A is possible at all by our method, this $G$ value is used to compute the material balance pseudotimes. After doing so, the normalized rate profiles are replotted versus the calculated material balance pseudotimes. This new plot is presented as Fig. 19.

The comparison of Figs. 16 and 19 shows a significant
improvement in smoothness in the latter one, so that a match of the
normalized rate profile of this well can now be attempted. The
results of the type curve matching procedures are shown in Figs.
20 and 21. In this case when Fedovich and McCray’s type
curves are used, the well production data are found to lie on a b=1
stem, indicating a harmonic decline as predicted from theory.
The agreement with the theoretical principles presented in this
work is a very important result because of the unfavorable
conditions involved in the analysis of this case. The match to the
harmonic stem allows the computation of a corrected gas in place
volume. Thus, the value of the gas in place is
\[
G = \frac{1}{115 \text{ D}(0.78 \text{ Mcf/ft}^3\text{psi})} \times \left( \frac{3.649 \times 10^8 \text{ psi}^{-3}}{1} \right) \times \left( 1 \right) \times \left( 1 \right)
\]

\[
G = 0.245 \times 10^6 \text{ Mcf}
\]

and also
\[
k = \frac{141.2 (1.166 \text{ RB/McF})(1.785 \times 10^{-2} \text{ cp.s})}{48 \text{ ft}} \times \left( \frac{0.3 \text{ ft}}{(0.5 \text{ ft})^2} \right) \times \left( 1 \right)
\]

\[
k = 0.306 \text{ mD}
\]

This corrected gas in place of 0.245 \times 10^6 \text{ Mcf} is not unreasonable
compared to the value provided by the algorithm. Also, any value of
permeability in the area is expected to be within tenths of
milidarcys, so the value of 0.306 mD represents a reasonable
estimate for this formation parameter. The knowledge of the skin
factor would improve the estimate of the permeability.

These conclusions make it possible to state that even for non-ideal
situations, the explicit calculation of gas in place with the new
algorithm provides an accurate result. Furthermore, the calculated
gas in place permits the computation of the pseudotypes which are
required for analysis of gas production data under actual field
conditions, the match to a harmonic stem and the later computation
of the permeability value.

SUMMARY AND CONCLUSIONS

SUMMARY

This work has introduced new methods of production data
analysis for the case of single phase flow of either oil or gas using
type curve analysis. The gas flow techniques have been presented in
"liquid equivalent forms". This ensures that the techniques for
analysis, or at least their form, are the same for either type of fluid.

CONCLUSIONS

1. The use of a material balance time function for the case of
single phase liquid production allows for the analysis of any
data set using the b=1 stem on Fedovich type curve regardless of
the rate or pressure conditions under which those data were
obtained. This modified time function must be taken as the
quotient between cumulative liquid production and flow rate.

2. The use of a modified definition of the conventional
pseudotype as proposed by Aggarwal12 that resembles a
material balance type of function permits a pseudo-steady-state
gas flow equation to be obtained which has the same form as
that for the liquid case. This modified definition has
previously been proven accurate by simulation techniques,
and now has been proven to be analytically correct and have
an exact theoretical definition.

3. An accurate technique has been devised that allows for the
explicit computation of gas in place from field data. This
technique makes the analysis of gas data using the modified
pseudotype function much simpler.

4. The harmonic type of decline (b=1) is the "all-inclusive"
solution because it is useful for any sort of analysis regardless of
the fluid type or the conditions under which the data are
obtained. The only requirement for the validity of this
statement is that the proper plotting variables be used in each
case, so that a match can be found.

5. The analysis and interpretation of production data are relatively
easy and can provide just about the same information as
conventional pressure transient analysis. Therefore
production data analysis should be used in reservoir
engineering because such analysis would save time and money
for it involves no cost in lost production.

6. The flow equations derived in this paper for both liquid and
gas which lead to harmonic declines are strictly valid for
boundary dominated flow. However, they constitute
reasonable approximations for transient flow conditions.

