Scattering Theory - General

(Read Roe, section 1.2)

<u>Definitions of flux, scattering cross section, intensity</u>

Flux *J* - for plane wave, energy/unit area/sec
- no. photons/unit area/sec

Scattering Theory - General

(Read Roe, section 1.2)

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$$J = A^*A = |A|^2$$

where A is the amplitude of the wave

Scattering Theory - General

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For a spherical wave scattered by a point:

J is energy/unit solid angle/sec

Scattering Theory - General

For a spherical wave scattered by a point:

J is energy/unit solid angle/sec

J_o is flux incident on point scatterer

Then we are interested in J/J_o and

$$J/J_o = d\sigma/d\Omega = \frac{\text{\# photons scattered into unit solid angle/sec}}{\text{incident beam flux}}$$

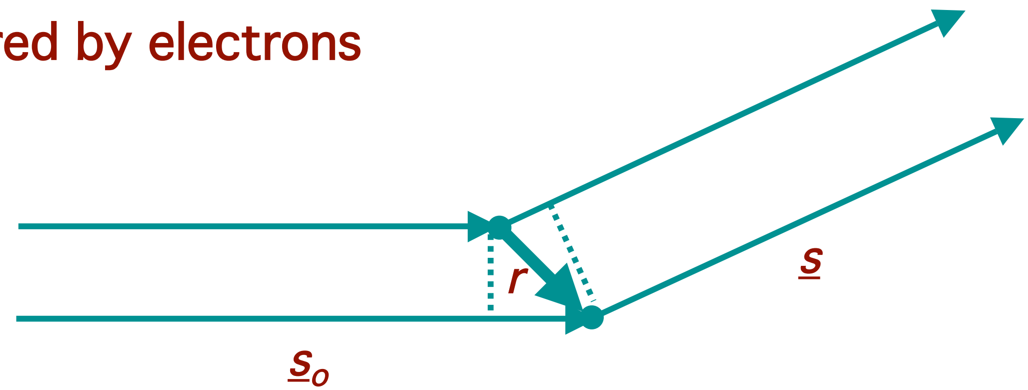
= differential scattering cross section

= intensity I

Scattering Theory - General

Interference and diffraction

X-rays scattered by electrons



Consider 2 identical points scattering wave $\underline{s}_o$ in the direction of $\underline{s}$

Phase difference $\Delta\phi$ betwn scattered waves is

$$\Delta\phi = (2\pi/\lambda)\,(\underline{s}_o \bullet \underline{r} - \underline{s} \bullet \underline{r})$$

Diffraction vector $\underline{S} = (\underline{s} - \underline{s}_o)/\lambda$

Scattering Theory - General

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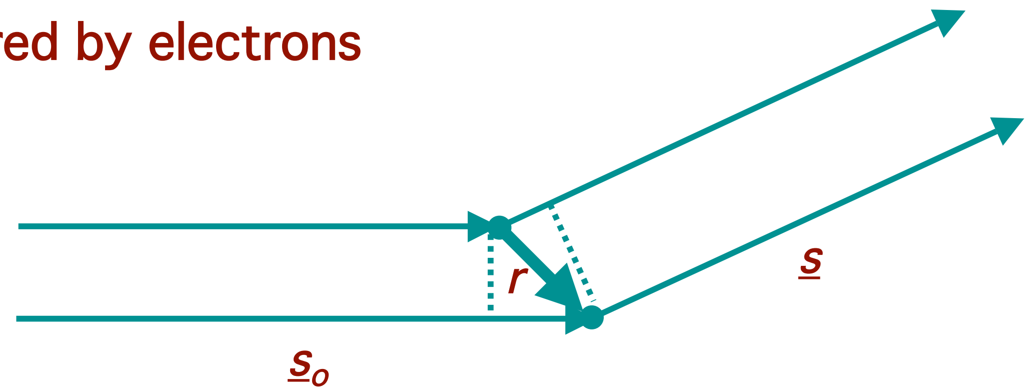

