

## Lattice planes

## Useful concept for crystallography \& diffraction

Think of sets of planes in lattice - each plane in set parallel to all others in set. All planes in set equidistant from one another
Infinite number of sets of planes in lattice

d interplanar spacing

## Lattice planes

Keep track of sets of planes by giving them names - Miller indices
(hkl)


## Miller indices (hkl)

Choose cell, cell origin, cell axes:


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Choose cell, cell origin, cell axes
Draw set of planes of interest:


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Find intercepts on cell axes:
origin
$1,1, \infty$


## Miller indices (hkl)

Choose cell, cell origin, cell axes
Draw set of planes of interest
Choose plane nearest origin
Find intercepts on cell axes
origin
$1,1, \infty$
Invert these to get (hkl)
(110)


## Lattice planes

## Exercises



## Lattice planes

Exercises
(001)


## Lattice planes

Exercises
(001).......intercepts: $\infty, \infty, 1$


## Lattice planes

Exercises
(011)


## Lattice planes

Exercises
(011).......intercepts: $\infty, 1,1$


## Lattice planes

Exercises
(113)


## Lattice planes

## Exercises

(113) .......intercepts: $1,1, \frac{1}{3}$


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Two things characterize a set of lattice planes: interplanar spacing (d) orientation (defined by normal)

## Strange indices

For hexagonal lattices - sometimes see 4-index notation for planes (hkil) where $i=-h-k$


## Zones

2 intersecting lattice planes form a zone


$$
\begin{aligned}
& \text { zone axis [uvw] is } \\
& u \hat{i}+v \hat{j}+w \hat{k}
\end{aligned}
$$

$$
\left|\begin{array}{lll}
i & j & k \\
h_{1} & k_{1} & l_{1} \\
h_{2} & k_{2} & l_{2}
\end{array}\right|
$$

plane (hkl) belongs to zone [uvw] if hu $+k v+l w=0$
if $\left(h_{1} k_{1} l_{1}\right)$ and $\left(h_{2} k_{2} l_{2}\right)$ in same zone, then
$\left(h_{1}+h_{2} \quad k_{1}+k_{2} \quad l_{1}+l_{2}\right)$ also in same zone.

## Zones

Example: zone axis for (111) \& (100) - [011]


zone axis [uvw] is $u i=v j+w 反$
$\left|\begin{array}{lll}i & j & k \\ h_{1} & k_{1} & l_{1} \\ h_{2} & k_{2} & l_{2}\end{array}\right|$
(011) in same zone? hu $+k v+l w=0$

$$
0 \cdot 0+1 \cdot 1-1 \cdot 1=0
$$

if $\left(h_{1} k_{1} I_{1}\right)$ and $\left(h_{2} k_{2} l_{2}\right)$ in same zone, then
$\left(h_{1}+h_{2} \quad k_{1}+k_{2} \quad l_{1}+l_{2}\right)$ also in same zone.

## Reciprocal lattice

## Real space lattice



## Reciprocal lattice

Real space lattice - basis vectors


## Reciprocal lattice

## Real space lattice - choose set of planes



## Reciprocal lattice

## Real space lattice - interplanar spacing d



## Reciprocal lattice

Real space lattice $\longrightarrow$ the (100) reciprocal lattice pt


## Reciprocal lattice

The (010) recip lattice pt


## Reciprocal lattice

The (020) reciprocal lattice point


## Reciprocal lattice

More reciprocal lattice points


## Reciprocal lattice

The (110) reciprocal lattice point


## Reciprocal lattice

Still more reciprocal lattice points


## Reciprocal lattice

## Reciprocal lattice notation



## Reciprocal lattice

## Reciprocal lattice for hexagonal real space lattice



## Reciprocal lattice

Reciprocal lattice for hexagonal real space lattice


## Reciprocal lattice

Reciprocal lattice for hexagonal real space lattice


## Reciprocal lattice

Reciprocal lattice for hexagonal real space lattice


## Reciprocal lattice

In general:

$$
\begin{aligned}
& a^{*}=\frac{b \times c}{a \cdot b \times c} \\
& b^{*}=\frac{c \times a}{a \cdot b \times c} \\
& c^{*}=\frac{a \times b}{a \cdot b \times c}
\end{aligned}
$$

