## $X$-ray diffraction



## $X$-ray diffraction

$$
\begin{array}{l|l}
\text { Braggs' law } & \lambda=2 d_{h k \mid} \sin \theta_{h k l}
\end{array}
$$



From this set of planes, only get reflection at one angle - $\theta$

## $X$-ray diffraction

$$
\begin{array}{l|l}
\text { Braggs' law } & \lambda=2 d_{h k l} \sin \theta_{h k l}
\end{array}
$$

another
set of
X-rays
lattice
planes

## $X$-ray diffraction

$$
\begin{array}{l|l}
\text { Braggs' law } & \lambda=2 d_{h k l} \sin \theta_{h k l}
\end{array}
$$



## Ewald construction

Think of set of planes reflecting in $x$-ray beam


## Ewald construction

Think of set of planes reflecting in $x$-ray beam
Center sphere on specimen origin $x$-ray beam is a sphere diameter

sphere radius $=1 / \lambda$

## Ewald construction

Think of set of planes reflecting in $x$-ray beam
Center sphere on specimen origin

$$
x \text {-ray beam is a sphere diameter }
$$

Construct lines as below


## Ewald construction

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Center sphere on specimen origin

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x \text {-ray beam is a sphere diameter }
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Construct lines as below


## Ewald construction


sphere radius $=1 / \lambda$
$\sin \theta=\frac{\mathrm{A} 0}{2 / \lambda}$

## Ewald construction


sphere radius $=1 / \lambda$
$\sin \theta=\frac{\mathrm{AO}}{2 / \lambda} \quad 2 \sin \theta=\mathrm{AO} \cdot \lambda$

## Ewald construction



## Ewald construction



$$
\text { sphere radius }=1 / \lambda
$$

$2 \sin \theta=A O \cdot \lambda \quad$ Braggs' law if $A 0=1 / d$ AO is normal to planes
If $O$ is origin then $A$ is a reciprocal lattice point

## Ewald construction



Criterion: if the origin of the reciprocal lattice is placed at 0 , then, for any reciprocal lattice point on the Ewald sphere, there be reflection along the direction from the center of the sphere to the point on the sphere.

Any point in the reciprocal lattice which does not lie on the sphere corresponds to sets of planes which are not in a position to reflect.

## Ewald construction



In general, reciprocal lattice points do not lie on the sphere.

## Ewald construction



In general, reciprocal lattice points do not lie on the sphere.

To observe the reflections, then, we must:

1. move the sphere
2. move the crystal (rotate)
3. change the size of the sphere

## Ewald construction

Most common in single crystal studies is to move (usually rotate) crystal

Consider crystal placed at sphere center oriented $w$ / planes of points in reciprocal lattice as below


## Ewald construction

Looking down on one plane of points.... the equatorial plane:


## Ewald construction

Looking down on one plane of points.... the equatorial plane

No points on sphere (here, in 2-D, a circle): must rotate reciprocal lattice to observe reflections.


## Ewald construction

Looking down on one plane of points...
the equatorial plane
Must rotate reciprocal lattice to observe reflections.


## Ewald construction

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Must rotate reciprocal lattice to observe reflections.


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Looking down on one plane of points...
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## Ewald construction

hkO reflected rays all lie in the equatorial plane.


## Ewald construction

hkO reflected rays all lie in the equatorial plane.
hk1 reflected rays lie on a cone.


## Ewald construction

hkO reflected rays all lie in the equatorial plane.
hk1 reflected rays lie on a cone.


## Ewald construction

Sheet of film or image paper wrapped cylindrically around crystal....


## Ewald construction

Sheet of film or image paper wrapped cylindrically around crystal.... looks like this after x-ray exposure of oscillating crystal .....when flattened:


## Ewald construction

To see reflections:

> | move sphere |
| :--- |
| move crystal |
| change sphere size |
| use polycrystalline sample |

real space
reciprocal space

only one set of planes one (hkl)


## Ewald construction



Ewald sphere
reciprocal lattice


## Ewald construction