7. The flow rate integral and flow rate integral derivative
functions are helpful to obtain a more accurate match to decline
type curves than using flow rate data alone. This is true for
production conditions as those actually found in the field.

NOMENCLATURE

**Dimensionless and Field Variables**

A = Drainage Area, \( \text{ft}^2 \)

b = Fedovich/Arps Decline Curve Exponent

b_{sat} = Constant in the Liquid Flow Equation Defined by

Eqs. A-9 or 3

b_{gas} = Constant in the Gas Flow Equation Defined by

Eqs. B-8 or 11

B_g = Gas Formation Volume Factor, RB/McF

B_{e1} = Gas Formation Volume Factor at Initial

Reservoir Pressure, RB/McF

B_e = Oil Formation Volume Factor, RB/STB

C_A = Reservoir Shape Factor

c_g = Gas Compressibility, \( \text{psi}^{-1} \)

c_e = Gas Compressibility at Average Reservoir

Pressure, \( \text{psi}^{-1} \)

C_P = Gas Compressibility at Original Reservoir

Pressure, \( \text{psi}^{-1} \)

c_t = Total System Compressibility, \( \text{psi}^{-1} \)

c_{II} = Total System Compressibility at Original

Reservoir Pressure, \( \text{psi}^{-1} \)

G = Original Gas in Place, McF

G_p = Cumulative Gas Production, McF

h = Formation Thickness, \( \text{ft} \)

k_a = Effective Permeability to Oil, mD

k_e = Effective Permeability to Gas, mD

n = Constant Defined by Eq. A-8 or 2

m = Constant Defined by Eq. B-7 or 10

M = Gas Molecular Weight

N = Original Oil in Place, STB

N_p = Cumulative Oil Production, STB

P = Pressure, \( \text{psi} \)

P = Average Reservoir Pressure, \( \text{psi} \)

P_{app} = Appropriate Pressure-like Type of Function for

Every Stem in the Modified Type Curve, \( \text{psi} \)

P_i = Initial Reservoir Pressure, \( \text{psi} \)

P_P = Normalized Pseudo-pressure, \( \text{psi} \)
### Special Subscripts
- \( a \) = "adjusted" variable for gas well test analysis. Use of these variables in gas well test analysis yields an equivalent liquid system.
- \( D \) = dimensionless variable
- \( M.P. \) = Match Point
- \( ps \) = Pseudo-steady-state