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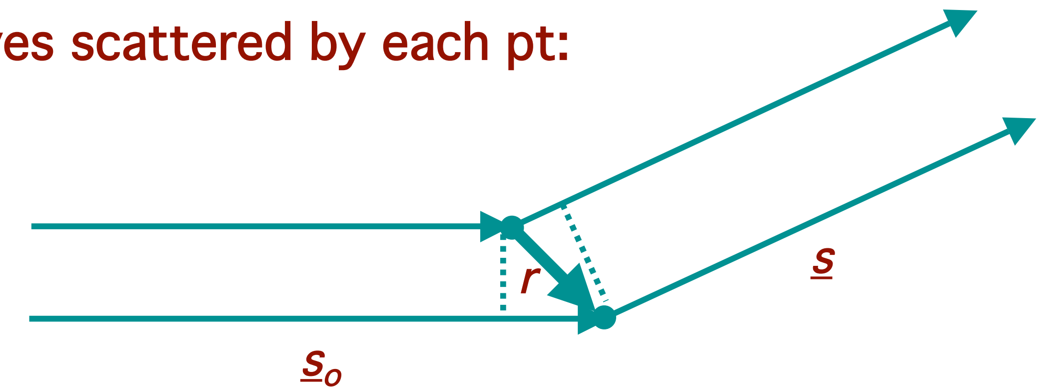
Diffraction vector $\underline{S} = (\underline{s} - \underline{s}_o)/\lambda$

$$|\,\underline{S}\,| = (2 \sin\theta)/\lambda$$

Scattering Theory - General

Interference and diffraction

Spherical waves scattered by each pt:



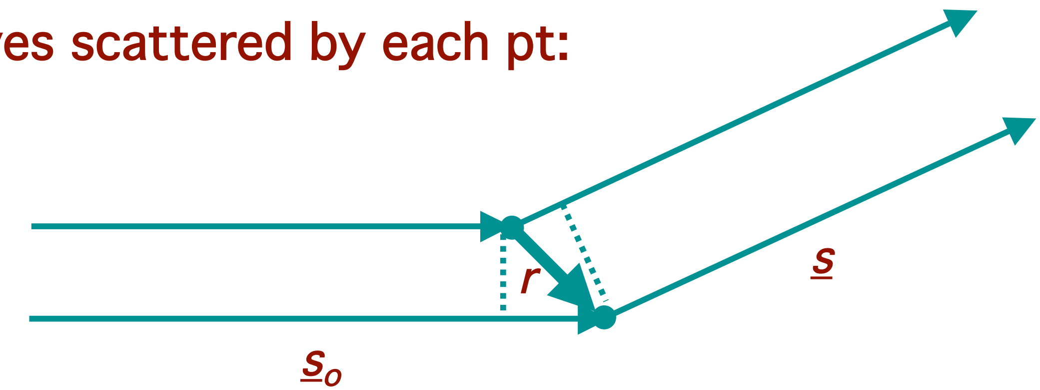
$$A_1 = A_o\, b\, \exp\,(2\pi i(\nu t - x/\lambda))$$
$$A_2 = A_1 \exp(i\Delta\phi) = A_o\, b\, (\exp\,(2\pi i(\nu t - x/\lambda)))\, \exp\,(-2\pi i\, \underline{S} \cdot \underline{r})$$

b = ‘scattering length’

Scattering Theory - General

Interference and diffraction

Spherical waves scattered by each pt:



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and

$$A = A_1 + A_2 = A_o\, b\, (\exp(2\pi i(\nu t - x/\lambda)))\, (1 + \exp(-2\pi i\, \underline{S} \cdot \underline{r}))$$

b = 'scattering length'

Scattering Theory - General

Interference and diffraction

$$A = A_1 + A_2 = A_o\, b\, (\exp\, (2\pi i(\nu t - x/\lambda)))\ (1 + \exp\, (-2\pi i\, \underline{S} \cdot \underline{r}))$$

$$J = A^*A = A_o^2 b^2\, (1 + \exp\, (2\pi i\, \underline{S} \cdot \underline{r}))\ (1 + \exp\, (-2\pi i\, \underline{S} \cdot \underline{r}))$$

So:

$$A(\underline{S}) = A_o\, b\, (1 + \exp\, (-2\pi i\, \underline{S} \cdot \underline{r}))$$

Scattering Theory - General

Interference and diffraction

$$A = A_1 + A_2 = A_o\, b\, (\exp(2\pi i(\nu t - x/\lambda)))\, (1 + \exp(-2\pi i\, \underline{S} \cdot \underline{r}))$$