### REFERENCES

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_r )</td>
<td>Normalized Pseudopressure at Average Reservoir Pressure, psia</td>
</tr>
<tr>
<td>( P_{r,ab} )</td>
<td>Normalized Pseudopressure at Abandonment Pressure, psia</td>
</tr>
<tr>
<td>( P_{ri} )</td>
<td>Normalized Pseudopressure at Initial Reservoir Pressure, psia</td>
</tr>
<tr>
<td>( P_{r,m} )</td>
<td>Normalized Pseudopressure at Wellbore Flowing Pressure, psia</td>
</tr>
<tr>
<td>( P_{wf} )</td>
<td>Wellbore Flowing Pressure, psia</td>
</tr>
<tr>
<td>( q )</td>
<td>Surface Flow Rate, STB/D or Mcf/D</td>
</tr>
<tr>
<td>( q_d )</td>
<td>Constant Pressure Dimensionless Rate Solution</td>
</tr>
<tr>
<td>( q_{dc} )</td>
<td>Dimensionless Decline Rate Function</td>
</tr>
<tr>
<td>( q_{dc} )</td>
<td>Dimensionless Decline Rate Integral</td>
</tr>
<tr>
<td>( q_{dc} )</td>
<td>Dimensionless Decline Rate Integral Derivative</td>
</tr>
<tr>
<td>( q_i )</td>
<td>Initial Rate for Oil or Gas, STB/D or Mcf/D</td>
</tr>
<tr>
<td>( q_o )</td>
<td>Oil Flow Rate, STB/D</td>
</tr>
<tr>
<td>( r_e )</td>
<td>Drainage Radius, ft</td>
</tr>
<tr>
<td>( r_w )</td>
<td>Effective Wellbore Radius (Includes Formation Damage Effects), ft</td>
</tr>
<tr>
<td>( t )</td>
<td>Time, days</td>
</tr>
<tr>
<td>( t_b )</td>
<td>Material Balance Time ( (N_o/q_o, \text{or} \ G_o/q_o) ), days</td>
</tr>
<tr>
<td>( t_p )</td>
<td>Conventional Normalized Pseudotime, days</td>
</tr>
<tr>
<td>( t_e )</td>
<td>Normalized Material Balance Pseudotime, days</td>
</tr>
<tr>
<td>( t_{DA} )</td>
<td>Dimensionless Time Based on Drainage Area and Normalized Conventional Pseudotime</td>
</tr>
<tr>
<td>( t_{DA} )</td>
<td>Dimensionless Time Based on Pseudotime</td>
</tr>
<tr>
<td>( t_{DM} )</td>
<td>Dimensionless Decline Time Based on Drainage Area and Normalized Time</td>
</tr>
<tr>
<td>( t_{DM} )</td>
<td>Dimensionless Decline Time Based on Drainage Area</td>
</tr>
<tr>
<td>( t_{DM} )</td>
<td>Dimensionless Decline Time Based on Material Balance Time</td>
</tr>
<tr>
<td>( t_{DM} )</td>
<td>Dimensionless Decline Time Based on Pseudotime</td>
</tr>
<tr>
<td>( T )</td>
<td>Appropriate Time-like Type of Function for Every Stem in the Modified Type Curve, days</td>
</tr>
<tr>
<td>( T_p )</td>
<td>Time for the Onset of Steady State, days</td>
</tr>
<tr>
<td>( T_r )</td>
<td>Temperature, °F</td>
</tr>
<tr>
<td>( z )</td>
<td>Real Gas Deviation Factor</td>
</tr>
<tr>
<td>( z_p )</td>
<td>Real Gas Deviation Factor at Average Reservoir Pressure</td>
</tr>
<tr>
<td>( z_i )</td>
<td>Real Gas Deviation Factor, at Initial Reservoir Pressure</td>
</tr>
<tr>
<td>( G )</td>
<td>Gas Specific Gravity</td>
</tr>
<tr>
<td>( k )</td>
<td>Carpe's Drawdown Parameter</td>
</tr>
<tr>
<td>( \mu )</td>
<td>Fluid Viscosity, cp</td>
</tr>
<tr>
<td>( \mu_g )</td>
<td>Gas Viscosity, cp</td>
</tr>
<tr>
<td>( \mu_r )</td>
<td>Gas Viscosity at Average Reservoir Pressure, cp</td>
</tr>
<tr>
<td>( \mu_{rp} )</td>
<td>Gas Viscosity at Initial Reservoir Pressure, cp</td>
</tr>
<tr>
<td>( \mu_o )</td>
<td>Oil Viscosity, cp</td>
</tr>
<tr>
<td>( \rho )</td>
<td>Gas Density, lb/mol</td>
</tr>
</tbody>
</table>
APPENDIX A - DEVELOPMENT AND USE OF THE MATERIAL BALANCE TIME FOR BOUNDARY DOMINATED LIQUID FLOW

DEVELOPMENT

The original material balance approach for boundary dominated flow (i.e., pseudosteady-state) was developed by Blasingame and Lee, and the following derivation builds up on their developments.

From the compressibility definition it can be shown that

\[ q_o = \frac{A_{ph}}{5.615 B_o} \left( \rho - \rho_f \right) \]  
(A-1)

Since the single phase liquid case is the one under consideration, \( \rho_f \) is assumed to be constant. Thus, the integration of Eq. A-1 produces

\[ \int_0^t \frac{q_o}{\rho_o} dt = \frac{A_{ph}}{5.615 B_o} \left( \rho - \rho_f \right) \]  
(A-2)

Also, since the integral on the left hand side corresponds to the cumulative production,

\[ N_p = \frac{A_{ph}}{5.615 B_o} \left( \rho - \rho_f \right) \]  
(A-3)

Solving this relation for the pressure difference involved, it follows that

\[ (\rho - \rho_f) = \frac{N_p}{5.615 B_o} A_{ph} \]  
(A-4)

which may be also rearranged as

\[ \frac{(\rho_f - \rho_f)}{\rho_o} = \frac{5.615 N_p}{A_{ph} C_o} \]  
(A-5)

and further rewritten as

\[ \frac{(\rho_f - \rho_f)}{\rho_o} = \frac{5.615 N_p}{A_{ph} C_o} \]  
(A-6)

Now, letting \( i = \frac{N_p}{\rho_o} \) be the material balance time function it follows that

\[ (\rho_f - \rho_f) = \frac{5.615 N_p}{A_{ph} C_o} \]  
(A-7)

Furthermore, if the dimensionless time is redefined as

\[ \tilde{t} = \frac{5.615 N_p}{A_{ph} C_o} \]  
(A-8)

then, the next to last relation becomes

\[ (\rho_f - \rho_f) = \frac{5.615 N_p}{A_{ph} C_o} \]  
(A-9)

The most important characteristic of Eq. A-4 is that it is always valid regardless of time, flow regime, or production scenario whether it is constant or variable bottomhole flowing pressure or constant or variable flow rate. This is due to the fact that Eq. A-4 is a material balance type of equation.

Now, it has been proved that for the scheme of constant rate production of single phase liquid the flow equation for the pressure response under boundary dominated flow may be written as

\[ \frac{q_o}{\rho_o} = \frac{k_h}{141.2 q_o B_o j_o} \frac{1}{2} \ln \left( \frac{4 A}{\rho_o C_o r_o^2} \right) \]  
(A-10)

Although Eq. A-5 was derived for constant rate (variable \( p_o \)), it has been shown in the literature to be a good approximation when the flowing bottomhole pressure is fixed (variable rate).

Thus, the addition of Eqs. A-4 and A-5 yields

\[ \frac{q_o}{\rho_o} = \frac{k_h}{141.2 q_o B_o j_o} \frac{1}{2} \ln \left( \frac{4 A}{\rho_o C_o r_o^2} \right) \]  
(A-11)

The previous considerations imply that Eq. A-5 is strictly valid for the pseudosteady-state flow regime and any rate or pressure profile. If Eqs. A-3 and A-6 are combined and the result rearranged, then

\[ \frac{q_o}{\rho_o} = \frac{1}{\rho_o} \frac{b_{ph}}{i} \]  
(A-12)

or finally,

\[ \frac{q_o}{\rho_o} = \frac{b_{ph}}{i} \]  
(A-13)

USE

The group on the L.H.S. of Eq. A-10 is exactly the dimensionless decline rate variable, \( q_{l,\text{ph}} \), as presented by Feoktistov1,2. The second term in the denominator of the R.H.S. group in the same equation is defined as, \( i_{ph} \), therefore

\[ i_{ph} = \frac{5.615 N_p}{A_{ph} C_o} \]  
(A-14)

The only difference between \( i_{ph} \) and the Feoktistov1,2 dimensionless decline time function, \( i_{ph} \), is that now real time \( t \) is substituted with \( i \). Since Feoktistov1,2 assumed a circular reservoir, the denominator on the R.H.S. of Eq. A-11 is given by \( \ln \left( \frac{t_o - t}{d} \right) \) which is the result of taking the appropriate \( C_o \) value for a circular reservoir. In addition, Feoktistov1,2 actually chose to use \( i/2 \) rather than \( 1/2 \) within the argument of the logarithm, because he found a better correlation in doing so.