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So:

$$A(\underline{S}) = A_o\, b\, (1 + \exp(-2\pi i\, \underline{S} \cdot \underline{r}))$$

If *n* identical scatterers:

$$A(\underline{S}) = A_o\, b \sum_{j=1}^{n} \exp(-2\pi i\, \underline{S} \cdot \underline{r}_j)$$

Scattering Theory - General

Interference and diffraction

If *n* identical scatterers:

$$A(\underline{S}) = A_o\, b \sum exp\,(-2\pi i\, \underline{S} \bullet \underline{r}_j))$$

For a cont$^{\underline{s}}$ assembly of identical scatterers:

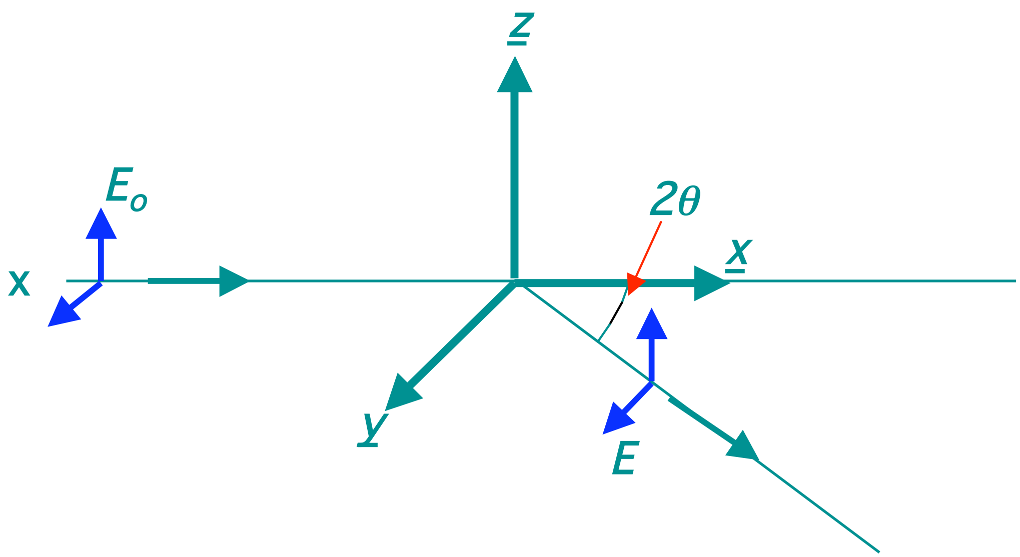
$$A(\underline{S}) = A_o\, b \int_V n(\underline{r})\, exp\,(-2\pi i\, \underline{S} \bullet \underline{r}_j))\, d\underline{r}$$

where $n(\underline{r})$ = # scatterers in $d\underline{r}$

Scattering Theory - General

Scattering by One Electron (Thomson)

Electrons scatter x-rays



$E_z = E_{oz}\, e^2/mc^2R$

$E_y = (E_{oy}\, e^2/mc^2R) \cos 2\theta$

Scattering Theory - General

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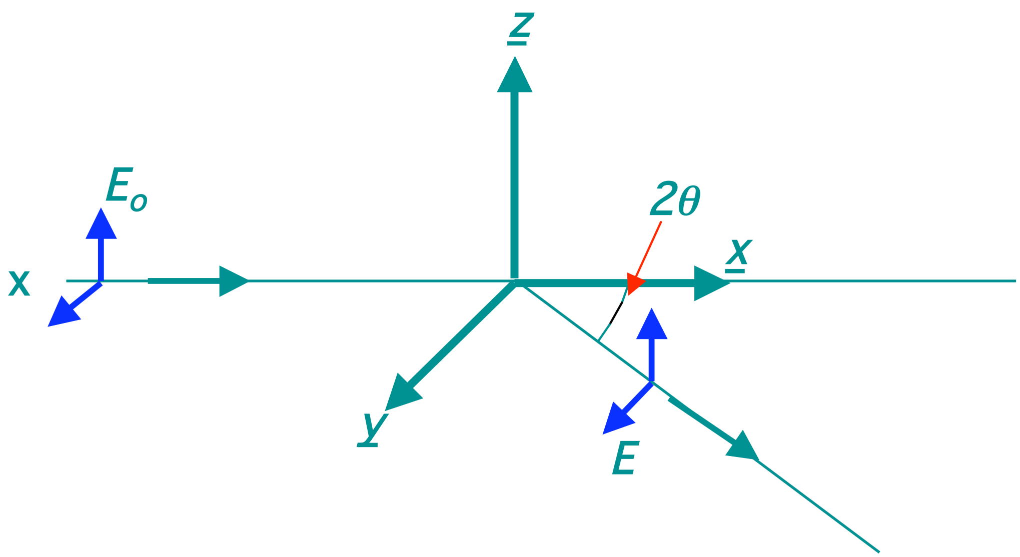