In any case, Eq. A-10 can be reduced to

\[ q_{ph} = \frac{1}{1 + i_{ph}} \]  
(A-15)

Eq. A-12 is a harmonic decline type of equation where the decline exponent is unity as required. This is evident by comparing Eq. A-12 to Arps' original definition of a hyperbolic decline which in dimensionless form is

\[ q_{ph} = \frac{1}{(1 + i_{ph})} \]  
(A-16)

When the decline exponent, \( b \), is taken as unity in Arps' equation,
a harmonic decline is obtained and Eqs. A-12 and A-13 would have exactly the same form.

Therefore if $i$ is correctly calculated, a scaled log-log plot of $rac{q_s}{(p_t - p_w)_{hr}}$ versus $i$ will overlay the $q_s$ vs. $i$ trend for a harmonic decline on the Fitts-Burnett curve. Once the match has been obtained, the relations to get $m$, $b_{hr}$, and $N$ are

$$b_{hr} = \frac{(q_s i_{hr})}{(p_t - p_w)_{hr}}$$  \hspace{0.5cm} (A-14)

$$m = \frac{1}{N_{ci}} = b_{hr} \frac{(q_s i_{hr})}{(p_t - p_w)_{hr}}$$  \hspace{0.5cm} (A-15)

$$N = \frac{1}{G_{ci}} \left( \frac{q_s}{(p_t - p_w)_{hr}} \right) \frac{d(p_t - p_w)_{hr}}{d(t)}$$  \hspace{0.5cm} (A-16)

where M.P. stands for a match point value.

**APPENDIX B - DEVELOPMENT, PROOF AND USE OF THE MATERIAL BALANCE PSEUDO-TIME FOR BOUNDARY DOMINATED GAS FLOW**

**DEVELOPMENT**

Blastingame and Lee\(^9\) proposed a modified gas flow equation based on modified pseudotime function to model the general variable rate/variable pressure drop case. This was done on an empirical basis using simulation to verify this approach. Following are the analytical proofs that constitute the sound theoretical background for Blastingame and Lee\(^9\)'s modified gas flow equation.

Thus, taking the material balance equation for liquid given by

$$\frac{(q_s - p_t)}{q_s} = \frac{1}{N} \frac{N_{ci}}{q_s} \frac{1}{N_{ci}}$$  \hspace{0.5cm} (B-1)

as the basis for their direction of thought, Blastingame and Lee\(^9\) proposed that, in the gas case,

$$\frac{(p_t - p_w)}{q_s} = \frac{1}{G} \frac{1}{i}$$  \hspace{0.5cm} (B-2)

is valid when the new pseudotime variable is a material balance type of relation given by

$$i = \frac{p_t}{q_s} \int_0^{p_t} \frac{d p}{G \mu \phi \rho (p_t - p_w)}$$  \hspace{0.5cm} (B-3)

The pseudopressure functions involved in Eq. B-2 are normalized types of variables, already published in the literature\(^14\) which are given by

$$p_p = \left( \frac{p_t}{p_t} \right) \frac{P_t}{P_t} dp$$

and

$$p_f = \left( \frac{p_t}{p_t} \right) \frac{P_t}{P_t} df$$

Eq. B-2 was coupled with the pseudosteady-state flow equation given by Al Husainy and Ramey\(^15,16\) for the flow of single phase gas, which is

$$\frac{(p_t - p_w)}{q_s} = 141.2 \frac{\mu_i}{k_i} \frac{B_i}{R} \frac{1}{2} \left[ 1 + \left( \frac{A_1}{A_2} \right) \right]$$  \hspace{0.5cm} (B-4)

Thus, the addition of Eq. B-3 and B-4 produced the following new gas equation for variable rate point transient flow

$$\frac{\Delta p}{q_s} = m_{hr} = b_{hr}$$  \hspace{0.5cm} (B-5)

where

$$\Delta p = (p_t - p_w)$$  \hspace{0.5cm} (B-6)

$$m_{hr} = G_{hr}$$  \hspace{0.5cm} (B-7)

$$b_{hr} = 141.2 \frac{\mu_i}{k_i} \frac{B_i}{R} \frac{1}{2} \left[ 1 + \left( \frac{A_1}{A_2} \right) \right]$$  \hspace{0.5cm} (B-8)

**PROOF**

The gas compressibility definition is

$$c_g = \frac{1}{\rho} \frac{d \rho}{d p} = \frac{1}{\rho} \frac{d \rho}{d T} \frac{d T}{d p}$$  \hspace{0.5cm} (B-9)

Evaluating this definition at the average reservoir pressure and eliminating constant terms, it follows that

$$\frac{d}{d p} \left( \frac{p_t}{c_g} \right) = \frac{p_t}{c_g}$$  \hspace{0.5cm} (B-10)

Also, solving the gas material balance equation for cumulative production and differentiating the resulting relation, it is readily seen that

$$q_t = \frac{G_{ci}}{p_t} \frac{d}{d t} \left( \frac{p_t}{c_g} \right)$$  \hspace{0.5cm} (B-11)

Now, if the chain rule is used on Eq. B-11, it follows that

$$q_t = \frac{G_{ci}}{p_t} \frac{d}{d t} \left( \frac{p_t}{c_g} \right)$$  \hspace{0.5cm} (B-12)

which, combined with Eq. B-10, produces

$$q_t = \frac{G_{ci}}{p_t} \frac{d}{d t} \left( \frac{p_t}{c_g} \right)$$  \hspace{0.5cm} (B-13)

If Eq. B-13 is substituted into Eq. B-3, it is seen that

$$i = \frac{G_{ci}}{q_s} \frac{G_{ci}}{p_t} \frac{d}{d t} \left( \frac{p_t}{c_g} \right)$$  \hspace{0.5cm} (B-14)

Assuming that the formation compressibility is negligible compared to the bulk compressibility, then $\frac{c_g}{c_g} = \frac{c_g}{c_g}$. Thus, using this approximation and changing the variable of integration from time to pressure, Eq. B-14 may be written as

$$i = \frac{G_{ci}}{q_s} \frac{G_{ci}}{p_t} \frac{d}{d p} \left( \frac{p_t}{c_g} \right)$$  \hspace{0.5cm} (B-15)

Furthermore, using the definition of normalized pseudopressure, Eq. B-15 may be represented by

$$i = \frac{G_{ci}}{q_s} \frac{p_t}{c_g}$$

which can be rearranged to yield

$$\frac{(p_t - p_w)}{q_s} = \frac{1}{G} i$$  \hspace{0.5cm} (B-16)
Eq. B-16 is identical to Eq. B-2. This fact constitutes the proof that Eq. B-3 is the exact definition of pseudotime for boundary dominated gas flow.

USE

The use of the tools developed above for decline curve analysis lies in the fact that Eq. B-5 can undergo exactly the same rearrangement process used for Eq. A-7. Thus, Eq. B-5 can be rewritten as

\[ \frac{q_{w1}}{b_{a,pret}} = \frac{1}{G_p} \left( \frac{1 - \frac{z_1}{z_j}}{1 - \frac{z_1}{z_j}} \right) \]  

Eq. B-17 and the liquid relation given by Eq. A-10 have identical forms and the former can be rewritten as

\[ q_{w1} = \frac{1}{1 + \alpha_{q, wt}} \]  

where

\[ q_{w1} = \frac{q_p}{b_{a,pret}} \]  

and

\[ \alpha_{q, wt} = \left( \frac{1}{b_{a,pret}} \right) \]  

literally, the dimensionless decline rate function is now in terms of pseudopressure rather than actual pressure and the dimensionless decline time is in terms of the material balance pseudotime. Eq. B-18 and the relation for liquid given by Eq. A-12 are identical in form, as is expected. Therefore, all of the comments regarding the comparison of Eq. A-12 to Arps' relations (i.e., a harmonic decline) apply here in the same fashion.