$E_z = E_{oz}\, e^2/mc^2R$

$E_y = (E_{oy}\, e^2/mc^2R) \cos 2\theta$

For an unpolarized beam:

$E_o^2 = \langle E_{oy}^{\;2} \rangle + \langle E_{oz}^{\;2} \rangle$ and $\langle E_{oy}^{\;2} \rangle = \langle E_{oz}^{\;2} \rangle = 1/2\, E_o^2 = J_o/2$

Scattering Theory - General

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Thus, the scattered flux is:

$$J_o\, (e^2/mc^2R)^2 (1 + \cos^2 2\theta)/2$$

Scattering Theory - General

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Thus, the scattered flux is:

$$J_o\, (e^2/mc^2R)^2\, \boxed{(1 + \cos^2 2\theta)/2}$$

polarization factor for unpolarized beam

Scattering Theory - General

Atomic Scattering Factor

$$f(\underline{S}) \equiv \text{scattering length}$$

and

$$f(\underline{S}) = \int n(\underline{r}) \exp(-2\pi i\, \underline{S} \bullet \underline{r})\, d\underline{r}$$

where $n(\underline{r})$ = average electron density distribution for an atom k

Scattering Theory - General

Atomic Scattering Factor

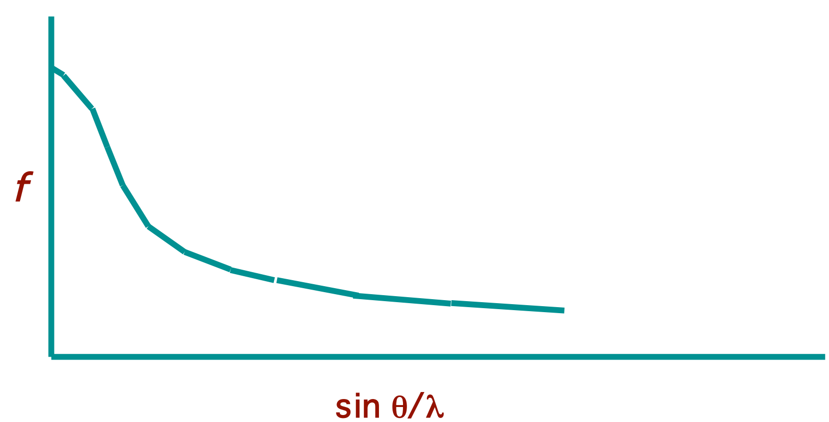
If electrons are grouped into atoms:

$$f(\underline{S}) \equiv \text{atomic scattering factor}$$

and

$$f_k(\underline{S}) = \int n(\underline{r}) \exp(-2\pi i\, \underline{S} \bullet \underline{r})\, d\underline{r}$$

where n(r) = average electron density distribution for an atom *k*



Scattering Theory - General

Scattering Amplitude *A(S)*

Finally, for N atoms:

$$A(\underline{S}) = A_o b_e \sum f_k(\underline{S}) \, exp\,(-2\pi i\, \underline{S} \bullet \underline{r}_k))$$

Scattering Theory - General

Scattering Amplitude *A(S)*

Finally, for N atoms:

$$A(\underline{S}) = A_o b_e \sum f_k(\underline{S}) \exp(-2\pi i\, \underline{S} \bullet \underline{r}_k)$$

Or, for a cont$^{\underline{s}}$ distribution of identical atoms whose centers are represented by $n_{at}(\underline{r})$:

$$A(\underline{S}) = A_o b_e f_k(\underline{S}) \int n_{at}(\underline{r}) \exp(-2\pi i\, \underline{S} \bullet \underline{r}_k)\, d\underline{r}$$