This indicates that when pseudopressure and the material balance pseudotime are used to model gas flow, the data trend must decline along the harmonic stem on Petkovski\-h\-l type curve. That is, if \( \phi \) is correctly calculated then a scaled log-log plot of \( q_{w1} \) versus \( \phi \) will overlay the \( q_{w1} \) versus \( \phi \) trend exactly for a \( \phi = \phi_0 \) stem on Petkovski\-h\-l type curve. The match point permits computation of the gas in place and formation properties from the following relations

\[ b_{a,pret} = \frac{q_{w1}(M.P.)}{q_p} \]  

\[ m_{o} = \frac{1}{G_p} b_{a,pret} \]  

\[ G = \frac{1}{M.P.} \left( \frac{q_p}{q_{w1}} \right) \]  

where \( M.P. \) stands for “Match Point”. The last three relations form the basis for analysis when this harmonic type curve technique is used.

APPENDIX C - AN EXPLICIT METHOD FOR COMPUTING GAS-IN-PLACE--AN AID FOR THE COMPUTATION OF PSEUDOTIME

In order to avoid the use of iterative schemes to compute the gas in place, \( G \), a new technique is presented to couple the gas material balance equation and the pseudosteady state flow relation. The final result of this combination is an algorithm that needs to be solved only once to estimate the value of gas in place. Solving the material balance equation for a volumetric dry gas reservoir for \( G \), it follows that

\[ \frac{1}{G_p} b_{a,pret} \left( \frac{1 - \frac{z_1}{z_j}}{1 - \frac{z_1}{z_j}} \right) \]  

Since \( G \) is a fixed value, Eq. C-1 can be written for two different points in time; that is, for two different values of average reservoir pressure. Thus, for times \( j \) and \( j + 1 \), Eq. C-1 can be arranged to yield

\[ \frac{1}{G_p} b_{a,pret} \left( \frac{1 - \frac{z_1}{z_j}}{1 - \frac{z_1}{z_j}} \right) = \left( \frac{P_j}{P_{j+1}} \right) \]  

Eq. C-2 may be then rearranged as follows

\[ \frac{P_j}{P_{j+1}} = \frac{P_j}{P_{j+1}} \left( \frac{1}{G_p} b_{a,pret} \left( \frac{1 - \frac{z_1}{z_j}}{1 - \frac{z_1}{z_j}} \right) \right) \]  

or even further as

\[ \frac{P_j}{P_{j+1}} = \frac{P_j}{P_{j+1}} \left( \frac{1}{G_p} b_{a,pret} \right) \]  

Now, from pseudosteady-state theory, the gas flow equation in terms of normalized pseudopressures is given by

\[ \frac{P_j}{P_{j+1}} = \frac{P_j}{P_{j+1}} \]  

where

\[ b_{a,pret} = 141.2 \frac{p_n^b}{k_h^b} \frac{1}{G_p} \frac{1}{M.P.} \]  

Eq. C-4 can also be written for two different points in time and solved for \( b_{a,pret} \) in both cases. Since \( b_{a,pret} \) is constant, the resulting relations may be equated to each other. Thus, for time levels \( j \) and \( j + 1 \) it follows that

\[ b_{a,pret} = b_{a,pret} \left( \frac{P_j}{P_{j+1}} \right) \]  

which may be rearranged as

\[ b_{a,pret} = \frac{b_{a,pret}}{P_j} \left( \frac{P_j}{P_{j+1}} \right) \]  

Eq. C-6, which is a flow type of equation, can be combined with Eq. C-3 to make it material balance correct. The proposed method to couple these two equations in order to obtain the average pressure profile and gas in place volume is given below

Procedures

1. A table of \( p, z, p_c, \) and \( p_p \) for the particular gas sample must be set up.

2. For the first time step \( (j = 1) \), it is assumed that \( p_{p1} = p_{p1} \).

That is, the first average pseudopressure is assumed to be equal to the initial reservoir pseudopressure. Then, \( p_{p1} \) can be recomputed from Eq. C-6 as

\[ p_{p2} = p_{p1} + \frac{q_{w1}}{q_{w1}} \left( \frac{P_j}{P_{j+1}} \right) \]  

where again \( p_{p1} = p_{p1} \).

3. Then, \( p_{p1} \) is obtained using a look-up procedure in the table that was set up in step 1. Thus \( p_{p1} \) can be estimated using the interpolated values.
4. Next, Eq. C-3 may be reentered with \( \frac{p_{z_{j+1}}}{z_{j+1}} \) (for the first case) and \( \frac{p_j}{z_j} \) (for the first case) can be back calculated as

\[
\frac{p_j}{z_j} = \frac{p_j}{z_j} \cdot \frac{G_{p, j+1}}{G_{p, j+1}} \left( \frac{p_j}{z_j} \cdot \frac{p_{z_{j+1}}}{z_{j+1}} \right)
\]

For the first time step, this calculation would be

\[
\frac{p_j}{z_j} = \frac{p_j}{z_j} \cdot \frac{G_{p, 1}}{G_{p, 2}} \left( \frac{p_j}{z_j} \cdot \frac{p_{z_2}}{z_2} \right)
\]

Again, \( \frac{p_j}{z_j} \) and \( \frac{p_{z_j}}{z_j} \) are obtained using a look-up procedure entering the table made in step 1 with the recently calculated \( \frac{p_j}{z_j} \).

5. Once more Eq. C-6 is used to recompute \( \frac{p_{z_{j+1}}}{z_{j+1}} \) with the last \( \frac{p_{z_j}}{z_j} \) found. For the first time step, \( \frac{p_{z_1}}{z_1} \) would be recomputed using the last \( \frac{p_{z_1}}{z_1} \).

6. Steps 2 through 4 must be repeated until no significant variation in two successive back substitutions for an average pressure at a given time step is obtained.

7. Once the right \( \frac{p_j}{z_j} \) is found \( G_{p, j} \) for the first case \( G_{p, j} \), the same procedure is applied to obtain the values for the next time level.

It is worth noting that the back calculation proposed explains why Eq. C-3 is solved for the \( j \) time level, whereas C-6 is solved for the \( j+1 \) time level.

In order to ensure that the results obtained through this analysis technique are correct, a plot of gas in place, \( G \), versus time may be made and a horizontal best fit trend should be constructed for \( G \).
Figure 1 - Fetkovich Carter Type Curve.
Figure 6 - Match of Normalized Flow Rate Integral and Integral Derivative on McCray Type Curve for a Simulated Case of Constant Bottomhole Pressure.
Figure 7 - Semilog Plot of Computed Gas in Place for a Simulated Example of Variable Flow Rate

Figure 8 - Cartesian Plot of Computed Gas in Place for a Simulated Example of Variable Flow Rate
Figure 9 - Match of Normalized Flow Rate Functions on Fetkovich and McCray Type Curves for a Simulated Case of Variable Flow Rate.
Figure 11 - Semilog Plot of Computed Gas in Place for a Simulated Example of Variable Bottomhole Pressure

Figure 12 - Cartesian Plot of Computed Gas in Place for a Simulated Example of Variable Bottomhole Pressure
Figure 13 - Match of Normalized Flow Rate Functions on Fetkovich and McCray Type Curves for a Simulated Case of Variable Bottomhole Flowing Pressure.
Figure 14 - Bottomhole Flowing Pressure Profile for Gas Well A.

Figure 15 - Semilog Flow Rate Profile for Gas Well A.
Figure 17 - Semilog Plot of Computed Gas in Place for Gas Well A.

Figure 18 - Cartesian Plot of Computed Gas in Place for Gas Well A.
Figure 19. Plot of Normalized Flow Rate versus Material Balance Pseudotime for Gas Well A.
Figure 20 - Match of Normalized Flow Rate Functions on Fetkovich and McCray Type Curves for Gas Well A.
Figure 21 - Match of Normalized Flow Rate Integral and Integral Derivative on the McCray Type Curve for Gas Well A.